Global Optimal Routing, Scheduling and Power Control for Multi-hop Wireless Networks with Interference

Javad Kazemitabar
Dept. of Elec. Eng. & Computer Science
University of California, Irvine
Email: skazemit@uci.edu

Vahid Tabatabaee
Institute for Systems Research
University of Maryland at College Park
Email: vahidt@umd.edu

Hamid Jafarkhani
Dept. of Elec. Eng. & Computer Science
University of California, Irvine
Email: hamidj@uci.edu

***Abstract*—We consider the problem of joint routing, scheduling and power control in multi-hop wireless networks. We use a linear relation between link capacity and signal to interference noise ratio in our formulation. In a previous work, using a duality approach, the optimal link scheduling and power control that minimizes the total average transmission power is found. We formulate this problem as a linear programming problem with exponential number of constraints. To cope with the exponential number of constraints, we propose an iterative algorithm based on the cutting plane method. The separation Oracle for the cutting plane algorithm turns out to be an element-wise concave optimization problem that can be effectively solved using branch and bound algorithm.**

We extend the same method to find the optimal *routing* scheduling and power control. Simulation results show that this methodology is more efficient and scalable compare to the previously proposed algorithm.

I. Introduction

Wireless mesh networks rely on multi-hop transmission to provide connectivity between end users. In such systems it is highly desirable to minimize the total power consumption to increase energy efficiency of the system and to minimize interference with other telecommunication systems working in the same environment. Three separate set of control variables that determine the energy consumption in a network are: (i) Routing which specifies path(s) that are used to transfer data between every source-destination (S-D) pair in the network and the fraction of traffic that on average is sent through each selected path. (ii) Scheduling which specifies the set of active links that transfer data at each time slot. (iii) Power Control which determines power transmission of active links at each time slot. For a given traffic matrix which specifies traffic demand between every source-destination pair our goal is to find the optimal routing, scheduling and transmission power that minimize the total power consumption in the network.

Cross-layer design for wireless networks are usually based on simplified MAC and PHY layer models, so that it can be formulated as a convex optimization problem. The models that consider interference, however, result in complicated mathematical relations between power and link capacity which are multi-variable and non-convex. Therefore, conventional convex optimization algorithms are not applicable anymore. Furthermore, to cope with the interdependence and coupling of variables in wireless networks, it is well-known that a cross-layer design approach that jointly optimizes routing, scheduling and power control can substantially reduce total power. In this work, we propose a cross-layer global optimal algorithm that minimizes the total power consumption in the network and consider a non-convex inter-dependent model for wireless links. The algorithm that we present here is centralized and we are working on distributed versions of it. However, the centralized algorithm can also be used for network provisioning and can also be used as a benchmark for heuristic distributed algorithms that proposed for cross-layer design.

Optimization approaches for wireless network resource allocation are developed in several papers. Our approach is similar to [1], where the problem of joint routing, scheduling and power control for wireless multi-hop network is considered. They present a centralized algorithm to find optimal scheduling and power control to minimize the total average power consumption in the network, subject to constraints regarding the peak power and minimum data rate over each link. In addition, using the dual variable as the link cost they propose a gradient based algorithm to find the optimal routing iteratively. However, the number in constraints of the scheduling and power allocation problem, which should be solved in every iteration, grows exponentially with the number of links. Hence, this approach is impractical for large size networks. We propose efficient relaxation methods and introduce an algorithm that simultaneously finds the optimal routing, scheduling and power allocation. Even though the worst case complexity of the relaxation methods can be exponential, these algorithms turn out to be very efficient for practical problems. In fact, by using the simulation results, we show that our proposed algorithms for routing, scheduling and power control is even much more efficient than the direct approach proposed in [1], [2] for scheduling and power control (with fixed routing).

The rest of the paper is structured as follows: In Section II, we describe the physical layer model and formulate the optimization problems. In Section III we describe our algorithm. Simulation results in addition to some comments about our algorithm are provided in Section IV. Section V concludes this paper.

