Slot auction model
Auctioneer optimization model
Airline optimization model
Illustration : a case study
Summary and future work
Problem Identification

Asynchronous non-uniform scheduling

ATL – Atlanta Airport
(FAA Airport Capacity Benchmark Report 2001)

TOTAL SCHEDULED OPERATIONS AND CURRENT OPTIMUM RATE BOUNDARIES

Underutilization

Schedule Facility Est.
Small aircraft makes inefficient use of slots

LAX arrivals, 1998 (Mark Hansen, Berkeley)

Aircraft in small category

25% flight share for 5% seat share
Auction Model Design issues

- **Feasibility**
  - package slot allocation for departure and arrival slots
  - incremental (airports and slots to auction off)

- **Optimality**
  - efficiency: throughput (enplanement opportunity) and delay
  - regulatory standards: safety, flight priorities
  - equity:
    - stability in schedule
    - airlines’ need to leverage investments
    - airlines’ competitiveness: new-entrants vs. incumbents

- **Flexibility**
  - primary market at strategic level
  - secondary market at tactical level
Objective:
- provide an optimum fleet mix at optimum safe arrival capacity
- ensure fair market access opportunity
- reduce queuing delay

Assumptions
- airlines could make use of slots they bid

Auction process:
- an interactive and iterative process to enable flexibility and optimization
- a mixture of simultaneous auction model and package model

Auction rules: Bidders are ranked using a linear combination of:
- flight OD pair
- #seats
- airline’s prior investment
- historical slot occupancy rate
- bid
Simultaneous multiple-round auction
- have discrete, successive rounds, with length of each round announced in advance. After each round closes, round results are processed and made public
  → Account for departure/arrival slots interdependence but subject to aggregation risks

Package bidding
- bidders submit bids for multiple combinations of lots rather than just individual lots. Package biddings are either accepted or rejected in their entirety
  → Eliminate aggregation risks
Auction Model Process

1. Determine factor weights
2. Determine time-windows to auction off
3. Another time window? No → End auction process, Yes → Call for bids for slots in the time window
4. Call for bids for slots in the time window → Submit information and bid
5. Winners determined? No → Incremental auction prioritizing most congested periods, Yes → Yes
Auctioneer Optimization Model

Airlines are ranked by a linear combination of:

- #seats
- flight OD pair
- airline’s prior investment
- historical slot occupancy rate
- monetary offer

**Ranking function:**

\[
\tau(B_{a,s}) = W^T \cdot B_{a,s}
\]

- \( S^T \): slot vector, \(|S^T|=\text{AAR} \)
- \( A \): airline vector
- \( W^T \): vector of factor weights
- \( B_{a,s} \): bid of airline \( a \) for slot \( s \)
Auctioneer Optimization Model

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Bidding matrix \( X = A \cdot S^T \)

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<tr>
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<td>5000</td>
<td>7500</td>
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Auctioneer Optimization Model

→ Objective function: Allocate slots to the highest ranked airlines

\[
\text{Max} \quad \sum_{a} \sum_{s} \tau (B_{a,s}) \cdot (X)_{a,s}
\]

Subject to:

\[
\begin{cases}
\sum_{a} (X)_{a,s} = 1 \quad \forall s \\
(M^{T})_{s} \cdot (X)_{a,s} \leq (B_{a,s})_{s} \quad \forall a, s \\
\sum_{s} (X)_{a,s} \leq 1
\end{cases}
\]

Variables:

- \( S^{T} \): slot vector, \(|S^{T}|=\text{AAR}\)
- \( A \): airline vector
- \( W^{T} \): vector of factor weights
- \( B_{a,s} \): bid of airline \( a \) for slot \( s \)
- \( X=A*S^{T} \): bidding matrix
- \( (X)_{a,s} = \begin{cases} 1 & \text{if airline } a \text{ is ranked highest for slot } s \text{ after a round} \\ 0 & \text{otherwise} \end{cases} \)
Airline Optimization Model

