Delay-minimal Transmission for Average Power Constrained Multi-access Communications

Jing Yang Sennur Ulukus
Department of Electrical and Computer Engineering
University of Maryland, College Park, MD 20742
yangjing@umd.edu *ulukus@umd.edu*

***Abstract*—We investigate the problem of minimizing the overall transmission delay of packets in a multi-access wireless communication system, where the transmitters have average power constraints. We use a multi-dimensional Markov chain to model the medium access control (MAC) layer behavior. The state of the Markov chain represents current queue lengths. Our goal is to minimize the average packet delay through controlling the probability of departure at each state, while satisfying the average power constraint for each queue. First, we formulate the problem as a constrained optimization problem. Next, we transform the problem into a standard linear programming problem. Then, we analyze the linear programming problem, and develop a procedure by which we determine the optimal solution analytically.**

I. Introduction

In many applications, the average delay packets experience is an important quality of service criterion. Our goal in this paper is to combine information theory and queueing theory to devise a transmission protocol which minimizes the average delay experienced by packets, subject to an average power constraint at each transmitter.

Similar goals have been undertaken by various authors in recent years. Reference [1] considers a time-slotted system with N queues and one server. In each slot, the controller allocates the server to one of the connected queues, such that the average delay in the system is minimized. The authors develop an algorithm named "longest connected queue (LCQ)," where the server is allocated to the longest of all connected queues at any given slot. The authors prove that in a symmetric system, LCQ algorithm minimizes the average delay. Reference [1] does not consider the issue of power consumption during transmissions.

Reference [2] combines information theory and queueing theory in multi-access communication over an additive Gaussian noise channel. Once a packet arrives, it is transmitted immediately with a fixed power. Each transmitter-receiver pair treats the other active pairs as noise. Therefore, the service rate for each packet is a function of the number of active users in the system. Reference [2] derives a relationship between the average delay and a fixed probability of error requirement.

References [3], [4], [5] and [6] consider the data transmission problem from both information theory and queueing theory perspectives. Reference [3] aims to minimize the average delay through rate allocation in a multi-access scenario in additive Gaussian noise, and develops the "Longer-Queue-Higher-Rate (LQHR)" allocation strategy in the symmetric multi-access case, which is shown to minimize the average delay of packets. This rate allocation corresponds to selecting an extreme point (i.e., a corner point) in the multi-access capacity region. Reference [4] considers the problem of rate/power control in a single-user communication over a fading channel. The objective is to minimize the average power with delay constraints. It formulates the problem into a dynamic programming problem and develops a delay-power tradeoff curve. Reference [5] uses dynamic programming to numerically calculate the optimal power/rate control policies that minimize the average delay in a single-user system under an average power constraint. Reference [6] formulates the power constrained average delay minimization problem into a Markov decision problem and analyzes the structure of the optimal solution for a single-user fading channel. As in [4], in these papers as well, because of the large number of possible rate/power choices at each stage, it is almost impossible to get analytical optimal solutions.

Reference [7] considers a cognitive multiple access system. In the model of [7], the primary user (PU) always transmits a packet during a slot whenever its queue is not empty. The secondary user (SU) always transmits when the PU is idle, and it transmits with some probability (which is a function of its own queue length) when the PU is active. The receiver operates at the corner point of the multiple access channel capacity region where the SU is decoded first and the PU is decoded next, so that even though the SU experiences interference from the PU, the PU is always decoded interference-free. Reference [7] aims to minimize the average delay through controlling the transmission probability of the SU. It formulates the problem as a one-dimensional Markov chain and derives an analytical result to minimize the average delay of the SU under an average power constraint.

In this paper, we generalize [7] to a two-user multi-access system, where both users have equal priority. Our goal is to minimize the average delay of the packets in the system under an average power constraint for each user. As in [4]–[7], we consider a discrete-time model. We divide the transmission time into time slots. Packets arriving at the transmitters are stored in the queues at each transmitter. In each slot, each user decides on a transmission rate based on the current lengths of both queues. Unlike [4]–[6], where the rate per slot is a continuous variable, we restrict the transmission rate for each user in a slot to be either zero or one packet per slot. We

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define the probabilities of choosing each transmission rate pair, which can be $(0,0)$, $(0,1)$, $(1,0)$ or $(1,1)$, for each given pair of queue lengths.