II. Modeling and Problem Formulation

Suppose there are N stationary nodes, labeled by the integers $1, 2, \cdots, N$. A set ε of $L_\varepsilon = |\varepsilon|$ transmission links, among the possible $N(N-1)$ links between nodes, make a network topology. These active links are chosen based on the distance, Signal to Interference Noise Ratio (SINR) or some other connectivity measure [3], [4], [5]. For simplicity, we assume in this paper that two nodes constitute a link if the distance between them is less than a threshold. For a given link $l = (i, j)$, the transmitter node i uses a signal power $P(l)$. The "path gain" from node i to node j is given by $G(i, j)$, and models the effects of signal attenuation due to distance, channel fading and shadowing, as well as antenna gain patterns. We assume that the path gains $G(i, j)$ are constant. The transmitting and receiving nodes of link l are denoted by $T(l)$ and $R(l)$ respectively. The received signal power at node $R(l)$ from the transmitter $T(l)$ thus is given by $P(l)G(T(l), R(l))$. However, signals emanating from other transmitters appear the receiver $R(l)$ as interference, and there is thermal noise as well. The signal to interference and noise ratio (SINR) for link l is defined as

$$\gamma(l) = \frac{G(T(l), R(l))P(l)}{\sum_{k \neq l} P(k)G(T(k), R(l)) + n_{R(l)}} \tag{1}$$

where n_j is the noise power at node j. Let us assume that the efficient bandwidth of channel l is $W(l)$. Assuming Gaussian noise plus interference, the maximum mutual information of link l will be $X(l) = W(l).log_2(1 + \gamma(l))$. In a low power regime, the SINR value, $\gamma(l)$, is very small. Therefore, we can use the linear approximation for log function and obtain

$$X(l) = W(l).\gamma(l) \tag{2}$$

A. Scheduling and power control

We start with reviewing the problem formulation given in [1]. For simplicity of exposition, we divide time into slots, each of equal duration and indexed by positive integers. Transmissions begin and end on slot boundaries. Generalizing the notation introduced earlier, let $X_t(l)$ and $P_t(l)$ be the data rate for link l in slot t, and transmission power for the transmitter $T(l)$ for link l in slot t, respectively. Let $P_t = (P_t(1), P_t(2), \cdots, P_t(L_\varepsilon))$ be the network power vector for slot t. Let $P^{max}(i)$ be the maximum transmission power for node i. Also, let $\varepsilon(i)$ be the links in ε that originate at node i. Each node must conform to the peak transmission power constraint in every slot:

$$\begin{array}{ll} 0 \leq & \sum_{l \in \varepsilon(i)} P_t(l) \leq P^{max}(i) \ and \\ 0 \leq & P_t(l), \text{ for all } t \geq \ 1 \text{ and } l \in \varepsilon \end{array} \tag{3}$$

The above constraints form a polytope in $\overrightarrow{P}$ space. Let us denote this polytope by $\mathbb{P}$. Using (1) and (2), the maximum achievable data rate for link l in slot t is

$$X_t(l) = W(l)\left(\frac{G(T(l), R(l))P_t(l)}{\sum_{k \neq l} G(T(k), R(l))P_t(k) + n_{R(l)}}\right) \tag{4}$$

The average rate of link l is then defined as $X_{avg}(l) = \liminf_{\ t \to \infty} \frac{1}{t} \sum_{k=1}^{t} X_k(l)$.
For each link l, let $C(l)$ be a given minimum required average data rate, i.e. we must have

$$X_{avg}(l) \geq C(l), \text{ for all } l \in \ \varepsilon \tag{5}$$

Define the required minimum average rate vector as $\overrightarrow{C} = (C(1), C(2), \cdots, C(L_\varepsilon))$.

