

# A MODELING APPROACH FOR THE PERFORMANCE MANAGEMENT OF HIGH SPEED NETWORK <sup>1</sup>

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## INTRODUCTION

As data networks have become larger, higher speed and more complex, there has arisen a growing need for advanced modeling, management and planning tools which will assist network operators maintain and improve network performance. This paper describes the analytical techniques used in a system tool that will simulate and analyze packet switched networks carrying bursty traffic and which can be used for a variety of networks and services such as frame relay, SMDS or B-ISDN. This network tool is based upon models of network elements. The information needed to drive the tool and describe its elements will vary from detailed to sketchy. Some information will be derived from historical measurements and some from expected characteristics of users based on responses to a questionnaire answered at the time of subscription to the data service.

By using the tool based on the analytical methods described in this paper, operators will be able to alter network elements and configurations in the model, analyze performance data, and then identify the consequences of network changes, including the evaluation of proposed configurations. The tool will also be used to compare the performance of different networks, to compare different configurations of the same network, and to evaluate network performance during specified intervals. In this paper, we describe the analytical models that were developed for this tool.

## A PACKET SWITCHED NETWORK

In general a packet switched network can be represented as a set of nodes interconnected by a set of links. This gives a topological characterization of the network. Customers can send traffic over the network by connecting to one of the nodes via an input interface and sending messages to an output interface at the same or some other node. This is illustrated in Figure 1. The nodes are the network switches, the links are the trunks connecting them or to customer access lines.

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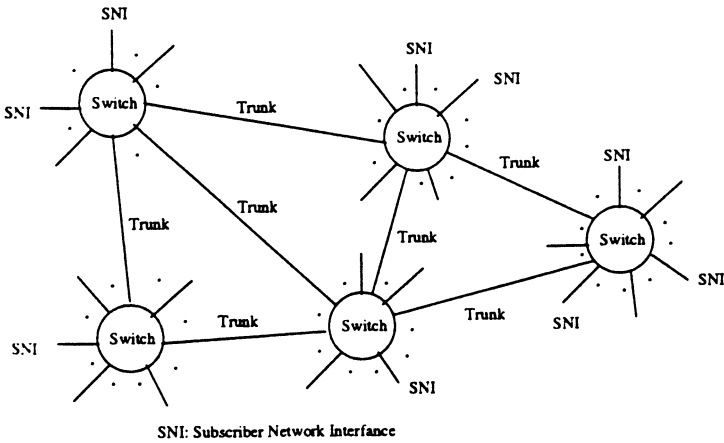


Figure 1. A Packet Switched Network

The input parameters that affect the performance of the entire network are described in items 1-3 below.

1. The traffic requirements that the entire network must accommodate. This can be described by a traffic matrix

$$\bar{R} = \begin{bmatrix} \bar{R}_{11} & \bar{R}_{12} & \dots & \bar{R}_{1N} \\ \bar{R}_{21} & \bar{R}_{22} & \dots & \bar{R}_{2N} \\ \vdots & \vdots & \dots & \vdots \\ \bar{R}_{N1} & \bar{R}_{N2} & \dots & \bar{R}_{NN} \end{bmatrix}$$

where the sub matrix  $\bar{R}_{\ell n} = [\bar{R}_{\ell n}(i, j)]$  is itself a traffic matrix whose elements  $\bar{R}_{\ell n}(i, j)$  give the average rate of traffic entering the network at the  $i$ th interface module of switch  $\ell$  which must be delivered to the  $j$ th interface module of switch  $n$ .

2. The burstiness of the arriving traffic. This can be characterized in a number of ways. A characterization of burstiness includes some statistical description of idle and bursty periods and the intensity of bursts. One way it can be characterized is in terms of the ratio of the variance to the squared mean of the inter-arrival times. This parameter  $v_a^2$ , called the squared coefficient of variation for inter-arrivals, is used by the QNA method[1][2] discussed later. An alternate description is in terms of the three parameters peak rate  $R_{peak}$ , utilization  $\rho$  and the average burst length  $b$ . This three parameter approach is used by the equivalent capacity method[3].
3. The associated packet length distribution matrices

$$\bar{L} = \begin{bmatrix} \bar{L}_{11} & \bar{L}_{12} & \dots & \bar{L}_{1N} \\ \bar{L}_{21} & \bar{L}_{22} & \dots & \bar{L}_{2N} \\ \vdots & \vdots & \dots & \vdots \\ \bar{L}_{N1} & \bar{L}_{N2} & \dots & \bar{L}_{NN} \end{bmatrix}$$

which gives the packet lengths for the traffic above and a corresponding matrix of squared coefficients of variation for packet lengths.