Our objective is to find a set of transmission probabilities that minimizes the average delay while satisfying the average power constraints for both users. As in [7], there are two main reasons that we introduce transmission probabilities: First, a randomized policy is more general than a deterministic policy; probability selections of 0 and 1 correspond to a deterministic policy, which is a special case of the randomized policy. Secondly, since we cannot choose arbitrary departure rates in each slot, the use of transmission probabilities enables us to control the average rate per slot at a finer scale. Compared to [4]–[6], our model has a more restricted policy space at each stage, however, this model enables us to construct a two-dimensional discrete-time Markov chain and eventually gives us a closed-form optimal solution.

In the rest of this paper, we first express the average delay and the average power consumed for each user as functions of the transmission probabilities and steady state distribution of the queue lengths. We then transform our problem to a linear programming problem, and derive the optimal transmission scheme analytically.

II. System Model

We consider a discrete-time additive Gaussian noise multiple-access system with two transmitters and one receiver. The received signal is

$$Y = X_1 + X_2 + Z \tag{1}$$

where X_i is the signal of user i, and Z is a Gaussian noise with zero mean and variance σ^2.

In this two-user system, the region of feasible received powers is [8]

$$\begin{aligned} P_1 &\geq \sigma^2(2^{2R_1} - 1) \\ P_2 &\geq \sigma^2(2^{2R_2} - 1) \\ P_1 + P_2 &\geq \sigma^2(2^{2(R_1+R_2)} - 1) \end{aligned} \tag{2}$$

For simplicity, we consider a symmetric two-user system, where both users have the same average power constraint P_{avg}.

In the MAC layer, we assume that packets arrive at the transmitters at a uniform size of B bits per packet. We partition the time into small slots such that we have at most one packet arrive and/or depart during each slot. Let $a_1[n]$ and $a_2[n]$ denote the number of packets arriving at the first and second transmitters, respectively, during time slot n; see Figure 1. We assume that the packet arrivals are i.i.d. from slot to slot. We also assume symmetric arrival rates to both queues:

$$Pr\{a_i[n] = 1\} = \theta, \quad Pr\{a_i[n] = 0\} = 1 - \theta, \quad i = 1, 2 \tag{3}$$

where θ is the common arrival rate.

There is a buffer with capacity N at each transmitter to store the packets. Let $d_1[n]$ and $d_2[n]$ denote the number of packets transmitted in time slot n. The queue length in each buffer evolves according to

$$q_i[n+1] = (q_i[n] - d_i[n])^+ + a_i[n] \quad i = 1, 2 \tag{4}$$

where $(x)^+$ denotes $\max(0, x)$.

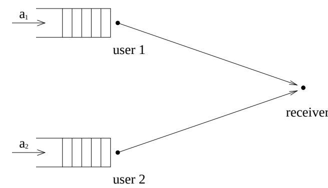

Fig. 1. The system model.

The departure rate for each queue in each slot is either zero or one packet per slot, and it depends on the current queue lengths. When both queues are empty, the departure rates for both queues should be zero packet per slot. In all other situations, the departure rates for both queues should not be zero packet per slot simultaneously. This is because, keeping both transmitters idle does not save any power, but causes unnecessary delay. Therefore, in these situations, there are three possible departure rate pairs: $(d_1, d_2) = (1,0), (0,1)$ or $(1,1)$. We enumerate them as d^1, d^2, d^3. When the first queue length is i and the second queue length is j, we define the probabilities of choosing each pair of these departure rates as g_{ij}^1, g_{ij}^2, g_{ij}^3, respectively. Note that $g_{ij}^1 + g_{ij}^2 + g_{ij}^3 = 1$. We also note that g_{ij}^k, $k = 1, 2, 3$, $i = 0, 1, \ldots, N$ and $j = 0, 1, \ldots, N$ are the main parameters we aim to choose optimally.