Objective function: Maximize revenue and ultimately maximize profit

Maximize \( \sum_s (P_s - B_s) \)

Subject to:

\[
B_s \leq M \cdot y_s \\
(B_0^T)_s \leq B_s + M \cdot (1 - y_s) \\
B_s \leq \alpha \cdot P_s \\
\left\{ \begin{array}{l}
\max \left( \frac{\tau(B_{a,s}) - \tau(B_{A,s})}{(W)_s} \right) \\
B_s' + \frac{a}{(W)_s} \cdot (B_0^T)_s \end{array} \right\} \cdot y_s \leq \alpha \cdot P_s \cdot y_s \\
\left\{ \begin{array}{l}
\max \left( \frac{\tau(B_{a,s}) - \tau(B_{A,s})}{(W)_s} \right) \\
B_s' + \frac{a}{(W)_s} \cdot (B_0^T)_s \end{array} \right\} \leq B_s + M \cdot (1 - y_s)
\]

Airlines’ package bidding constraints

Variables:

| \( \{B_s\} \) | set of monetary bids |
| \( \{P_s\} \) | airline expected profit by using a slot |
| \( M \) | big positive value |
| \( y_s \) | binary value |
| \( y_s \) | \( \begin{cases} 
1 & \text{if airline bids for slot } s \\
0 & \text{otherwise} 
\end{cases} \) |
| \( B_0^T \) | airport threshold vector |
| \( \alpha \) | airline threshold fraction |
| \( B_s' \) | old bid for slot s in previous round |
Illustration: an example

Winner-determining factors

<table>
<thead>
<tr>
<th>Factors</th>
<th>Weight</th>
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<tbody>
<tr>
<td>Number of seats</td>
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<tr>
<td>Previous Airline infrastructure investment</td>
<td>0.25</td>
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<tr>
<td>Historic slot occupancy frequency</td>
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<tr>
<td>OD-Pair</td>
<td>0.13</td>
</tr>
<tr>
<td>Bid</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Ranking function:

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Bidding matrix, initial round

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<td>6600</td>
<td>5200</td>
</tr>
<tr>
<td>NW (0.25)</td>
<td>S (30) 0 (0.4)</td>
<td></td>
<td></td>
<td>H (205) 1 (0.25)</td>
<td></td>
<td></td>
<td>L (128) 0 (0.35)</td>
<td></td>
</tr>
<tr>
<td>AA (0.19)</td>
<td>L (147) 0 (0.5)</td>
<td></td>
<td></td>
<td>H (283) 1 (0.4)</td>
<td></td>
<td></td>
<td>L (291) 1 (0.35)</td>
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<td>UA (0.15)</td>
<td>S (18) 0 (0.25)</td>
<td></td>
<td></td>
<td>H (392) 1 (0.3)</td>
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<td></td>
<td>S (18) 0 (0.2)</td>
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<td></td>
<td></td>
<td>S (30) 0 (0.15)</td>
<td></td>
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<td>L (150) 0 (0.35)</td>
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<tr>
<td>SW (0)</td>
<td>S (18) 0 (0.15)</td>
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<td>S (30) 0 (0.06)</td>
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<td>S (18) 0 (0.1)</td>
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<tr>
<td>US (0.21)</td>
<td>S (30) 0 (0.35)</td>
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<td>L (120) 0 (0.5)</td>
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<td>S (30) 0 (0.45)</td>
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<tr>
<td>DAL (0.11)</td>
<td>S (18) 0 (0.2)</td>
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<td>L (85) 0 (0.2)</td>
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<td>H (202) 1 (0.3)</td>
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%investment

Aircraft type (#seats) OD pair (Slot occupancy rate)
### Illustration: an example

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<tr>
<td>AA (0.19)</td>
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<td>CA (0.13)</td>
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#### Air carrier monetary bids

