Dynamic Stochastic Model for a Single Airport Ground Holding Problem

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Literature

- Static Integer Programming
  - Many papers on deterministic problem

- Stochastic problem studied in Richetta and Odoni (1993); Hoffman (1997); Ball et al. (2003)

- Equity issues addressed by Vossen et al. (2002)
**Literature**

- Dynamic Models: Richetta and Odoni (1994)
  - Inability to revise previously assigned ground delays

- Objective function is to minimize expected delay cost

- No longer Linear Programming Problem if non-linear measure of delay introduced

- Can handle specific type of scenario tree
Research Contributions

- Dynamic Stochastic Model for SAGHP
  - Ability to revise ground delays of some flights (non-departed)
  - Can handle any generalized scenario tree

- Alternative Objective Functions
  - Expected Squared Deviation from RBS Allocation

- Multi-Criteria Optimization
Capacity Scenario Tree

- Scenario 1 (p=0.3)
- Scenario 2 (p=0.2)
- Scenario 3 (p=0.4)
- Scenario 4 (p=0.1)

P(Scenario 2)=0.2 → 0.4 → 1

T:
1 2 3 ... τ₁ τ₂ τ₃ ...
Decision Making Process
\[ X_{f,t}^q = \begin{cases} 1 & \text{if flight f is planned to arrive by time period t under scenario q;} \\ 0 & \text{otherwise} \end{cases} \]
Model Formulation

- **Objective Function**
  - Min. Expected Total Cost of Delay (sum of ground and airborne delays)

- **Major Constraints**
  - Number of arrivals during any time interval (period) less than airport capacity

  - Coupling Constraints: decisions cannot be based on a particular scenario until it is completely realized
Model Parameters and Input Data

\{1..T + 1\} : set of time periods of uniform duration, \( T \) being the planning horizon

\( \Phi = \{1..F\} : \text{Set of Flights} \)

\( \text{Dep}_f \in \{1..T\} : \text{scheduled departure time period of flight } f \)

\( \text{Arr}_f \in \{1..T\} : \text{scheduled arrival time period of flight } f \)

\( \lambda : \text{Cost ratio between airborne and ground delay} \)
Θ : set of capacity scenarios

\( P_q \) : Probability of occurrence of scenario \( q \in \Theta \)

\( M_t^q \) : Airport arrival capacity at time period \( t \) under capacity scenario \( q \)

\( M_{T+1}^q \) is set to a high value for all \( q \in \Theta \)
\[ B = \text{total number of branches of the scenario tree}; \quad B \geq |\Theta| \]

\[ N_i = \text{number of scenarios represented by } i^{th} \text{ branch}; \quad i \in \{1..B\} \]

The scenarios represented by branch \( i \) is given by set

\[ \Omega_i = \{ S_1^i, \ldots, S_k^i, \ldots, S_{N_i}^i \}, \quad S_k^i \in \Theta \]

The time periods corresponding to start and end nodes of a branch are given by \( o_i \) and \( \mu_i; \quad i \in \{1..B\} \)
\[ B = 7; \Theta = \{\chi_1, \chi_2, \chi_3, \chi_4\}; |\Theta| = 4 \]

\[ i = 2; N_2 = 2; \Omega_2 = \{\chi_1, \chi_2\}; o_2 = \tau_1; \mu_2 = (\tau_2 - 1) \]

\[ i = 1; N_1 = 4; \Omega_1 = \{\chi_1, \chi_2, \chi_3, \chi_4\}; o_1 = 1; \mu_1 = (\tau_1 - 1) \]

\[ i = 5; N_5 = 1; \Omega_5 = \{\chi_2\}; o_5 = \tau_2; \mu_5 = T \]
**Decision Variables**

\[ X_{f,t}^q = \begin{cases} 
1 & \text{if flight } f \text{ is planned to arrive by the end of} \\
& \text{time period } t \text{ under scenario } q; \\
0 & \text{otherwise} 
\end{cases} \]

\[ q \in \Theta, f \in \Phi, \quad t \in \{Arr_f..T + 1\} \]

\[ Y_{f,t}^q = \begin{cases} 
1 & \text{if flight } f \text{ is released for departure by the end of} \\
& \text{time period } t \text{ under scenario } q; \\
0 & \text{otherwise} 
\end{cases} \]

\[ q \in \Theta, f \in \Phi, \quad t \in \{Dep_f..T + 1\} \]

\[ W_q = \text{number of aircraft subject to airborne queuing delay} \]

\[ \text{at time } t \text{ for one or more time periods, under scenario } q \]
**Objective Function**

\[
\text{Min } \sum_{q \in \{1..Q\}} P_q \times \left\{ \sum_{f \in \{1..F\}} \sum_{t=\text{Arr}_f}^{T+1} (t - \text{Arr}_f) \times (X_{f,t}^q - X_{f,t-1}^q) \right\} + \lambda \times \sum_{t=1}^{T} W_t^q \]

**Constraints**

**Decision Variables Non Decreasing**

\[
X_{f,t}^q - X_{f,t-1}^q \geq 0; \quad \forall f \in \Phi, q \in \Theta, t \in \{\text{Arr}_f .. T + 1\}
\]