The state space of the Markov chain consists of all possible pairs of queue lengths. We denote the state as $\mathbf{q} \triangleq (q_1, q_2)$. When both of the queues are empty, i.e., $\mathbf{q}[n] = (0,0)$, transmitters have no packet to send, and from (4), $\mathbf{q}[n+1] = \mathbf{a}[n]$. The corresponding transition probabilities in this case are:

$$\begin{aligned} Pr\{\mathbf{q}[n+1] = (0,0) | \mathbf{q}[n] = (0,0)\} &= (1-\theta)^2 \\ Pr\{\mathbf{q}[n+1] = (1,0) | \mathbf{q}[n] = (0,0)\} &= \theta(1-\theta) \\ Pr\{\mathbf{q}[n+1] = (0,1) | \mathbf{q}[n] = (0,0)\} &= \theta(1-\theta) \\ Pr\{\mathbf{q}[n+1] = (1,1) | \mathbf{q}[n] = (0,0)\} &= \theta^2 \end{aligned} \tag{5}$$

When one of the queues is empty, there is only one possible departure rate pair, which is either $(0,1)$ or $(1,0)$, depending on which queue is empty. Therefore, from our argument above, the departure probabilities should not be free parameters, but must be chosen as $g_{i0}^1 = g_{0j}^2 = 1$. The corresponding transition probabilities are:

$$\begin{aligned} Pr\{\mathbf{q}[n+1] = (i-1,0) | \mathbf{q}[n] = (i,0)\} &= (1-\theta)^2 \\ Pr\{\mathbf{q}[n+1] = (i-1,1) | \mathbf{q}[n] = (i,0)\} &= \theta(1-\theta) \\ Pr\{\mathbf{q}[n+1] = (i,0) | \mathbf{q}[n] = (i,0)\} &= \theta(1-\theta) \\ Pr\{\mathbf{q}[n+1] = (i,1) | \mathbf{q}[n] = (i,0)\} &= \theta^2 \end{aligned} \tag{6}$$

A similar argument is valid when the first queue is empty, i.e., $\mathbf{q}[n] = (0, j)$. Transition probabilities in this case can be written similar to (6).

When neither of the queues is empty, i.e., for $\mathbf{q}[n] = (i, j)$, where $1 \leq i, j \leq N$, the transition probabilities are:

$$\begin{aligned} Pr\{(i-1, j-1) | (i,j)\} &= g_{ij}^3 (1-\theta)^2 \\ Pr\{(i-1, j+1) | (i,j)\} &= g_{ij}^1 \theta(1-\theta) \\ Pr\{(i+1, j-1) | (i,j)\} &= g_{ij}^2 \theta(1-\theta) \\ Pr\{(i, j+1) | (i,j)\} &= g_{ij}^1 \theta^2 \\ Pr\{(i+1, j) | (i,j)\} &= g_{ij}^2 \theta^2 \\ Pr\{(i-1, j) | (i,j)\} &= g_{ij}^3 \theta(1-\theta) + g_{ij}^1 (1-\theta)^2 \end{aligned}$$

$$Pr\{(i,j-1)|(i,j)\} = g_{ij}^3\theta(1-\theta) + g_{ij}^2(1-\theta)^2$$
$$Pr\{(i,j)|(i,j)\} = (g_{ij}^1 + g_{ij}^2)\theta(1-\theta) + g_{ij}^3\theta^2 \quad (7)$$

In this paper, we assume that the average power constraints are large enough to prevent any packet losses and maintain a stable system. In order to prevent overflows, we always choose to transmit one packet from a queue whenever its length reaches N. Therefore, we have $g_{iN}^1 = g_{Nj}^2 = g_{NN}^3 = 1$.

Let us define the steady state distribution of this Markov chain as $\boldsymbol{\pi} = [\pi_{00}, \pi_{01}, \cdots, \pi_{0N}, \pi_{10}, \cdots, \pi_{NN}]$. Then, the steady state distribution must satisfy

$$\boldsymbol{\pi}\mathbb{P} = \boldsymbol{\pi}, \qquad \boldsymbol{\pi}\mathbf{1} = 1 \quad (8)$$

where $\mathbb{P}$ is the transition matrix defined by the transition probabilities (5)-(7). We can express the average number of packets in the system as $\sum_{i,j}\pi_{ij}(i+j)$. According to Little's law [9], for our problem, the average delay D is equal to

$$D = \frac{1}{2\theta}\sum_{i,j}\pi_{ij}(i+j) \quad (9)$$

where 2θ is the average arrival rate for the two-user multiple-access system.

