Optimal Distortion-Power Tradeoffs in Gaussian Sensor Networks

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Abstract— **We investigate the optimal performance of dense sensor networks by studying the joint source-channel coding problem. The overall goal of the sensor network is to take measurements from an underlying random process, code and transmit those measurement samples to a collector node in a cooperative multiple access channel with imperfect feedback, and reconstruct the entire random process at the collector node. We provide lower and upper bounds for the minimum achievable expected distortion when the underlying random process is Gaussian. In the case where the random process satisfies some general conditions, we evaluate the lower and upper bounds explicitly and show that they are of the same order for a wide range of sum power constraints. Thus, for these random processes, under these sum power constraints, we determine the achievability scheme that is order-optimal, and express the minimum achievable expected distortion as a function of the sum power constraint.**

I. Introduction

With the recent advances in the hardware technology, small cheap nodes with sensing, computing and communication capabilities have become available. In practical applications, it is possible to deploy a large number of these nodes to sense the environment. In this paper, we investigate the optimal performance of a dense sensor network by studying the joint source-channel coding problem. The sensor network is composed of N sensors, where N is very large, and a single collector node. The overall goal of the sensor network is to take measurements from an underlying random process $S(t)$, $0 \leq t \leq T_0$, code and transmit those measured samples to a collector node in a cooperative multiple access channel with imperfect feedback, and reconstruct the entire random process at the collector node. We investigate the minimum achievable expected distortion and the corresponding achievability scheme when the underlying random process is Gaussian and the communication channel is a cooperative Gaussian multiple access channel with feedback (potentially imperfect).

Scaglione and Servetto [1] investigated the scalability of the sensor networks. The goal of the sensor network in [1] was that each sensor reconstructs the data measured by all of the sensors using sensor broadcasting. In this paper, we focus on the case where the reconstruction is required only at the collector node. Also, in this paper, the task is not the reconstruction of the data the sensors measured, but the reconstruction of the underlying random process.

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Gastpar and Vetterli [2] studied the case where the sensors observe a noisy version of a linear combination of L Gaussian random variables which all have the same variance, code and transmit those observations to a collector node, and the collector node reconstructs the L random variables. In [2], the expected distortion achieved by applying separation-based approaches was shown to be exponentially worse than the lower bound on the minimum achievable expected distortion. In this paper, we study the case where the data of interest at the collector node is not a finite number of random variables, but a random process. We assume that the sensors are able to take noiseless samples. Our upper bound is also developed by using a separation-based approach, but it is shown to be of the same order as the lower bound, for a wide range of power constraints for random processes that satisfy some general conditions.

El Gamal [3] studied the capacity of dense sensor networks and found that all spatially band-limited Gaussian processes can be estimated at the collector node, subject to any non-zero constraint on the mean squared distortion. In this paper, we study the minimum achievable expected distortion for space-limited, and thus, not band-limited, random processes, and we show that the minimum achievable expected distortion decreases to zero as the number of nodes increases, unless the sum power constraint is unusually small.

In [4], we studied this problem when the underlying process was Gauss-Markov. We found the minimum achievable expected distortion and the corresponding order-optimal achievability scheme for a wide range of sum power constraints. In this work, we extend our results to the case where the underlying process is a Gaussian random process which satisfies some general conditions. We first provide lower and upper bounds for the minimum achievable expected distortion for arbitrary Gaussian random processes for which the Karhunen-Loeve expansion exists. Then, we focus on the case where the Gaussian random process also satisfies some general conditions, evaluate the lower and upper bounds explicitly, and show that they are of the same order, for a wide range of sum power constraints. Thus, for these random processes, under a wide range of sum power constraints, we determine an order-optimal achievability scheme, and identify the minimum achievable expected distortion.

II. System Model

The collector node wishes to reconstruct a sequence of random processes $\{S^{(l)}(t), 0 \leq t \leq T_0\}_{l=1}^{n}$, i.i.d. in time, where l denotes the time, n is the block length, and t denotes the spatial position. At each time instant, the random process $S(t)$ is assumed to be Gaussian with zero-mean and a continuous autocorrelation function $K(t,s)$. The N sensor nodes are placed at positions $0 = t_1 \leq t_2 \leq \cdots \leq t_N = T_0$, and observe samples $\mathbf{S}_N = (S(t_1), S(t_2), \cdots, S(t_N))$. For simplicity and to avoid irregular cases, we assume that the sensors are equally spaced. The distortion measure is the squared error,

$$d(s(t), \hat{s}(t)) = \frac{1}{T_0}\int_0^{T_0} (s(t) - \hat{s}(t))^2 dt \tag{1}$$