The average power consumed by the transmitter of link l is then $P_{avg}(l) = \limsup_{\ t \to \infty} \frac{1}{t} \sum_{k=1}^{t} P_k(l)$
Define the average power vector as $\overrightarrow{P_{avg}} = (P_{avg}(1), P_{avg}(2), \cdots, P_{avg}(L_\varepsilon))$. There may or may not exist a sequence of network power vectors $P_1, P_2, \cdots$ that satisfy (2) and (5). If there does exist a sequence of such network power vectors, our aim is to minimize a linear function of P_{avg}. An example of such a linear function is simply the total average power

$$h(P_{avg}) = \sum_{l \in \varepsilon} P_{avg}(l) \tag{6}$$

The $Scheduling\ and\ Power\ Control\ Problem$ is then defined as

$$\min \ h(P_{avg}) \text{ subject to (2) and (5)} \tag{7}$$

Let us define the value of this cost function as a function of $\overrightarrow{C}$ denoted by $H(\overrightarrow{C})$. We can absorb constraint (5) into the cost function and define the potential function

$$V(\overrightarrow{P}, \beta) = h(\overrightarrow{P}) + \sum_{l \in \varepsilon} \beta(l)[C(l) - X(l)] \tag{8}$$

Using duality methods, we can show that [1]

$$H(\overrightarrow{C}) = \max_{\beta \geq 0}\{ \ \min_{\overrightarrow{P}} \ V(\overrightarrow{P_{avg}}, \beta) \text{ subject to (2)}\} \tag{9}$$

Computation of (9) involves optimizing over all schedules of network power vectors satisfying the peak power constraint in every slot. However, since the potential function V is linear in $\overrightarrow{P}$,$\overrightarrow{C}$, and $\overrightarrow{X}$, it follows that (9) can be computed by an optimization over a single slot [1], [2]. Therefore, we can just focus on solving

$$\begin{array}{l} \max_{\beta \geq 0}\{\min_{\overrightarrow{P}} V(\overrightarrow{P}, \beta)\} \\ \text{s.t.} \\ \overrightarrow{P} \in \mathbb{P} \end{array} \tag{10}$$

It can be shown [1] that V is element-wise concave in terms of $\overrightarrow{P}$. Therefore, the minimization problem takes its optimal value at the extreme points of polytope $\mathbb{P}$. In other words, the optimal schedule should be in such a way that at most one of the links emanated from each node is active. That link, if there is any, should be sending data at full allowable power [2].

For simplicity, we show the mth extreme point of $\mathbb{P}$ by P_{ext}^m. Considering M as the total number of extreme points, we will need to solve the following max-min problem:

$$H(\overrightarrow{C}) = \max_{\beta \geq 0} \left\{ \min_m V(P_{ext}^m, \beta) : 1 \leq m \leq M \right\} \tag{11}$$

This optimization problem can be solved by a linear program:

$$
\begin{aligned}
&\max T \\
&\text{s.t.} \\
&T - \sum_{l\in\epsilon} P^m_{ext}(l) - \sum_{l\in\epsilon} \beta(l)(C(l) - X^m(l)) \leq 0 \\
&\qquad\qquad m = 1, \cdots, M
\end{aligned} \tag{12}
$$

The problem with this method is, M, the number of extreme points (constraints) grows exponentially with the number of links in the network, and therefore it is not possible to find the solution for large values of M. In the next section, we introduce an efficient mechanism to solve this optimization problem.

Let us denote the extreme points that minimize $V(\overrightarrow{P}, \beta)$ by P^i and their total number by K. Assuming feasibility, it can be shown that there exists a vector $\overrightarrow{\lambda}$ such that [1]:

$$
\begin{aligned}
&\textstyle\sum_{i=1}^K \lambda_i = 1 \\
&X_{avg}(l) = \textstyle\sum_{i=1}^K \lambda_i X^i(l) = C(l)
\end{aligned} \tag{13}
$$

The value of λ_i indicates the relative frequency that the power vector P^i should be utilized in an optimal policy.

Therefore, the main challenge is to find the optimal solutions of (12). In our algorithm, as will be shown in the next section, we insert the constraints iteratively to take advantage of the fact that we need at most $L_\epsilon + 1$ transmission modes in the optimal solution.

B. *Scheduling, Power Control, and Routing*

In the power control-scheduling problem link rates, $C(l)$, were fixed and the algorithm finds the optimal power allocation vectors and their scheduling frequency. The routing problem should find the optimal link rates for a given end-to-end traffic demand matrix as well.