III. Problem Formulation

The transmission rate for both transmitters during a slot is either one packet per slot or zero packet per slot. Equivalently, the transmission rate is either B/τ bits/channel use or 0 bits/channel use, where τ is the number of channel uses in each slot. Next, let us consider the power consumptions during each slot. When only one user transmits, since there is no interference from the other transmitter, the transmitted power for the active user needs to satisfy

$$P \geq \sigma^2(2^{2R} - 1) \triangleq \alpha \quad (10)$$

where $R = B/\tau$. In order to minimize the power, we choose the transmit power for the active user as α. When both users transmit simultaneously, from (2), their powers should additionally satisfy

$$P_1 + P_2 \geq \sigma^2(2^{4R} - 1) \triangleq \beta \quad (11)$$

The feasible power region is shown in Figure 2.

Any operating point on the dominant face of the power region gives the same sum transmit power. We assume that the operating point is $P_1 = \beta_1$, $P_2 = \beta_2$, where (β_1, β_2) is a point on the dominant face, i.e., $\beta_1 + \beta_2 = \beta$. Thus, for any state $(i,j) \neq (0,0)$, the average power consumed for the first queue is $g_{ij}^1\alpha + g_{ij}^3\beta_1$, while the average power consumed for the second queue is $g_{ij}^2\alpha + g_{ij}^3\beta_2$. Our goal is to find the transmission policy, i.e., the probabilities g_{ij}^ks, along with the operating point (β_1, β_2), such that the average delay is minimized, subject to an average power constraint for each user. Therefore, our problem can be expressed as:

$$\min_{\mathbf{g},\beta_1,\beta_2} \quad \frac{1}{2\theta}\sum_{i,j}\pi_{ij}(i+j) \quad (12)$$
$$\text{s.t.} \quad \sum_{i,j}\pi_{ij}(g_{ij}^1\alpha + g_{ij}^3\beta_1) \leq P_{avg} \quad (13)$$
$$\sum_{i,j}\pi_{ij}(g_{ij}^2\alpha + g_{ij}^3\beta_2) \leq P_{avg} \quad (14)$$
$$\boldsymbol{\pi}\mathbb{P} = \boldsymbol{\pi}, \quad \boldsymbol{\pi}\mathbf{1} = 1 \quad (15)$$

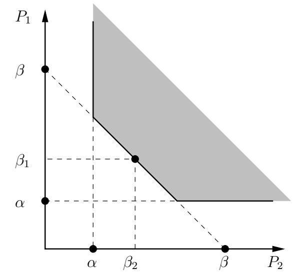

Fig. 2. Feasible power region.

We note that the state transition matrix $\mathbb{P}$ is filled with variables in (5)-(7) which depend on g_{ij}^ks. Also, through (15), π_{ij}s depend on g_{ij}^ks, as well.

IV. Analysis of the Problem

Note that $g_{ij}^1 + g_{ij}^2 + g_{ij}^3 = 1$ for any $(i,j) \neq (0,0)$, therefore $\pi_{ij} = \pi_{ij}(g_{ij}^1 + g_{ij}^2 + g_{ij}^3)$. Define $x_{00} = \pi_{00}$, $x_{ij}^k = \pi_{ij}g_{ij}^k$, $k = 1,2,3$, $i = 0,1,\ldots,N$, $j = 0,1,\ldots,N$. Our aim is to find optimal g_{ij}^ks. However, as we will see, our analysis will be more tractable with variables x_{ij}^k. Let us construct a vector of all of our unknowns $\mathbf{x} = [x_{00}, x_{01}^1, x_{01}^2, x_{01}^3, \ldots, x_{NN}^3]^T$. We also note that under a policy preventing any overflows, which requires a large enough P_{avg}, all packets arriving at a buffer are eventually transmitted out. Therefore, we have the average arrival rate equal to the average departure rate, i.e.,

$$\sum_{i,j}(x_{ij}^1 + x_{ij}^3) = \theta, \qquad \sum_{i,j}(x_{ij}^2 + x_{ij}^3) = \theta \quad (16)$$