Each sensor node and the collector node, denoted as node 0, is equipped with one transmit and one receive antenna. At any time instant, let X_i and Y_i denote the signals transmitted by and received at, node i, and let h_{ji} denote the channel gain from node j to node i. Then, the received signal at node i can be written as,

$$Y_i = \sum_{j=0, j\neq i}^{N} h_{ji} X_j + Z_i, \qquad i = 0, 1, 2, \cdots, N \tag{2}$$

where $\{Z_i\}_{i=0}^{N}$ is a vector of $N+1$ independent and identically distributed, zero-mean, unit-variance Gaussian random variables. Therefore, the channel model of the network is such that all nodes hear a linear combination of the signals transmitted by all other nodes at that time instant. We assume that the channel gain h_{ij} is bounded, i.e.,

$$\bar{h}_l \leq h_{ij} \leq \bar{h}_u, \qquad i, j = 0, 1, \cdots, N \tag{3}$$

where $\bar{h}_u$ and $\bar{h}_l$ are positive constants independent of N. This model is quite general and should be satisfied easily: by the conservation of energy, $h_{ij}^2 \leq 1, i, j = 0, 1, \cdots, N$, and since all nodes are within finite distance from each other, the channel gains should be lower bounded as well. This channel gain model differs from the one in [4], and is more reasonable. Since the collector node is also equipped with a transmit antenna, there is feedback from the collector node to the sensor nodes. However, in a wireless environment, this feedback is imperfect, in the sense that it is corrupted by channel noise, as well as interference from other simultaneously transmitting nodes.

We assume that K channel uses are allowed per realization of the random process in time for the reconstruction, where K is a finite positive integer independent of the number of sensors N and block length n. We also assume that all sensors share a sum power constraint of $P(N)$ which is a function of N. The two most interesting cases for the sum power constraint are $P(N) = NP_{\text{ind}}$ where each sensor has its individual power constraint P_{ind}, and $P(N) = P_{\text{tot}}$ where all sensors share a constant total power constraint P_{tot}. Our goal is to determine the scheme that achieves the minimum achievable expected distortion D^N at the collector node for a given total transmit power constraint $P(N)$, and also to determine the rate at which this distortion goes to zero as a function of the number of sensor nodes and the power constraint.

All logarithms are base e. Due to space limitations, all proofs are omitted here and can be found in [5].

III. The Class of Gaussian Random Processes

For a Gaussian random process $S(t)$ with continuous autocorrelation, we perform the Karhunen-Loeve expansion

$$S(t) = \sum_{k=0}^{\infty} S_k \phi_k(t) \tag{4}$$

to obtain the ordered eigenvalues $\{\lambda_k\}_{k=0}^{\infty}$, and corresponding eigenfunctions $\{\phi_k(t), t \in [0, T_0]\}_{k=0}^{\infty}$.

Let $\mathcal{A}$ be the set of Gaussian random processes that satisfy the following conditions:

1. There exist nonnegative constants d_l, d_u and nonnegative integers c_l, c_u and $K_0 \geq c_u + 1$ such that when $k > K_0$,

$$\frac{d_l}{(k + c_l)^x} \leq \lambda_k \leq \frac{d_u}{(k - c_u)^x} \tag{5}$$

for some constant $x > 1$. The condition that $x > 1$ is without loss of generality, because for all continuous autocorrelations, the eigenvalues decrease faster than k^{-1}.

2. In addition to continuity, $K(t,s)$ satisfies the Lipschitz condition of order $1/2 < \alpha \leq 1$, i.e., there exists a constant B such that

$$|K(t_1, s_1) - K(t_2, s_2)| \leq B\left(\sqrt{(t_1 - t_2)^2 + (s_1 - s_2)^2}\right)^{\alpha} \tag{6}$$

for all $t_1, s_1, t_2, s_2 \in [0, T_0]$.