By routing the traffic load, described above, through the network we can obtain the traffic load on each of the  $N$  switches. In order to perform this routing it is necessary to first model the delays and packet losses associated with the switches.

# CLASSIFICATION AND CHARACTERIZATION OF TRAFFIC

In high-speed packet switched network architectures, several classes of traffic streams with widely varying traffic characteristics are statistically multiplexed and share common switching, buffering and transmission resources. Because of the potentially dramatic differences in the statistical behavior of connections and the associated problems posed for bandwidth management and traffic control, we will classify the different users and applications that require packet switching service.

We will identify a number of different classes of traffic, which should include most network users (either directly or as a mixture of such traffic types, so that their overall traffic will be determined by the percentage of each of the different classes they use), according to their traffic patterns, behavior and characteristics. For each of these classes we will determine traffic parameters which will help us to develop a realistic model for the source traffic in the analytical model. We present a number of services in association with different users that require packet switching service. We identify expected traffic patterns, behavior and characteristics for these users.

**1. LAN Interconnection Traffic:** LANs support a wide variety of users and applications, with different performance requirements and traffic characteristics. Possible applications include distributed file systems and databases, host-peripheral connections, demand paging, parallel processor interconnection, computer servers, disk less work stations, electronic mail, file transfer, etc. These applications are supported by the Network Disk (ND) protocol, the TCP and the UDP protocols among others. According to recent results of LAN traffic analysis studies conducted at Bellcore[4][5][6], the protocols above have the following traffic characteristics.

Protocol	Traffic
TCP	80% of 50 bytes, 10% of 500 bytes, 10% of 1Kbyte
UDP	80% of 150 bytes, 10% of 1.5 Kbytes
ND	80% of 1550 bytes, 20% of 50 bytes

The utilization due to ND traffic is comparable to that of UDP traffic, while TCP traffic represents a small fraction of the total network traffic. The combination of the above traffic types results in a "trimodal" distribution for the overall traffic. The largest percentage of packets are less than 200 bytes in size while there are considerable spikes at lengths of 1 Kbyte and 1550 bytes.

## 2. Video applications

**Video Telephony, Multimedia Teleconferencing:** The data rate ranges from 1.5 Mbits/sec to 140 Mbits/sec depending on the encoding, the compression, and the service quality requirements.

**Compressed Real Time Packet Video:** Transmission of good quality compressed video requires at least 1.5 Mb/sec with small video packet loss and delay [7]. Given the study in reference [8] and for 1.5 Mbits/sec transmission rates, we conclude that a video frame can be segmented into 4 to 5 packets of 1500 bytes with inter-arrival times of approximately 8 msec.

**3. Teletex( Correspondence exchange):** With a transmission rate of 9.6 Kbps, page data volume of 20 Kbits and packet sizes of 1500 bytes, if we consider one page as a burst, then the mean burst period is about 2 secs while the packet inter-arrival time is 1 sec.

4. **Facsimile:** A bit rate of 2 Mb/s (for compressed data) and more (for uncompressed data) is desirable in order to achieve acceptable transmission times for compatibility with the operation speeds of document preparation equipment of the near future.

5. **High-resolution graphics – Electronic Imaging (Medical Imaging, CAD-CAM applications):** Based on the typical transmission speed of work stations which is about 0.5 Mb/s and using as a packet size the maximum allowable Ethernet packet of about 1530 bytes, while assuming that a single image or graph represents a burst, we conclude that the burst length is on the order of minutes and the mean packet inter-arrival time is approximately 25 msec.

## THE QNA METHOD OF NETWORK ANALYSIS

The Queuing Network Analyzer (QNA) is a software tool developed at AT&T Bell Telephone Laboratories to evaluate the performance of a queuing network. It is based on work described by W. Whitt [1][2]. Each node is viewed as a single GI/G/m queue. The analysis is approximated by considering the first and second moments of the inter-arrival time, packet length and service time distribution. It gives an approximate performance measure at the network level, instead of investigating the performance of particular queues at each node. We decided to use this approach in our modeling effort because it allows the characterization of traffic merged from many component streams as well as traffic split from a common stream.

The parameters that characterize the flow of packets to a switch are the means and squared coefficients of variation of the inter-arrival time and packet length; namely  $E\{T_a\}$  (or the average arrival rate  $\lambda = 1/E\{T_a\}$ ),  $E\{L\}$ ,  $v_a^2$  and  $v_L^2$ . Here  $v^2$  is the variability parameter or squared coefficient of variation, which is defined to be  $\text{Var}(T)/E^2(T)$ . The service is characterized by an average service time  $E\{X\}$  (or the average service rate  $\mu = 1/E\{X\}$ ) and  $v_X^2$ . The service discipline is assumed to be first-in-first-out.