When the average power constraints of the users are very large, the transmitters can always transmit a packet from the nonempty queues. In this case, the Markov chain will have only four non-transient states: $(0,0), (0,1), (1,0), (1,1)$, with the stationary distribution $\pi_{00} = (1-\theta)^2$, $\pi_{01} = \pi_{10} = \theta(1-\theta)$, $\pi_{11} = \theta^2$. The average power consumption for each queue is $P_{1csmp} = \theta(1-\theta)\alpha + \theta^2\beta_1$, $P_{2csmp} = \theta(1-\theta)\alpha + \theta^2\beta_2$. Consequently, the total average power consumption is $P_{csmp} = 2\theta(1-\theta)\alpha + \theta^2\beta$. Therefore, if the average power constraint P_{avg} is greater than $P_{csmp}/2$, then we can always find a pair (β_1, β_2) such that $P_{avg} \geq P_{1csmp}$, $P_{avg} \geq P_{2csmp}$. This implies that the given average power constraints will be loose. The corresponding average delay will be one slot, which is the minimal delay we can achieve.

Therefore, from now on, we will focus on the case where the given P_{avg} is less than $P_{csmp}/2$ computed above. In this case, both power constraints in (13) and (14) should be tight, and all four of the constraints (13)-(15) in our optimization problem will be equality constraints. By solving these equations, we have $\beta_1 = \beta_2 = \beta/2$, and we also obtain the values of x_{00} and $\sum_{i,j}x_{ij}^k$, $k = 1,2,3$. Thus, we transform our optimization problem in (12)-(15), which was in terms of g_{ij}^ks, into

$$\min_{\mathbf{x}} \quad \sum_{i,j}\left(\sum_{k=1}^{3}x_{ij}^k(i+j)\right) \quad (17)$$
$$\text{s.t.} \quad x_{00} = 1 - \frac{2\theta(\beta-\alpha) - 2P_{avg}}{\beta - 2\alpha} \quad (18)$$
$$\sum_{i,j}x_{ij}^1 = \sum_{i,j}x_{ij}^2 = \frac{\theta\beta - 2P_{avg}}{\beta - 2\alpha} \quad (19)$$

$$\sum_{i,j} x_{ij}^3 = \frac{2P_{avg} - 2\theta\alpha}{\beta - 2\alpha} \tag{20}$$

$$\mathbb{Q}\mathbf{x} = \mathbf{0} \tag{21}$$

which is in terms of x_{ij}^ks. Here, $\mathbb{Q}$ is a $(N+1)^2 \times (4(N+1)^2 - 3)$ matrix defined by matrix $\mathbb{P}$. We get the equations in (21) from (15) by substituting $\pi_{ij} g_{ij}^k$ for x_{ij}^k. The optimization problem in (17)-(21) is a linear programming problem. In addition, we observe that, in the objective function, all of the x_{ij}^ks with the same sum of indices share the same weight $i+j$. This motivates us to group the x_{ij}^ks along the diagonals of the two-dimensional Markov chain and define their sum, for the nth diagonal, as

$$y_n = \sum_{i=0}^{n} (x_{i,n-i}^1 + x_{i,n-i}^2), \quad t_n = \sum_{i=0}^{n} x_{i,n-i}^3 \tag{22}$$

We also get $2N$ flow-in-flow-out equations between the diagonal groups. For $n = 0, 1$, we have

$$x_{00}\left(\theta^2 + 2\theta(1-\theta)\right) = (y_1 + t_2)(1-\theta)^2$$

$$(x_{00} + y_1)\theta^2 = (y_2 + t_3)(1-\theta)^2 + t_2\left(1-\theta^2\right)$$

and for $n = 2, 3, \ldots, 2N-2$, we have

$$y_n\theta^2 = (y_{n+1} + t_{n+2})(1-\theta)^2 + t_{n+1}\left(1-\theta^2\right)$$

$$y_{2N-1}\theta^2 = t_{2N}\left(1-\theta^2\right) \tag{23}$$

Figure 3 illustrates the transitions between diagonal groups for a system with $N = 3$.