**Planned Departure Time of Flights**

\[
Y_{f,t}^q = \begin{cases} 
X_{f,t}^q + \text{Arr}_f - \text{Dep}_f ; & \text{if } t + \text{Arr}_f - \text{Dep}_f \leq T \\
1 & \text{otherwise}
\end{cases}
\]
Arrival Capacity

\[ W_{q_{t-1}}^q - W_t^q + \sum_{f \in \Phi} \left( X_{q, f, t}^q - X_{q, f, t-1}^q \right) \leq M_q^q ; \quad t \in \{1..T+1\}, q \in \Theta \]

Feasibility Conditions

\[ W_0^q = W_{T+1}^q = 0 \]

\[ X_{q, f, T+1}^q = 1 \quad \forall f \in \Phi, q \in \Theta \]

Coupling Constraints for Ground Holding Decision Variables

\[ Y_{f, t}^q = Y_{f, t}^{q_1} = \ldots = Y_{f, t}^{q_{N_i}} ; \quad f \in \Phi, t \in \{1..T\}; S_k^i \in \Omega_i : N_i \geq 2 \text{ and } \sigma_i \leq t \leq \mu_i \]
**Static vs Dynamic Formulation**

- Static Stochastic Model (Ball et al 2003, Richetta-Odoni 1993) is a special case of dynamic model.

- The decisions are taken once at the beginning of day and not revised later.

\[ X^q_{f,t} \text{ is same for all } q \in \Theta; \text{ i.e., superscript } q \text{ in the decision variables can be dropped.} \]

Therefore, \( X^q_{f,t} \) can be denoted as \( X_{f,t}; \forall q \in \Theta \)

Similarly, \( Y^q_{f,t} = Y_{f,t}; \forall q \in \Theta \)
Example

- 4 Scenarios, 4 Decision Stages
- 13 Time Periods
- 13 Flights
- Cost Ratio $\lambda = 5$

Probability Mass Function: $P\{\xi_1\} = 0.5; P\{\xi_2\} = 0.3; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1$
Scenario Tree

\( \xi_1 \)
\( \xi_2 \)
\( \xi_3 \)
\( \xi_4 \)
<table>
<thead>
<tr>
<th></th>
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Expected Cost of Delay

- Richetta-Odoni
- Mukherjee-Hansen

- Airborne Delay
- Ground Delay
Case 1: Baseline

Probability Mass Function

\[ P\{\xi_1\} = 0.4; P\{\xi_2\} = 0.2; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.1; P\{\xi_6\} = 0.1 \]

Cost Ratio \( \lambda = 3 \)
Scenario Tree for Baseline Case
Other Cases

Case 2: Change in Cost Ratio. $\lambda = 25$

Case 3: Change in PMF

$P\{\xi_1\} = 0.1; P\{\xi_2\} = 0.1; P\{\xi_3\} = 0.1; P\{\xi_4\} = 0.1; P\{\xi_5\} = 0.2; P\{\xi_6\} = 0.4$

Case 4: Early Branching. Scenarios are realized 30 minutes earlier
Results: Baseline Case

- Mukherjee-Hansen Model
  - Ground delays more severe
  - Less airborne delays
  - Total expected cost least

- Delay reduction compared to Static Model
  - 10% in Mukherjee-Hansen Model
  - 2% in Richetta-Odoni
## Planned Arrival Rates in Baseline Case

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Mukherjee-Hansen Model</th>
<th>Richetta-Odoni Model</th>
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<tr>
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<td>25</td>
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<tr>
<td>9:15 AM-9:30 AM</td>
<td>16</td>
<td>5</td>
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<td>9:30AM-9:45AM</td>
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<td>26</td>
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<td>9:45AM-10:00AM</td>
<td>20</td>
<td>25</td>
</tr>
<tr>
<td>10:00AM-10:15AM</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>
Cases 2 and 3

- No airborne delays
- Static model plans for worst scenario
- Dynamic models adaptive to changing conditions
- Delays are higher in case 3 due to high probability of worse conditions
Case 4: 30 Minutes Early Information

- Static model produces same delays as in baseline case
- Value of early information
- Mukherjee-Hansen Model
  - Least delays
  - Absorbs most of the delay by ground holding
Perfect Information Case
Alternative Objective Functions

Minimizing Expected Squared Deviation from RBS Allocation

\[
\text{Min } \sum_{q \in \{1..Q\}} P\{q\} \times \left\{ \sum_{t = \text{Arr}_f}^{T + 1} \left( t - \text{RBS}_f \right)^2 \times (X_{f,t} - X_{f,t-1}) \right\} + \lambda \times \sum_{t = 1}^{T} \sum_{q} W_q
\]
Multi-Criteria Optimization

\[ \text{Min} \quad \sum_{q \in \{1..Q\}} P\{q\} \times \left[ \sum_{f \in \Phi} \left( \sum_{t = \text{Arr}_f}^{T+1} (t - \text{Arr}_f) \times (X_{f,t}^q - X_{f,t-1}^q) \right) + \sum_{f \in \Phi} \left( \sum_{t = \text{Arr}_f}^{T+1} \left( t - \text{RBS}_f^q \right)^2 \times (X_{f,t}^q - X_{f,t-1}^q) \right) \right] + \lambda \times \sum_{t=1}^T W_t^q \]
Work in Progress

- Reformulating the model as a minimum cost network flow problem.

- Ability to handle time varying unconditional probabilities of the capacity scenarios
Acknowledgements

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