The basic methodology is to merge all traffic flows (characterized by the above four parameters) entering an input interface of a node or queue into one combined flow. After service, the parameters of the departure flow, different from those of the input, can also be approximately characterized. This four parameter characterization can be used for all flows within the network. The output of QNA gives the mean and variance of the delay at each queue by using approximate GI/G/m formulas.

Let

$W$  be the packet waiting time of a queue;

$\lambda$  be the packet arrival rate to the queue;

$X$  be the packet service time of the queue, which is equal to the packet length  $L$  (in bits or bytes) divided by the service rate  $R_s$  (in bits or bytes per second) of the queue,  $L/R_s$ ;

$\rho = \lambda E\{X\}$  be the load on the queue;

$v_T^2$  be the squared coefficient of variation of the packet inter-arrival time;

$v_X^2$  be the squared coefficient of variation of the packet service time, which is equal to that of the packet length  $v_L^2$ .

The mean waiting time of the queue is approximated by

$$E\{W\} = g \frac{\rho E\{X\} (v_T^2 + v_X^2)}{2(1 - \rho)} \quad (1)$$

where  $g = \exp\left(-\frac{2(1 - \rho)(1 - v_T^2)^2}{3\rho(v_T^2 + v_X^2)}\right)$  if  $v_T^2 < 1$  and  $g = 1$  if  $v_T^2 > 1$ , is a function of the traffic and service parameters.

Once the traffic parameters for a queue have been estimated, we can use formula (1) to determine an approximation to the mean waiting time. After the queue service, we want to know the departure traffic parameters, which are input parameters to the next queue. If there is no loss in the queuing system, the packet rate, the mean packet length and the variability parameter of the packet length are the same as those of the input traffic parameters, but the variability parameter of the packet inter-departure time,  $v_D^2$ , is changed. This can be estimated by Marshall's formula [9],

$$v_D^2 \approx v_T^2 + 2\rho^2 v_X^2 - \frac{2\rho(1-\rho)E\{W\}}{E\{X\}} \quad (2)$$

which can be further approximated as

$$v_D^2 \approx (1-\rho^2)v_T^2 + \rho^2 v_X^2 \quad (3)$$

simply by letting  $g = 1$  in the estimation of  $E\{W\}$  and substituting it into (2). Equation (3) is the approximation used by [1].

If the departure traffic is split into many traffic streams after the queue service, the QNA method also gives a way to calculate the parameters for the split traffic stream. The packet rate, packet length and the variability parameter of the packet length of each stream are easily calculated, because those parameters are dependent on the traffic to each stream and independent of the queue server. Again, we have to estimate the variability parameter of the packet inter-arrival time to each stream. Suppose that a packet goes to the  $i$ th output stream with probability  $P_i$ , then, the variability parameter of the packet inter-arrival time,  $v_{T_i}^2$ , is

$$v_{T_i}^2 = 1 - P_i + P_i v_D^2 \quad (4)$$

These parameter estimation methods and the method for combining traffic are the basic techniques used in the QNA method. We will refer to these formulas quite often in the later discussion.

## MODELING DELAY

Many switches are based on shared bus or polling architectures and can be modeled by using a round robin service discipline (i.e., the server visits each station in order and serves at most one packet during each visit). A typical switch of this type, used for Frame Relay Service, is shown in Fig.2. It consists of three parts: an input module, a switch module and an output module. An input packet (or a frame of data) from a Customer Premises Equipment (CPE) access line or a trunk connected to the switch is detected and verified by an Access Processor (AP) in the input module. Valid packets are forwarded to the transmit buffer of the cyclic server and wait for the switch server to switch them to their destination output module. Packets arriving at the output module are stored in the transmit cache for the AP to perform the HDLC processing. They are then multiplexed and transferred to the output line. The switch discipline is round robin and at most one packet is switched when a queue is polled. In the output module, the packet service rate depends mainly on the transmission rate of the output line. A queuing model for the switch module is shown in Fig. 3.

There are three queues which must be modeled; the input queue, the switch queue and the output queue. There are also some packet transfer delays in or between the modules which are proportional to the packet size and can be placed anywhere in the queuing model. For simplicity, we place all transfer delays which occur before the switch module into the input module and the transfer delays which occur after the switch module into the output module.