We multiply both sides of the n-th equation in (23) with z^n and sum over n. Then, we take the second derivative of the sum with respect to z and let $z = 1$,

$$\sum_{n=1}^{2N} t_n n = \frac{1}{2(1-\theta)}\left(x_{00}\theta^2 + (1-\theta)^2 \left(\sum_{n=1}^{2N} y_n\right) + \left(1-\theta^2+2(1-\theta)^2\right)\left(\sum_{n=1}^{2N} t_n\right) - \left((1-\theta)^2 - \theta^2\right)\left(\sum_{n=1}^{2N} y_n n\right)\right) \tag{24}$$

From the definition of y_n and t_n in (22), and using (19)-(20), we note

$$\sum_{n=1}^{2N} y_n = \sum_{i,j}(x_{ij}^1 + x_{ij}^2) = \frac{2(\theta\beta - 2P_{avg})}{\beta - 2\alpha} \triangleq \Psi \tag{25}$$

$$\sum_{n=1}^{2N} t_n = \sum_{i,j} x_{ij}^3 = \frac{2P_{avg} - 2\theta\alpha}{\beta - 2\alpha} \triangleq \Phi \tag{26}$$

This, together with (24), implies that our objective function in (17) can be written as

$$\sum_{n=1}^{2N}(y_n + t_n)n = \frac{1}{2(1-\theta)}\sum_{n=1}^{2N} y_n n + C \tag{27}$$

where C is a constant, and $\frac{1}{2(1-\theta)}$ is positive. Therefore, minimizing the original objective function in (17) is equivalent to minimizing $\sum_{n=1}^{2N} y_n n$.

V. The Two-step Optimization Scheme

We propose to solve our original optimization problem in two steps. In the first step, we will consider the optimization

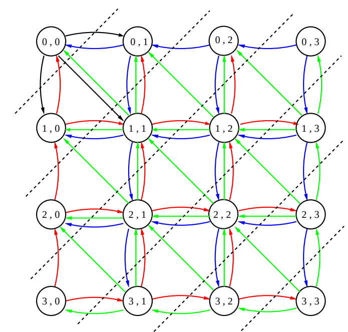

Fig. 3. The transitions between diagonal groups when $N = 3$.

problem in terms of y_ns and t_ns, where the objective function is $\sum_{n=1}^{2N} y_n n$, and the constraints are (25), (18), and (23). The objective function of this optimization problem is exactly the same as that of our original optimization problem in (17)-(21), however, our constraints are more lenient than those of (17)-(21). These imply that, the result we obtain in the first step, in principle, may not be feasible for the original problem.

Therefore, in the second step we will allocate y_ns and t_ns we obtain from the first step to x_{ij}^ks in such a way that the remaining independent transition equations in (21) are satisfied. We note that (16) can be derived from (21), therefore, once (21) is satisfied, (16) will be satisfied. Together with (25), we can make sure that (19) and (20) are satisfied. Therefore, if we can find a valid allocation in the second step, we will conclude that the solution found in the first step is a feasible solution to our original problem. In addition, once we prove the optimality of the solution in the first step, it will be globally optimal for the original problem.

First, we will minimize $\sum_{n=1}^{2N} y_n n$ subject to (25), (18), and (23). This means that we will allocate Ψ to y_ns in a way to minimize $\sum_{n=1}^{2N} y_n n$. This will require us to allocate larger values to y_ns with smaller n, while making sure that (25), (18), and (23) are satisfied. Let us define $\eta = \frac{\theta^2 + 2\theta(1-\theta)}{(1-\theta)^2}$, $\delta = \frac{\theta^2}{(1-\theta)^2}$, $\rho = \frac{1-\theta^2}{(1-\theta)^2}$. Examining (23), we note that for fixed x_{00}, maximizing $y_1, y_2, \ldots$ requires us to set $t_2, t_3, \ldots$ to zero. Therefore, we choose