Now, assume we have a set of source-destination (S-D) pairs $\{(i, j)\}$. Suppose that for each S-D pair the desired traffic demand is known. Solution to the joint routing, scheduling and power control problem specifies links' average traffic rates, power vectors and scheduling frequency of power vectors such that the given traffic demand and maximum power constraints are satisfied. Our objective is to find the solution with minimum average power consumption. Let us then define $v_{ij}(l)$ as the portion of flow in link l that belongs to the S-D pair $\{(i, j)\}$. The mathematical statement of the joint routing scheduling, and power control problem will be as follows,

$$
\begin{aligned}
&\min \textstyle\sum_l P_{avg}(l) \\
&\text{s.t.} \\
&a.\quad 0 \leq \textstyle\sum_{l\in\varepsilon(i)} P_t(l) \leq P^{max}(i) \text{ and} \\
&b.\quad 0 \leq\ P_t(l), \text{ for all } t \geq\ 1 \text{ and } l \in \epsilon \\
&c.\quad \textstyle\sum_{l\in\mathcal{I}_p} v_{ij}(l) - \sum_{l\in\mathcal{O}_p} v_{ij}(l) \\
&= \begin{cases} 0 & p \neq\ i \ or\ j \\ -d_{ij} & n = i \\ d_{ij} & n = j \end{cases} \\
&d.\quad \textstyle\sum_{ij} v_{ij}(l) = C(l) \\
&e.\quad X_{avg}(l) \geq C(l), \text{ for all } l \in \epsilon
\end{aligned} \tag{14}
$$

where $\mathcal{I}_n$ and $\mathcal{O}_n$ are sets of links entering and leaving node n respectively. Equations (12c) and (12d) define a polytope that we denote by $\mathbb{C}$. For a rate vector $\overrightarrow{C}$ to be a route, it must be on this polytope.

III. Power Control, Scheduling and Routing Algorithms

In this section we will introduce a new algorithm that can find the optimal power control, scheduling, and routing for large networks. We first explain the power control-scheduling algorithm for the fixed routing and then extend the algorithm to compute the optimal power control, scheduling, and routing simultaneously.

A. *The power control-scheduling algorithm*

Recall that to compute the optimal scheduling and power control for a given routing we have to solve the optimization problem given in (12). The problem with this method is that the number of extreme points (constraints), M, grows exponentially with the number of links in the network, and therefore it is not possible to find the solution for large values of M.

A common approach to deal with large number of constraints in linear programs is cutting plane methods [6] [1]. Instead of dealing with all constraints, the cutting plane algorithm considers a subset of constraints I and form the relaxed problem:

$$
\begin{aligned}
&\max T \\
&\text{s.t.} \\
&T - \sum_{l\in\epsilon} P^m(l) - \sum_{l\in\epsilon} \beta(l)(C(l) - X^m(l)) \leq 0 \\
&\qquad\qquad m \in I
\end{aligned} \tag{15}
$$

We use Kelly's convex cutting plane algorithm [6] which is basically an iterative algorithm for introducing new constraints into the constraint subset. We initialize the constraint set I_1 by selecting one of the extreme points of $\mathbb{P}$ arbitrarily and forming its corresponding constraint.

At each iteration, e.g. k, we solve the relaxed optimization problem (15) with constraint set I_k instead of I. Let (T_k, β_k) be the optimal solution of the relaxed problem. There are two possibilities: (1) If (T_k, β_k) is a feasible solution for the original optimization problem, then we are done and we have found an optimal solution for that problem too. (2) If (T_k, β_k) is not feasible we have to find a violated constraint and add it to the constraint set to form I_{k+1} and start the next iteration. We need a method to check if (T_k, β_k) is a feasible solution.

Definition: Suppose that we have a candidate solution for a linear program (LP). A separation ***Oracle*** [7] determines if the solution is feasible. If the solution is not feasible, the separation Oracle finds a hyperplane that separates the solution from the feasible region.