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**Round 1**

Withdraw from auction for these slots.
### Illustration: an example

#### Bid score matrix, initial round

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**Withdraw from auction for these slots**
## Illustration: an example

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<td>1.18</td>
<td>0.96</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
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<td>2.2</td>
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</tr>
<tr>
<td>SW (0)</td>
<td>0.62</td>
<td>0.83</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
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<tr>
<td>US (0.21)</td>
<td>1.34</td>
<td>2.96</td>
<td>1.68</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
<td>1.11</td>
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</tr>
<tr>
<td>DAL (0.11)</td>
<td>0.87</td>
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<td></td>
<td></td>
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</tr>
</tbody>
</table>

### Air carrier monetary bids

<table>
<thead>
<tr>
<th>Slots</th>
<th>8:00:00</th>
<th>8:02:30</th>
<th>8:05:00</th>
<th>8:07:30</th>
<th>8:10:00</th>
<th>8:12:30</th>
<th>8:15:00</th>
<th>8:17:30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min Monetery Bid</td>
<td>5000</td>
<td>7500</td>
<td>8600</td>
<td>10000</td>
<td>12000</td>
<td>8300</td>
<td>6600</td>
<td>5200</td>
</tr>
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<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
<td>3.18</td>
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<tr>
<td>AA (0.19)</td>
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<td>3.98</td>
<td>3.98</td>
<td>3.98</td>
<td>3.98</td>
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<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
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<tr>
<td>CA (0.13)</td>
<td>1.18</td>
<td>0.96</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
<td>2.2</td>
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<td>2.2</td>
</tr>
<tr>
<td>SW (0)</td>
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<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
<td>0.62</td>
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<tr>
<td>US (0.21)</td>
<td>1.34</td>
<td>2.96</td>
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<td>1.11</td>
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<tr>
<td>DAL (0.11)</td>
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</table>

### Withdraw from auction for these slots

## Round 1

Break ties by adding 10% of airline profit to bids.
## Air carrier monetary bids

<table>
<thead>
<tr>
<th>Slots</th>
<th>8:00:00</th>
<th>8:02:30</th>
<th>8:05:00</th>
<th>8:07:30</th>
<th>8:10:00</th>
<th>8:12:30</th>
<th>8:15:00</th>
<th>8:17:30</th>
</tr>
</thead>
<tbody>
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<td>Min Monetery Bid</td>
<td>5000</td>
<td>7500</td>
<td>8600</td>
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<td>SW (0)</td>
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## Bid score matrix

<table>
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<tr>
<th>Slots</th>
<th>8:00:00</th>
<th>8:02:30</th>
<th>8:05:00</th>
<th>8:07:30</th>
<th>8:10:00</th>
<th>8:12:30</th>
<th>8:15:00</th>
<th>8:17:30</th>
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<tbody>
<tr>
<td>Min Monetery Bid</td>
<td>5000</td>
<td>7500</td>
<td>8600</td>
<td>10000</td>
<td>12000</td>
<td>8300</td>
<td>6600</td>
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<td>AA (0.19)</td>
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<td>UA (0.15)</td>
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<td>SW (0)</td>
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<td></td>
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<td>US (0.21)</td>
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</table>