$$y_1 = x_{00}\eta \tag{28}$$

$$y_2 = (x_{00} + y_1)\delta \tag{29}$$

$$y_n = y_{n-1}\delta, \quad t_n = 0, \quad n = 1, 2, \ldots, n^* \tag{30}$$

where n^* is the largest integer satisfying $\sum_{n=1}^{n^*} y_n < \Psi$. Let $\Delta = \Psi - \sum_{n=1}^{n^*-1} y_n$. We need to check that all of the group transition equations are satisfied. In the following we assume that $n^* \geq 3$. If $n^* = 1, 2$, the allocation will be in a slightly different form, which we omit here due to space limitations.

If $\Delta = y_{n^*}\delta\rho/(\delta+\rho)$, then let

$$y_{n^*+1} = \Delta, \qquad t_{n^*+2} = y_{n^*+1}\delta/\rho \tag{31}$$

We can verify that after this allocation, group transition equations (23) are satisfied. We also note that Ψ is allocated to $\{y_n\}_{n=1}^{n^*+1}$, among which, $\{y_n\}_{n=1}^{n^*}$ attain their maximum possible values. Therefore, the objective function achieves its minimal possible value for the first step.

If $\Delta > y_{n^*}\delta\rho/(\delta+\rho)$, we assign Δ to y_{n^*+1} and y_{n^*+2}

proportionally. Specifically, we let

$$
\begin{aligned}
y_{n^*+1} &= \frac{\Delta(\rho+\delta) + y_{n^*}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \\
y_{n^*+2} &= \frac{\Delta(\rho+\delta)\rho - y_{n^*}\delta\rho^2}{\rho^2+\delta\rho+\delta+\rho} \\
t_{n^*+2} &= \frac{y_{n^*}\delta(\delta\rho+\delta+\rho) - \Delta(\rho+\delta)}{\rho^2+\delta\rho+\delta+\rho} \\
t_{n^*+3} &= \frac{\Delta(\rho+\delta)\delta - y_{n^*}\delta^2\rho}{\rho^2+\delta\rho+\delta+\rho}
\end{aligned} \tag{32}
$$

Since $y_{n^*}\delta > \Delta > y_{n^*}\delta\rho/(\delta+\rho)$, we can verify that each value above is positive, and the sum constraint and the group transition equations are satisfied. Among the non-zero $\{y_n\}_{n=1}^{n^*+2}$, although $\{y_n\}_{n=1}^{n^*}$ attain their maximum, y_{n^*+1} does not. Therefore, different from the first scenario, in this case, we cannot immediately claim that the result is optimal. We will give the mathematical proof for the optimality of this assignment in a longer, journal version of this paper.

If $\Delta < y_{n^*}\delta\rho/(\delta+\rho)$, we need to remove some value from y_{n^*} and assign it to y_{n^*+1} to satisfy the equations. Define $\Delta' = \Delta + y_{n^*}$. We use Δ', y_{n^*-1} instead of Δ and y_{n^*} in (32), to obtain the allocation for y_{n^*} and y_{n^*+1}. We can verify in this case also that each value is positive, and the sum constraint and the group transition equations are satisfied. Similar to the second case, we cannot immediately claim that this result is optimal because after the adjustment, y_{n^*} does not achieve its maximum value.

Next, in our second step, we focus on the assignment of the y_ns and t_ns found in the first step to x_{ij}^ks. In order to make things easier to handle, in the second step, we allocate nonzero values only to a small number of states. Specifically, when n is odd, we assign nonzero values to states $\left(\frac{n+1}{2}, \frac{n-1}{2}\right)$ and $\left(\frac{n-1}{2}, \frac{n+1}{2}\right)$; when n is even, we allocate y_n to states $\left(\frac{n}{2}+1, \frac{n}{2}-1\right)$, $\left(\frac{n}{2}-1, \frac{n}{2}+1\right)$ and $\left(\frac{n}{2}, \frac{n}{2}\right)$, and we allocate t_n to state $\left(\frac{n}{2}, \frac{n}{2}\right)$ only. The dots in Figure 4 show the states which are assigned non-zero values. All of the remaining states are assigned zero values, which implies that they are transient states with zero steady state probability.