[1] In fact, cutting plane methods are usually used in the more general context of convex programming

We formulate the following Oracle optimization problem:

$$\begin{array}{l} \min \left(\sum_{l} P(l) + \beta_k(l)(C(l) - X(P(l))) \right) \\ \text{s.t.} \\ \overrightarrow{P} \in \mathbb{P} \end{array} \tag{16}$$

As explained before, subject to feasibility, there is always an optimal solution in the extreme points of the power polytope, $\mathbb{P}$. Therefore, similar to the concave programming problems, branch and bound algorithms converge in finite number of iterations, and in fact, in most practical cases the algorithm finds the optimal solution fast. We use GAMS [8] software which is based on the branch and bound algorithm. The library of GAMS that analytically finds the global optimum is BARON [9]. BARON uses a modified branch and bound method named as branch and reduce. The linear program as well as the core code is performed by MATLAB software. In the simulation results we elaborate more on the computation time of the algorithm. The essence of the algorithm is as follows.

In summary, at iteration k, we solve the following LP:

$$\begin{array}{l} \max T \\ T - \sum\limits_{l \in \epsilon} P^m(l) - \sum\limits_{l \in \epsilon} \beta(l)(C^m(l) - X^m(l)) \leq 0 \\ m = 1, \cdots k \end{array} \tag{17}$$

Let β_k be the solution of the above LP in iteration k, which will be used in the following non-linear Oracle problem to test optimality:

$$\begin{array}{l} \min_{\overrightarrow{P}} \left(\sum_l P(l) + \sum_l \beta_k(l)(C(l) - X(l)) \right) \\ \text{s.t.} \\ \overrightarrow{P} \in \mathbb{P} \end{array} \tag{18}$$

We then add the new $\overrightarrow{P}$ vector to form a new constraint for the next iteration. In the following we prove that the algorithm converges in finite iterations to the optimal solution.

Lemma 1: The proposed algorithm converges after finite number of iterations.

Proof: See [10]

Lemma 2: After convergence the value of both linear and nonlinear (global) optimizers will be equal to $H(\overrightarrow{C})$.

Proof: See [10]

Theorem 1: The set of power modes after convergence, $\{\overrightarrow{P^1}, \cdots, \overrightarrow{P^m}\}$, satisfy the rate requirement. In other words, there is a non-negative time sharing, $\lambda_1, \cdots, \lambda_m$ that

$$\begin{array}{ll} a. & \sum \lambda_i = 1 \\ b. & \sum \lambda_i . \overrightarrow{X}^i = \overrightarrow{C} \\ c. & \sum \lambda_i \sum_l P^i(l) = H(\overrightarrow{C}) \end{array} \tag{19}$$

Proof: See [10]

B. Power Control, Scheduling and Routing

We extend the power control-scheduling algorithm developed in section III.A to find the optimal routing too. The routing algorithm described in [1] has scalability problems, since it has to consider all extreme points of the power polytope explicitly and relies on iterative gradient based updates of the routing parameters. Both these issues are resolved in our algorithm.

The problem formulation is given in (14). In Section II.A, we explained that by using duality and due to linearity in P and X, we need to only solve the optimization problem given in (10). Similarly, we can use the duality theorem for linear programs and linearity of (14) in P, X and C to show that for the joint power control, scheduling, and routing problem it is sufficient to solve the following optimization problem.

$$\begin{array}{l} \max_{\beta \geq 0} \{ \min_{\overrightarrow{P}, \overrightarrow{C}} V(\overrightarrow{P}, \beta, \overrightarrow{C}) \} \\ \text{s.t.} \\ \overrightarrow{P} \in \mathbb{P} \text{ and } \overrightarrow{C} \in \mathbb{C} \end{array} \tag{20}$$

Here, we introduce $\overrightarrow{C}$ as one of the arguments of function $V()$, since it is not constant and we seek to find its optimal value. The problem can be rewritten as an LP:

$$\begin{array}{l} \max T \\ \text{s.t.} \\ T - V(\overrightarrow{P}, \beta, \overrightarrow{C}) \leq 0 \quad \text{for all } \overrightarrow{P} \in \mathbb{P} \text{ and } \overrightarrow{C} \in \mathbb{C} \end{array} \tag{21}$$