## Auction winners

<table>
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<tr>
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<th>8:05:00</th>
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<th>8:12:30</th>
<th>8:15:00</th>
<th>8:17:30</th>
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<tbody>
<tr>
<td>Minimum Bid</td>
<td>5000</td>
<td>7500</td>
<td>8600</td>
<td>10000</td>
<td>12000</td>
<td>8300</td>
<td>6600</td>
<td>5200</td>
</tr>
<tr>
<td>NW (0.25)</td>
<td>S (30) 0 (0.4)</td>
<td>H (205) 0.25</td>
<td>L (128) 0.35</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>AA (0.19)</td>
<td>L (147) 0.5</td>
<td>H (283) 1.4</td>
<td>L (291) 1.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>UA (0.15)</td>
<td>S (18) 0.25</td>
<td>H (392) 1.3</td>
<td>S (18) 0.2</td>
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<tr>
<td>CA (0.13)</td>
<td>S (30) 0.35</td>
<td>S (30) 0.15</td>
<td>L (150) 0.35</td>
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</tr>
<tr>
<td>SW (0)</td>
<td>S (18) 0.15</td>
<td>S (30) 0.06</td>
<td>S (18) 0.1</td>
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<tr>
<td>US (0.21)</td>
<td>S (30) 0.35</td>
<td>L (120) 0.5</td>
<td>L (85) 0.2</td>
<td>S (30) 0.45</td>
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<tr>
<td>DAL (0.11)</td>
<td>S (18) 0.2</td>
<td>H (202) 1.3</td>
<td>S (30) 0.4</td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Summary and future work

- Generic market-based approach to airport demand management allows to evaluate many alternatives by varying the weighting factors,
- Legal and procedural challenges would require rigorous cost-benefit evaluation.

Future work
- Modeling airlines’ behavior,
- Cross-airport package bidding,
- Conduct sensitivity analysis for different auction formats (with different values of factor weights)
back-up slides
Lack of competition

- Hirschman-Herfindahl Index (HHI) is a standard measure of market concentration.
- The Department of Justice uses it to measure the competition within a market place.
- HHI = (100 * s_i)^2 where s_i is the market share of airline i.
- Ranging between 100 (perfect competitiveness) and 10000 (perfect monopoly).
- In a market place with an index over 1800, the market begins to demonstrate a lack of competition.
Potential solutions

- Congestion pricing
  - monitoring and updating constraints
- Promote use of larger aircraft
  - airline economic constraints
- Improve local infrastructure
  - environmental constraints
- Rerouting flights
  - market constraints
Design scope

- Scope:
  - Strategic auction for arrival slots at individual airports

- Objective:
  - Provide an optimum fleet mix at optimum safe capacity
  - Ensure fair market access opportunity
  - Reduce queuing delay

- Definitions:
  - Slot: The concession or the entitlement to use runway capacity of a certain airport by an air carrier on a specific date and at a specific time [CITE: EEC COM(2001)335]
  - Grandfather right: Air carriers having historical use rate of a slot of at least 80% will have precedence.
Auctioneer Optimization Model

Variables:

- $S^T$: slot vector, $|S^T|=AAR$
- $A$: airline vector
- $W^T$: vector of factor weights
- $B_{a,s}$: bid of airline $a$ for slot $s$
- $X = A*S^T$: bidding matrix

$(X)_{a,s} = \begin{cases} 1 & \text{if airline } a \text{ is ranked highest for slot } s \\ 0 & \text{otherwise} \end{cases}$

- $M^T$: initial monetary offer vector

Ranking function: $\tau(B_{a,s}) = W^T * B_{a,s}$

Objective function: $\text{Max } \sum_a \sum_s \tau(B_{a,s}) * (X)_{a,s}$

Subject to: $\sum_a (X)_{a,s} = 1 \quad \forall s$

$(M^T)_s * (X)_{a,s} <= (B_{a,s})_s \quad \forall a, s$

Safe separation b/w leading and trailing aircraft:

Mean of the calculated inter-arrival times by aircraft weight categories

<table>
<thead>
<tr>
<th>Leading</th>
<th>Small</th>
<th>Large</th>
<th>Heavy</th>
<th>B757</th>
</tr>
</thead>
<tbody>
<tr>
<td>Small</td>
<td>1.33</td>
<td>1.13</td>
<td>1.07</td>
<td>1.1</td>
</tr>
<tr>
<td>Large</td>
<td>2.74</td>
<td>1.21</td>
<td>1.07</td>
<td>1.1</td>
</tr>
<tr>
<td>Heavy</td>
<td>3.91</td>
<td>2.467</td>
<td>1.73</td>
<td>2.26</td>
</tr>
<tr>
<td>B757</td>
<td>3.35</td>
<td>1.92</td>
<td>1.69</td>
<td>1.7</td>
</tr>
</tbody>
</table>