3. For $k = 0, 1, \cdots$, the function $\phi_k^2(s)$ and the function $K(t,s)\phi_k(s)$ as a function of s satisfy the following condition: there exist positive constants B_1, B_2, B_3, B_4, $\beta \leq 1$, $\gamma \leq 1$, and nonnegative constant τ, independent of k such that

$$|\phi_k^2(s_1) - \phi_k^2(s_2)| \leq B_3(k + B_4)^{\tau}|s_1 - s_2|^{\gamma} \tag{7}$$

and for all $t \in [0, T_0]$

$$|K(t, s_1)\phi_k(s_1) - K(t, s_2)\phi_k(s_2)| \leq B_2(k + B_1)^{\tau}|s_1 - s_2|^{\beta} \tag{8}$$

for all $s_1, s_2 \in [0, T_0]$.

The reasons why these conditions are needed for explicit evaluation of the lower and upper bounds on the minimum achievable expected distortion will be clear from the proofs, which are not presented here due to space limitations. We provide some intuition as to why these conditions are needed. Condition 1 shows that we consider random processes that have eigenvalues λ_k which decrease at a rate of approximately k^{-x}. The lower (upper) bound on the eigenvalues in (5) will be used to calculate the lower (upper) bound on the minimum achievable expected distortion. Conditions 2 and 3 are needed because instead of the random process itself that is of interest to the collector node, the collector node, at best, can know only the sampled values of the random process. How well the entire process can be approximated from its samples is critical in obtaining quantitative results. Lipschitz conditions describe the quality of this approximation well. For instance, by condition 3, we require the variation in the eigenfunctions ϕ_k^2 to be no faster than order k^τ. We note that the well-known trigonometric basis satisfies this condition. We also note that our conditions are quite general. Many random processes satisfy these conditions including the Gauss-Markov process, Brownian motion process, etc.

IV. A Lower Bound on the Minimum Achievable Expected Distortion

A. Arbitrary Gaussian Random Processes

The lower bound is obtained by assuming that all of the sensor nodes know the random process exactly, and, the sensor network forms an N-transmit 1-receive antenna point-to-point system to transmit the random process to the collector node. Let C_u^N be the capacity of this point-to-point system and $D_p(R)$ be the distortion-rate function of the random process $S(t)$ [6]. In this point-to-point system, the separation principle holds and feedback, imperfect or perfect, does not increase the capacity, and therefore

$$D^N \geq D_p(C_u^N) \tag{9}$$

To evaluate $D_p(C_u^N)$, we first find the rate distortion function, which is the inverse function of $D_p(R)$, of $S(t)$ [6, Section 4.5] as,

$$R(\theta) = \sum_{k=0}^{\infty} \max\left(0, \frac{1}{2}\log\left(\frac{\lambda_k}{\theta}\right)\right) \tag{10}$$

and

$$D(\theta) = T_0^{-1}\sum_{k=0}^{\infty} \min(\theta, \lambda_k) \tag{11}$$

It can be seen that the function $R(\theta)$ is a strictly decreasing function of θ when $\theta \leq \lambda_0$. Hence, in this region, the inverse function of $R(\theta)$ exists, which we will call $\theta(R), R \geq 0$. Next, we find C_u^N, the capacity of the N-transmit 1-receive antenna point-to-point system [7] over K channel uses as,

$$C_u^N = \frac{K}{2}\log\left(1 + P(N)\sum_{i=1}^{N} h_{i0}^2\right) \tag{12}$$

Then, we have

$$D_p(C_u^N) = D(\theta(C_u^N)) \tag{13}$$

Thus, for arbitrary Gaussian random processes, a lower bound on the minimum achievable expected distortion is

$$D_l^N = D_p(C_u^N) \tag{14}$$

B. The Class of Gaussian Random Processes in $\mathcal{A}$

Next, we evaluate $D_p(C_u^N)$ for the class of Gaussian random processes in $\mathcal{A}$. We will divide our discussion into two separate cases based on the sum power constraint. For the first case, $P(N)$ is such that

$$\lim_{N\to\infty} (NP(N))^{-1} = 0 \tag{15}$$

is satisfied. The cases where $P(N) = NP_{\text{ind}}$ and $P(N) = P_{\text{tot}}$ are included in $P(N)$ satisfying (15). Note that C_u^N increases monotonically in N. Since we are interested in the number $\theta(C_u^N)$, we will analyze the function $\theta(R)$ where R is very large. Correspondingly, this means that we will analyze the function $D(\theta)$ where θ is very small.