Recall that the function $V()$ is element wise concave in $\overrightarrow{P}$ and linear in $\overrightarrow{C}$. Hence, the optimal solution can be found in extreme points of polytopes $\mathbb{P}$ and $\mathbb{C}$. Similar to the power control-scheduling case we use the Kelly's cutting plane method to devise an iterative algorithm as follows:

At Iteration k we solve the following master linear program with k constraints:

$$\begin{array}{l} \max T \\ T - \sum\limits_{l \in \epsilon} P^m(l) - \sum\limits_{l \in \epsilon} \beta(l)(C^m(l) - X^m(l)) \leq 0 \\ \qquad\qquad\qquad\qquad m = 1, \cdots k \end{array} \tag{22}$$

Let (T_k, β_k) be the optimal solution of (22) at Iteration k; β_k, will be used in the following non-linear Oracle problem

$$\begin{array}{l} \min_{\overrightarrow{P}, \overrightarrow{C}} \left(\sum_l P(l) + \sum_l \beta_k(l)(C(l) - X(l)) \right) \\ \text{s.t.} \\ \overrightarrow{C} \in \mathbb{C} \text{ and } \overrightarrow{P} \in \mathbb{P} \end{array} \tag{23}$$

If the Oracle optimal value is less than T_k, then the corresponding solution $(\overrightarrow{P}^{k+1}, \overrightarrow{C}^{k+1})$ forms a violated constraint that should be added to the master LP constraint set for the next iteration. Otherwise, the algorithm has converged.

We can further simplify the last step as follows. The potential function in (23) is separable to two functions. The first one, i.e. $\sum_l \beta_k(l) C(l)$, is the sum of weighted $C(l)$s and can be minimized via *Dijkstra* algorithm [11]. The second one is simply the potential function in the previous subsection minus a constant value. Therefore, it does not require a new technique to find the minimum.

Similar to the scheduling-only case, since the optimal solution is in the extreme points of the power and routing polytopes, the cutting plane algorithm converges in finite number of steps.

After convergence, the optimal routing vector and the optimal transmission strategy will be superposition of all the $\overrightarrow{C}^k$ vectors and $\overrightarrow{P}^k$ vectors respectively. The corresponding coefficients are similar and are found in the same manner as described in the proof of Theorem 1.

IV. Simulation Results and Discussion

We consider a square area of 200 by 200 meters for our simulations. The node locations are selected with a uniform random generator in this area. We consider networks with 7 to 50 nodes in our simulations. However, in most of the experiments the number of nodes ranges from 7 to 30. There is an edge between two nodes of the network graph if the distance between them is less than a specific value denoted by $d_{\max}$. The number of edges ranges from 9 to 102 in our experiments, with 9 corresponding to a network with 7 nodes and 102 to a network with 50 nodes. Each edge represents two uni-directional communication links in the network. Therefore, the number of communication links is twice the number of edges of the network graph. For example, in the network of Fig. 1 we have 102 edges which is equivalent to 204 uni-directional links. The path-loss and shadowing parameter, $G(i,j)$, is considered proportional to inverse square of the distance between nodes i and j. The algorithm, however, works for any other choice of $G(i,j)$.

The efficient bandwidth of links is 1 MHz, and the noise power .000010 Watt. The maximum power of a node is 1 Watt. For each network, we randomly pick the source-destination pairs. The number of source-destination pairs in a network is about 20% of the total number of nodes. The end-to-end data-rate from each source to its corresponding destination is 10 Kbps, so that in all experiments a feasible solution exists. However, the data rate value does not affect the properties and efficiency of the algorithm. For each network size, multiple random networks were generated and the results were consistent among different runs. Here, we report the average results of at least 3 experiments for each network size. Three separate algorithms are studied here: (1) optimal scheduling and power control using an LP with all extreme points constraint with minimum hop routing, (2) our proposed algorithm for scheduling and power control with minimum hop routing and (3) our proposed joint routing, scheduling and power control algorithm.

In Fig. 1 the source and destination pairs and the links that are utilized in the optimal routing solution for each pair are specified. We represent each source node by $\triangleright$ and the destination node by $\square$. The same color is used for these symbols and the links that are in the optimal route between them. Note that in this case the optimal routing has selected only one path (no multi-path routing) and there is tendency to select common links for different source-destination pairs. The same trend is observed in other experiments as we will discuss later.