(Mark Hansen)
Auctioneer Optimization Model

Variables:

\( S^T \quad \text{slot vector, } |S^T|=\text{AAR} \)
\( A \quad \text{airline vector} \)
\( W^T \quad \text{vector of factor weights} \)
\( B_{a,s} \quad \text{bid of airline } a \text{ for slot } s \)
\( X=A*S^T \quad \text{bidding matrix} \)

\( (X)_{a,s} = \begin{cases} 1 & \text{if airline } a \text{ is ranked highest for slot } s \\ 0 & \text{otherwise} \end{cases} \)

\( M^T \quad \text{initial monetary offer vector} \)

Ranking function:
\( \tau(B_{a,s}) = W^T * B_{a,s} \)

Objective function:
\[ \max \sum_a \sum_s \tau(B_{a,s}) * (X)_{a,s} \]

Subject to:
\[ \sum_a (X)_{a,s} = 1 \quad \forall s \]
\[ (M^T)_s * (X)_{a,s} \leq (B_{a,s})_s \quad \forall a, s \]

Safe separation b/w leading and trailing aircraft:
\[ \sum_s (X)_{a,s} * f(AT(a, s), AT(a', s+1)) \leq 60 \]

Mean of the calculated inter-arrival times by aircraft weight categories

<table>
<thead>
<tr>
<th>Leading</th>
<th>Small</th>
<th>Large</th>
<th>Heavy</th>
<th>B757</th>
</tr>
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<tr>
<td>Small</td>
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<td>1.7</td>
</tr>
</tbody>
</table>

(Mark Hansen)
Auctioneer Optimization Model

Objective function:
\[
\text{Max } \sum_a \sum_s \tau(B_{a,s}) \cdot (X)_{a,s}
\]

Subject to:

Capacity constraint: One slot allocated to one flight
\[
\sum_a (X)_{a,s} = 1 \quad \forall s
\]

Ranking function:
\[
\tau(B_{a,s}) = W^T \cdot B_{a,s}
\]

Variables:
- \(S^T\) slot vector, \(|S^T|=\text{AAR}\)
- \(A\) airline vector
- \(W^T\) vector of factor weights
- \(B_{a,s}\) bid of airline a for slot s
- \(X=A*S^T\) bidding matrix
- \((X)_{a,s} =\begin{cases} 1 & \text{if airline a is ranked highest for slot s after a round} \\ 0 & \text{otherwise} \end{cases}\)
Objective function:

$$\text{Max } \sum_a \sum_s \tau(B_{a,s}) * (X)_{a,s}$$

Subject to:

$$\sum_a (X)_{a,s} = 1 \quad \forall s$$

Minimum bid: At any round, monetary offers should be equal or greater than initial thresholds

$$(M^T)_s * (X)_{a,s} \leq (B_{a,s})_s \quad \forall a, s$$

Ranking function:

$$\tau(B_{a,s}) = W^T * B_{a,s}$$

Variables:

- $S^T$: slot vector, $|S^T|=AAR$
- $A$: airline vector
- $W^T$: vector of factor weights
- $B_{a,s}$: bid of airline $a$ for slot $s$
- $X=A*S^T$: bidding matrix
- $(X)_{a,s} = \begin{cases} 1 & \text{if airline } a \text{ is ranked highest for slot } s \text{ after a round} \\ 0 & \text{otherwise} \end{cases}$
- $M^T$: initial monetary offer vector
Auctioneer Optimization Model