Lemma 1 *For large enough R, we have*

$$\theta(R) \geq d_l\left(\frac{x}{4}\right)^x R^{-x} \tag{16}$$

Lemma 2 *For small enough θ, we have*

$$D(\theta) \geq \frac{d_l^{\frac{1}{x}}}{2T_0}\theta^{1-\frac{1}{x}} \tag{17}$$

We are now ready to calculate the distortion. When N is large enough and the sum power constraint $P(N)$ satisfies (15), using (17), (16) and the fact that

$$C_u^N \leq \frac{K}{2}\log\left(1 + \bar{h}_u^2 NP(N)\right) \tag{18}$$

a lower bound on the achievable distortion is

$$\Theta\left((\log(NP(N)))^{1-x}\right) \tag{19}$$

For the second case, $P(N)$ is such that (15) is not satisfied. C_u^N is either a constant independent of N or goes to zero as N goes to infinity. Examining (10), we see that $\theta(C_u^N)$ is bounded below by a constant independent of N, and hence, $D\left(\theta\left(C_u^N\right)\right)$ is a constant and does not go to zero as N increases. Therefore, combining this case with (19), for all possible power constraints $P(N)$, a lower bound on the distortion is

$$\Theta\left(\min\left((\log(NP(N)))^{1-x}, 1\right)\right) \tag{20}$$

When the sum power constraint grows almost exponentially with the number of nodes, the lower bound on the minimum achievable expected distortion decreases inverse polynomially with N. Even though this provides excellent performance, it is impractical since sensor nodes are low energy devices and it is often difficult, if not impossible, to replenish their batteries.

When the sum power constraint is such that (15) is not satisfied, the transmission power is so low that the communication channels between the sensors and the collector node are as if they do not exist. The estimation error is on the order of 1, which is equivalent to the collector node blindly estimating $S(t) = 0$ for all $t \in [0, T_0]$. Even though the consumed power $P(N)$ is very low in this case, the performance of the sensor network is unacceptable; even the lower bound on the minimum achievable expected distortion does not decrease to zero with the increasing number of nodes.

For practically meaningful sum power constraints, including the cases of $P(N) = NP_{\text{ind}}$ and $P(N) = P_{\text{tot}}$, the lower bound on the minimum achievable expected distortion decays to zero at the rate of

$$(\log N)^{1-x} \tag{21}$$

V. An Upper Bound on the Minimum Achievable Expected Distortion

A. Arbitrary Gaussian Random Processes

Any distortion found by using any achievability scheme will serve as an upper bound on the minimum achievable expected distortion. We consider the following separation-based achievable scheme: First, we perform distributed rate-distortion coding at all sensor nodes using [8, Theorem 1]. After obtaining the indices of the rate-distortion codes, we transmit the indices as independent messages using the amplify-and-forward method introduced in [9]. The distortion obtained using this scheme will be denoted as D_u^N.

We apply [8, Theorem 1], generalized to N sensor nodes in [10, Theorem 1], to obtain an achievable rate-distortion point.

Theorem 1 *If the individual rates are equal, the following sum rate and distortion are achievable,*

$$D_a^N(\theta') = \frac{1}{T_0}\int_0^{T_0}\left(K(t,t) - \frac{T_0}{N-1}\boldsymbol{\rho}_N^T(t)\left(\Sigma_N' + \theta' I\right)^{-1}\boldsymbol{\rho}_N(t)\right)dt \tag{22}$$

$$R_a^N(\theta') = \sum_{k=0}^{N-1}\frac{1}{2}\log\left(1 + \frac{\mu_k^{(N)'}}{\theta'}\right) \tag{23}$$

where $\boldsymbol{\rho}_N(t)$ is an $N \times 1$ column vector with the i-th entry being $K(t,(i-1)/(N-1)T_0)$. Σ_N' is an $N \times N$ matrix with the (i,j)-th entry being $T_0/(N-1)\,K((i-1)/(N-1)T_0,(j-1)/(N-1)T_0)$, and $\mu_0^{(N)'}, \mu_1^{(N)'}, \cdots, \mu_{N-1}^{(N)'}$ are the eigenvalues of Σ_N'.

We further evaluate $D_a^N(\theta')$ in the next lemma.