In Fig. 2 we have plotted the amount of power consumed. The first curve in this figure represents the amount of power consumed when using our joint routing and scheduling method. The algorithm corresponding to the other curve uses a minimum route and then finds the best schedule using our scheduling-only algorithm. Our algorithm consumes, on average, 15 percent less power. It is one of conclusions of this work that, in a wireless network the minimum-hop route is not the power-optimal one.

The number of constraints directly affects the running time of the algorithm. Fig. 3 compares the running time of the

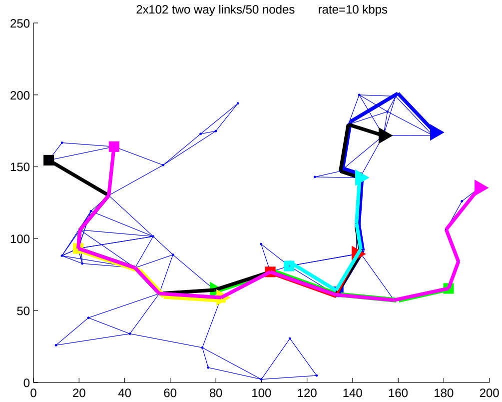

Fig. 1. One of the simulated networks with 50 nodes

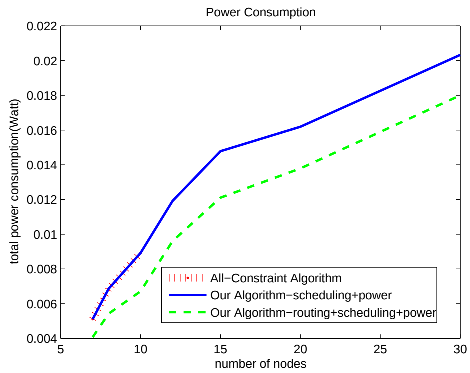

Fig. 2. Comparison of the total consummed power

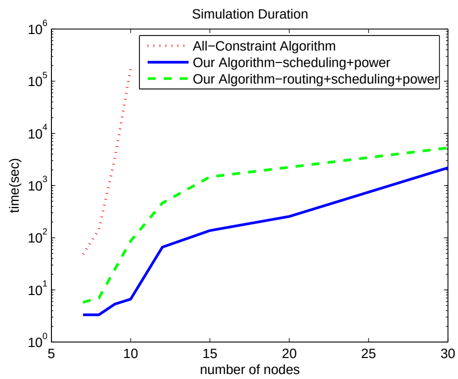

Fig. 3. Comparison of the time it takes for each algorithm to converge

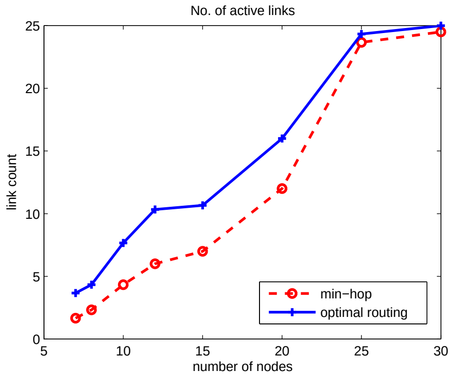

Fig. 4. Comparison of the number of active links

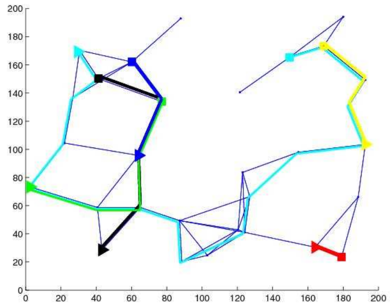

Fig. 5. Generated routes for a network with 51 two-way links (102 single-way links)

algorithms[2]. We were not able to run the all-constraint method for networks with more than 12 edges. Besides the time, memory is also an issue for the all-constraint method. For example, a 15-edged network has at least $2^{15\times 2} = 1Gig$ extreme points that needs to be stored. Both algorithms introduced in this paper are much more time efficient than the previously proposed algorithms.