Objective function:

$$\text{Max } \sum_{a} \sum_{s} \tau(B_{a,s}) \ast (X)_{a,s}$$

Subject to:

$$\sum_{a} (X)_{a,s} = 1 \quad \forall s$$

$$(M^T)_s \ast (X)_{a,s} \leq (B_{a,s})_5 \quad \forall a, s$$

Airline constraints: Only one slot is needed in a set of adjacent slots

$$\sum_{s} (X)_{a,s} \leq 1$$

Ranking function:

$$\tau(B_{a,s}) = W^T \ast B_{a,s}$$

Variables:

- $S^T$: slot vector, $|S^T|=AAR$
- $A$: airline vector
- $W^T$: vector of factor weights
- $B_{a,s}$: bid of airline $a$ for slot $s$
- $X= A^S^T$: bidding matrix
- $(X)_{a,s} = \begin{cases} 1 & \text{if airline a is ranked highest for slot s after a round} \\ 0 & \text{otherwise} \end{cases}$
- $M^T$: initial monetary offer vector
Auctioneer Optimization Model

Objective function:

\[
\text{Max } \sum_{a} \sum_{s} \tau(B_{a,s}) \cdot (X)_{a,s}
\]

Subject to:

\[
\begin{align*}
    \sum_{a} (X)_{a,s} &= 1 \quad \forall s \\
    (M^T)_{s} \cdot (X)_{a,s} &\leq (B_{a,s})_{s} \quad \forall a, s
\end{align*}
\]

Airlines’ package bidding constraints

Ranking function:

\[
\tau(B_{a,s}) = W^T \cdot B_{a,s}
\]

Variables:

- \(S^T\): slot vector, \(|S^T|=\text{AAR}\)
- \(A\): airline vector
- \(W^T\): vector of factor weights
- \(B_{a,s}\): bid of airline \(a\) for slot \(s\)
- \(X=A^S\): bidding matrix
- \((X)_{a,s}=\begin{cases} 1 & \text{if airline } a \text{ is ranked highest for slot } s \text{ after a round} \\
0 & \text{otherwise} \end{cases}\)
- \(M^T\): initial monetary offer vector
Airline Optimization Model

Objective function: Maximize profit

Maximize \( \sum_s (P_s - B_s) \)

Subject to:

- Airline either bids for slot \( s \) (monetary offer greater than minimum threshold) or not.

\[
B_s \leq M \cdot y_s \\
(B_0^T)_s \leq B_s + M \cdot (1 - y_s)
\]

Variables:

- \( \{B_s\} \) set of monetary bids
- \( \{P_s\} \) airline expected profit by using a slot
- \( M \) big positive value
- \( y_s \) binary value
- \( y_s = \begin{cases} 1 & \text{if airline bids for slot } s \\ 0 & \text{otherwise} \end{cases} \)
- \( B_0^T \) airport threshold vector
Objective function: Maximize profit

\[ \text{Maximize } \sum_s (P_s - B_s) \]

Subject to:

\[ B_s \leq M \times y_s \]

\[ (B_0^T)_s \leq B_s + M \times (1 - y_s) \]

When offering bid for slot s, monetary amount should not pass a threshold

\[ B_s \leq \alpha \times P_s \]

Variables:

- \( \{B_s\} \) set of monetary bids
- \( \{P_s\} \) airline expected profit by using a slot
- \( M \) big positive value
- \( y_s \) binary value
- \( y_s = \begin{cases} 1 & \text{if airline bids for slot s} \\ 0 & \text{otherwise} \end{cases} \)
- \( B_0^T \) airport threshold vector
- \( \alpha \) airline threshold fraction
Airline Optimization Model

Objective function: Maximize profit

Maximize \[ \sum_s (P_s - B_s) \]

Subject to:

\[ B_s \leq M * y_s \]

\[ (B^T_0)_s \leq B_s + M * (1 - y_s) \]

\[ B_s \leq \alpha * P_s \]

Given \( B_s \)’, the bid airline offered in the previous round, in order to be ranked highest in this round, airline has to increase at least:

\[ (B^T_0)_s = (W)_5 \]

\[ \max_a (\tau(B_{a,s})) - \tau(B_{A,s}) * (B^T_0)_s = \max_a (\tau(B_{a,s})) - \tau(B_{A,s}) \]