Lemma 3 *For all Gaussian random processes, we have*

$$D_a^N(\theta') = O\left(\max\left(A^{(N)}, B^{(N)}, \frac{1}{T_0}\sum_{k=0}^{N-1}\left(\frac{1}{\theta'} + \frac{1}{\mu_k^{(N)'}}\right)^{-1}\right)\right) \tag{24}$$

where $A^{(N)}$ and $B^{(N)}$ are defined as

$$A^{(N)} = \frac{2}{T_0}\sum_{i=1}^{N-1}\int_{\frac{i-1}{N-1}T_0}^{\frac{i}{N-1}T_0}\left(\boldsymbol{\rho}_N\left(\frac{i-1}{N-1}T_0\right) - \boldsymbol{\rho}_N(t)\right)_i dt + \frac{1}{T_0}\sum_{i=1}^{N-1}\int_{\frac{i-1}{N-1}T_0}^{\frac{i}{N-1}T_0}\left(K(t,t) - K\left(\frac{i-1}{N-1}T_0, \frac{i-1}{N-1}T_0\right)\right)dt \tag{25}$$

and

$$B^{(N)} = \frac{2}{T_0}\sum_{i=1}^{N-1}\int_{\frac{i-1}{N-1}T_0}^{\frac{i}{N-1}T_0}\left\|\boldsymbol{\rho}_N\left(\frac{i-1}{N-1}T_0\right) - \boldsymbol{\rho}_N(t)\right\| dt \tag{26}$$

respectively.

Lemma 3 tells us that the expected distortion achieved by using the separation-based scheme is upper bounded by the maximum of three types of distortion. We define the third distortion as

$$D_b^N(\theta') = \frac{1}{T_0}\sum_{k=0}^{N-1}\left(\frac{1}{\theta'} + \frac{1}{\mu_k^{(N)'}}\right)^{-1} \tag{27}$$

Now, we determine an achievable rate for the communication channel from the sensor nodes to the collector node. The channel in its nature is a multiple access channel with potential cooperation between the transmitters and imperfect feedback from the collector node. The capacity region for this channel is not known. We get an achievable sum rate, with identical individual rates, for this channel by using the idea presented in [9]. This result is a generalization of [3, Theorem 1] from a constant power constraint to a more general power constraint.

Theorem 2 *When the sum power constraint $P(N)$ is such that there exists an $\epsilon > 0$ where*

$$\lim_{N\to\infty} P(N)N^{\frac{1}{2}-\epsilon} > 1 \tag{28}$$

the following sum rate is achievable over K channel uses,

$$C_a^N = \frac{d_0 K}{2}\log(NP(N)) \tag{29}$$

where d_0 is a positive constant, independent of N. Otherwise, C_a^N approaches a positive constant or zero as $N \to \infty$.

Theorem 2 shows that when the sum power constraint is such that (28) is satisfied, the achievable rate increases with N. Otherwise, the achievable rate is either a positive constant or decreases to zero, which will result in poor estimation performance at the collector node.

The function $R_a^N(\theta')$ is a strictly decreasing function of θ', thus, the inverse function exists, which we will denote as $\theta_a^N(R)$. Hence, to find D_u^N, we first find $\theta_a^N\left(C_a^N\right)$, and then,

$$D_u^N = D_a^N\left(\theta_a^N\left(C_a^N\right)\right) \tag{30}$$

We will perform this calculation when the underlying random process is in $\mathcal{A}$.

B. The Class of Gaussian Random Processes in $\mathcal{A}$

We analyze all three types of distortion for Gaussian random processes in $\mathcal{A}$.

Lemma 4 *For Gaussian random processes in $\mathcal{A}$, we have*

$$A^{(N)} = O\left(N^{-\alpha}\right) \tag{31}$$

$$B^{(N)} = O\left(N^{\frac{1}{2}-\alpha}\right) \tag{32}$$

The results in Lemma 4 depend crucially on condition 2 in the definition of $\mathcal{A}$ in Section III.

Using the results of Lemma 4, we have

$$D_u^N = O\left(\max\left(N^{\frac{1}{2}-\alpha}, D_b^N\left(\theta_a^N\left(C_a^N\right)\right)\right)\right) \tag{33}$$

It remains to evaluate $D_b^N\left(\theta_a^N\left(C_a^N\right)\right)$. We first define two sequences ϑ_L^N and ϑ_U^N which satisfy

$$\lim_{N\to\infty} \frac{1}{\vartheta_L^N N^{\min\left(x, \frac{x\gamma}{\tau}, \frac{\alpha x}{x-1}, \frac{\beta x}{x+\tau}\right)}} = 0, \quad \lim_{N\to\infty} \vartheta_U^N = 0 \tag{34}$$