If we compare running time curves of the two algorithms introduced here, it becomes clear that finding the optimal routing can be very time consuming. In the following we compare the total power consumption of the optimal routing with the minimum hop routing when we use optimal scheduling and power control in both cases to quantify the performance gain achieved with optimal routing.

Fig. 4 shows the number of active links in each method. As can be seen the number of links used by the joint routing and scheduling method is more than that of the minimum-hop method.

To further study optimal routing characteristics, Fig. 5 shows the topology and optimal routes for a network with 102 links. The optimal route for each source-destination pair consists of a *single* path.

[2]All simulations were performed by a 2.6 GHz Compaq Xeon computer, with 256 kb cache and 1 GB of memory.

These results suggest that the interaction between routing, scheduling and power control is very complex and it is very important to consider appropriate physical and MAC layer models in the design of algorithms.

V. Conclusion

We proposed a new global optimization algorithm for joint power control, routing and scheduling to minimize the power consumption in multi-hop wireless networks. The previously proposed centralized algorithms are too complex and cannot be used for large size networks. The distributed algorithms are in general approximation algorithms. Further, they are based on the assumption of a node exclusiveness interference model [12]-[14]. In most practical systems this is not a valid assumption. Our algorithm is a cutting plane algorithm that is proved to obtain the global optimum. Simulation results suggest that the algorithm converges very fast and can be used for large size networks. Even though our algorithm is centralized it can provide valuable insight regarding the characteristics of the optimal solutions and capacity of the networks.

Acknowledgment

This work was supported in part by an ARO Multi-University Research Initiative (MURI), grant # W911NF-04-1-0224.

References

[1] R. Cruz and A. Sanathanam, “Optimal Routing, Link Scheduling and Power Control in Multi-hop Wireless Networks,” *Proceedings of IEEE Infocom 2003*.
[2] A. Sanathanam, “Joint Optimization of Radio Resources in Wireless Multi-Hop Networks,” *PhD. Thesis*, University of California San Diego.
[3] J. Kazemitabar, H. Yousefi’zadeh, and H. Jafarkhani, “Impact of Physical Layer Parameters on Connectivity of Ad Hoc Networks,”*Proceedings of IEEE ICC 2006*.
[4] H. Jafarkhani, H. Yousefi’zadeh, and J. Kazemitabar, “Capacity-Based Connectivity of MIMO Fading Ad Hoc Networks,”*Proceedings IEEE Globecom 2005*.
[5] H. Yousefi’zadeh, H. Jafarkhani, and J. Kazemitabar, “Outage Probability Metrics of Connectivity for MIMO Fading Ad-Hoc Networks,” *Proceedings of IEEE Milcom 2005*.
[6] J. E. Kelly, Jr., “The Cutting Plane Method for Solving Convex Programs,” *Journal of the Society for Industrial and Applied Mathematics*, vol. 8, no. 4, December 1960.
[7] http://www-math.mit.edu/∼vempala/18.433/L18.pdf
[8] www.gams.com
[9] http://archimedes.scs.uiuc.edu/baron.html
[10] J. Kazemitabar, V. Tabatabaee, and H. Jafarkhani, “Global Optimal Routing, Scheduling and Power Control for Multi-hop Wireless Networks with Interference,” *Technical Report*, accessible at www.ece.uci.edu/∼skazemit/Technical_Report.pdf
[11] A. Tanenbaum, *Computer Networks* Fourth Edition, Prentice Hall 2002.
[12] L. Lin, X. Lin, N. Shroff, “Low-Complexity and Distributed Energy Minimization in Multi-hop Wireless Networks,” *Proceedings of Infocom 2007*, May 2007.
[13] M.J. Neely, “Energy Optimal Control for Time Varying Wireless Networks,” *IEEE Transactions on Infomation Theory*, vol. 52, no. 2, pp. 2915-2934, July 2006.
[14] V. Bharghavan, A. Demers, S. Shenker, L. Zhang, “MACAW: A Media Access Protocol for Wireless LAN’s ,” *Proceedings of the ACM SIGCOMM’94*, August 1994.