The values of nonzero x_{ij}^ks are determined by the transition equations. First of all, for the nonzero-valued states, we need to make sure that they only transit to other nonzero-valued states. Otherwise, the transition equations for the zero-valued states will not be satisfied. Second, motivated by the symmetric setting of the system, while assigning y_n and t_n to x_{ij}^ks within the groups, we will follow a symmetric allocation as much as possible. Because of these two reasons, for group n where n is odd, we split y_n evenly between $x^1_{\frac{n+1}{2},\frac{n-1}{2}}$ and $x^2_{\frac{n-1}{2},\frac{n+1}{2}}$, and we split t_n evenly between $x^3_{\frac{n+1}{2},\frac{n-1}{2}}$ and $x^3_{\frac{n-1}{2},\frac{n+1}{2}}$ when $t_n \neq 0$. For group n where n is even, we pick $x^1_{\frac{n}{2}+1,\frac{n}{2}-1}$ and $x^2_{\frac{n}{2}-1,\frac{n}{2}+1}$ equal, and equal to $\frac{1}{2}(y_n+t_{n+1})\theta(1-\theta)$. Similarly, we pick $x^2_{\frac{n}{2},\frac{n}{2}}$ and $x^1_{\frac{n}{2},\frac{n}{2}}$ equal and equal to $\frac{1}{2}y_n - \frac{1}{2}(y_n + t_{n+1})\theta(1-\theta)$. Finally, we let $x^3_{\frac{n}{2},\frac{n}{2}} = t_n$, if $t_n \neq 0$.

It is easy to verify that the transition equations for all of the states are satisfied, and x_{ij}^ks are nonnegative as well. In summary, there always exists a feasible allocation to satisfy all of the transition equations with y_ns and t_ns found via the assignment procedure in the first step.

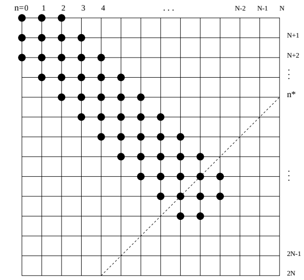

Fig. 4. Allocation pattern within groups.

Since $g_{ij}^k = \frac{x_{ij}^k}{\sum_{k=1}^3 x_{ij}^k}$, once we obtain the values of x_{ij}^ks, we can compute the transmission probabilities g_{ij}^k. Let $\bar{n}$ denote the largest group index n such that $y_n \neq 0$. From our allocation in the first step, we know that $\bar{n}$ equals either n^*+1 or n^*+2, depending the value of Δ. For the states with nonzero-valued x_{ij}^k, if $i+j < \bar{n}$, then, when $i > j$, we have $g_{ij}^1 = 1$; when $i < j$, we have $g_{ij}^2 = 1$; when $i = j$, we have $g_{ii}^1 = g_{ii}^2 = 1/2$ and $g_{ii}^3 = 0$. If $i + j > \bar{n}$, then we have $g_{ij}^3 = 1$. If $i + j = \bar{n}$, the transmission probabilities are determined by the values of $y_{\bar{n}}$, $t_{\bar{n}}$ and $t_{\bar{n}+1}$, and g_{ij}^1, g_{ij}^2, g_{ij}^3 will be arbitrary numbers in $[0, 1]$.

The states with zero-valued x_{ij}^k are transient states. Therefore, the actual values of the transmission probabilities assigned to these states do not impact the stationary distribution, and the average delay. For these transient states, in order to be consistent with the recurrent states, when $i + j \leq \bar{n}$, we simply let $g_{ij}^1 = 1$ if $i > j$, and let $g_{ij}^2 = 1$ if $i < j$; when $i + j > \bar{n}$, we let $g_{ij}^3 = 1$.

Our allocation indicates that there exists a threshold number $\bar{n}$. If the sum of the two queue lengths is greater than $\bar{n}$, both users should transmit during the slot. If the sum of the two queue lengths is less than $\bar{n}$, only the user with the longer queue transmits one packet in a time slot; if in this case both queues have the same length, each queue transmits one packet while the other one keeps silent with probability $1/2$.

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