Variables:

- \( \{B_s\} \): set of monetary bids
- \( \{P_s\} \): airline expected profit by using a slot
- \( M \): big positive value
- \( y_s \): binary value
- \( \alpha \): airline threshold fraction
- \( B_s' \): old bid for slot s in previous round
- \( B^T_0 \): airport threshold vector
- \( \tau \): air traffic density
- \( \sigma \): air traffic density vector
- \( \bar{s} \): air traffic density factor
Objective function: Maximize profit

Maximize \( \sum_s (P_s - B_s) \)

Subject to:

\[
B_s \leq M \cdot y_s
\]

\[
(B^T_0)_s \leq B_s + M \cdot (1 - y_s)
\]

\[
B_s \leq \alpha \cdot P_s
\]

Given \( B_s' \), the bid airline offered in the previous round, in order to be ranked highest in this round, airline has to increase at least:

\[
\max \left( \tau(B_{a,i}) - \tau(B_{A,i}) \right) + \frac{\max(B_i + \frac{\alpha}{(W)_s} \cdot (B^T_0)_i)}{\min(y_s \cdot B_s)} \geq \max(\alpha \cdot P_s)
\]

Variables:

- \( \{B_s\} \) set of monetary bids
- \( \{P_s\} \) airline expected profit by using a slot
- \( M \) big positive value
- \( y_s \) binary value
- \( \alpha \) airline threshold fraction
- \( B_s' \) old bid for slot s in previous round

Airline Optimization Model
Objective function: Maximize profit

Maximize \( \sum_s (P_s - B_s) \)

Subject to:

\[ B_s \leq M \times y_s \]

\[ (B_0^T)_s \leq B_s + M \times (1 - y_s) \]

\[ B_s \leq \alpha \times P_s \]

\[ \max \left( \frac{\tau(B_{a,i}) - \tau(B_{A,i})}{(W)_5} \right) \times (B_0^T)_i \times y_i \leq \alpha \times P_i \times y_i \]

\[ B_i^* + \frac{\alpha}{(W)_5} \times (B_0^T)_i \leq B_i + M \times (1 - y_i) \]

Variables:

\( \{B_s\} \) set of monetary bids
\( \{P_s\} \) airline expected profit by using a slot
\( M \) big positive value
\( y_s \) binary value
\( y_s = \begin{cases} 1 & \text{if airline bids for slot } s \\ 0 & \text{otherwise} \end{cases} \)
\( B_0^T \) airport threshold vector
\( \Sigma \) airline threshold fraction
\( B_s' \) old bid for slot s in previous round
Airline Optimization Model

Objective function: Maximize profit

Maximize \( \sum_s (P_s - B_s) \)

Subject to:

\[
\begin{align*}
B_s & \leq M * y_s \\
(B_0^T)_s & \leq B_s + M * (1 - y_s) \\
B_s & \leq \alpha * P_s \\
\left( \max \left( \frac{\nu(B_a,i) - \nu(B_A,i)}{a \cdot (W)_5} \right) \right) & \leq B_i + M * (1 - y_i)
\end{align*}
\]

Airlines’ package bidding constraints

Variables:

\[
\begin{align*}
\{B_s\} & \quad \text{set of monetary bids} \\
\{P_s\} & \quad \text{airline expected profit by using a slot} \\
M & \quad \text{big positive value} \\
y_s & \quad \text{binary value} \\
y_s & = \begin{cases} 
1 & \text{if airline bids for slot } s \\
0 & \text{otherwise}
\end{cases} \\
B_0^T & \quad \text{airport threshold vector} \\
\sum_i & \quad \text{airline threshold fraction} \\
B_s' & \quad \text{old bid for slot } s \text{ in previous round}
\end{align*}
\]