Lemma 5 *For large enough N and R in the interval of*

$$\left[\frac{2d_u^{\frac{1}{x}} x^2}{x-1}\left(\vartheta_U^N\right)^{-\frac{1}{x}}, \frac{x d_l^{\frac{1}{x}}}{8}\left(\vartheta_L^N\right)^{-\frac{1}{x}}\right] \tag{35}$$

we have

$$\frac{x^x d_l}{8^x R^x} \le \theta_a^N(R) \le \frac{2^x x^{2x} d_u}{(x-1)^x R^x} \tag{36}$$

Hence, for all $P(N)$ that satisfy

$$\lim_{N\to\infty} \frac{NP(N)}{e^{N^{\min\left(1, \frac{\gamma}{\tau}, \frac{\alpha}{x-1}, \frac{\beta}{x+\tau}\right)}}} = 0 \tag{37}$$

and (28), we have C_a^N in the interval of (35) and Lemma 5 applies.

Now we upper bound $D_b^N(\theta')$.

Lemma 6 *For $\theta' \in [\vartheta_L^N, \vartheta_U^N]$ for large enough N, we may upper bound $D_b^N(\theta')$ as*

$$D_b^N(\theta') \le \frac{4d_u^{\frac{1}{x}}}{T_0}\left(\frac{x+1}{x-1}\right)\theta'^{1-\frac{1}{x}} \tag{38}$$

Proofs of Lemmas 5 and 6 use all conditions 1, 2 and 3 in the definition of $\mathcal{A}$ in Section III.

Hence, when $P(N)$ is such that (37) and (28) are satisfied, using (38) and (36) and the fact that when R is in the interval of (35), $\theta_a^N(R)$ is in $[\vartheta_L^N, \vartheta_U^N]$, we have

$$D_b^N(\theta_a^N(C_a^N)) \le \Theta\left((\log(NP(N)))^{1-x}\right) \tag{39}$$

From (33), an upper bound on the minimum achievable expected distortion is

$$\Theta\left((\log(NP(N)))^{1-x}\right) \tag{40}$$

when the sum power constraint satisfies (28) and

$$\lim_{N\to\infty} \frac{NP(N)}{e^{N^{\min\left(1, \frac{\gamma}{\tau}, \frac{\alpha}{x-1}, \frac{2\alpha-1}{2(x-1)}, \frac{\beta}{x+\tau}\right)}}} = 0 \tag{41}$$

This upper bound on the minimum achievable expected distortion coincides with the the lower bound when $P(N)$ satisfies (28) and (41). The practically interesting cases of $P(N) = NP_{\text{ind}}$ and $P(N) = P_{\text{tot}}$ fall into this region. In both of these cases, the minimum achievable expected distortion decreases to zero at the rate of

$$(\log N)^{1-x} \tag{42}$$

which is the same as the lower bound in (21).

VI. Conclusion

We investigate the performance of dense sensor networks by studying the joint source-channel coding problem. We provide lower and upper bounds for the minimum achievable expected distortion when the underlying random process is Gaussian. When the random process satisfies some general conditions, we evaluate the lower and upper bounds explicitly, and show that they are both of order $(\log(NP(N)))^{1-x}$ for a wide range of sum power constraints ranging from $N^{-\frac{1}{2}}$ to $\frac{e^{N^{\min\left(1, \frac{\gamma}{\tau}, \frac{\alpha}{x-1}, \frac{2\alpha-1}{2(x-1)}, \frac{\beta}{x+\tau}\right)}}}{N}$. Therefore, for random processes that satisfy these general conditions, and under these sum power constraints, we have found that an order-optimal scheme is a separation-based scheme, that is composed of distributed rate-distortion source coding [8] and amplify-and-forward channel coding [9], and the imperfect feedback link does not need to be utilized.

The most interesting cases of $P(N) = NP_{\text{ind}}$ and $P(N) = P_{\text{tot}}$ fall into this region, and for these cases, the minimum achievable expected distortion decreases to zero at the rate of $(\log N)^{1-x}$. Hence, the power constraint $P(N) = P_{\text{tot}}$ performs as well as $P(N) = NP_{\text{ind}}$ "order-wise", and therefore, in practice we may prefer to choose $P(N) = P_{\text{tot}}$. In fact, any $P(N) \ge N^{-1/2+\epsilon}$, for example, $P(N) = N^{-1/3}$, performs as well as $P(N) = NP_{\text{ind}}$ and $P(N) = P_{\text{tot}}$. This means that if we choose $P(N) = N^{-1/3}$, as we add more and more sensors, the sum power constraint can go to zero without affecting the performance "order-wise".

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