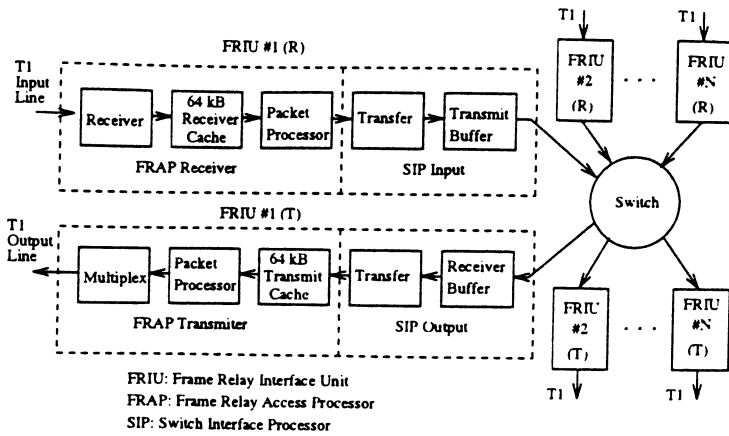


Figure 2. Switch Transport Process

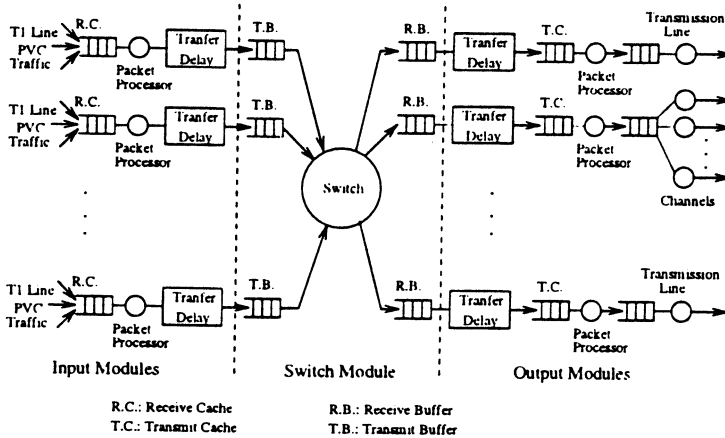


Figure 3. Equivalent Model for the Switch

The transfer delays at the input and output modules are assumed to be equal, because similar data transfers are necessary before and after the switch module.

### Input Traffic Description

Traffic to a switch node either comes from a previous node through a trunk or enters at the node through an access line from CPEs. A source CPE sends its traffic through the network via a Permanent Virtual Circuit (PVC) to the destination CPE. PVC  $k$  (denoted by  $PVC_k$ ), which originates at input line (or channel)  $i$  of a node and goes to output line (or channel)  $j$ , has four parameters: the packet arrival rate  $\alpha_k$ , the squared coefficient of variation of packet inter-arrival times  $v_{A_k}^2$ , the mean packet length  $E\{L_k\}$  and the squared coefficient of variation of the packet length  $v_{L_k}^2$ . Those four parameters are observed before the traffic is transmitted to the input module, hence, when many PVCs share an input line, the input traffic is the PVCs' combined traffic with four parameters: the packet arrival rate  $\lambda_{1i}$ , the squared coefficient of variation of the packet inter-arrival time  $v_{T_{1i}}^2$ , the mean packet length  $E\{L_{1i}\}$  and the squared coefficient of variation of the packet length  $v_{L_{1i}}^2$ . A simple

way to estimate the parameters of the combined traffic is by the QNA method [1].

If an input line is a trunk or a full access line, all PVCs coming through the line are merged in the line with a line service rate of 1.344 Mbps (for a T1 one line).

When PVCs come through channel  $m$  of input module  $i$ , similarly, the combined four parameters,  $\lambda_{1im}$ ,  $E\{L_{1im}\}$ ,  $v_{T_{1,m}}^2$  and  $v_{L_{1,m}}^2$  for that channel can be found by considering each channel as an input line with the service rate of the channel. The channel traffic streams are further combined and form the traffic of input module  $i$ . Thus, there are two merges for the input traffic through channelized lines.

### Delay Model of the Input Module

An input module  $i$  has a packet processor, which processes each incoming packet with a constant processing time  $E\{X_{1i}\}$ . The load on the queue is  $\rho_{1i} = \lambda_{1i}E\{X_{1i}\}$ . The waiting time of the packet processor is<sup>1,2</sup>

$$E\{W_{1i}\} = g_{1i} \frac{\rho_{1i}E\{X_{1i}\}(v_{T_{1i}}^2 + v_{X_{1i}}^2)}{2(1 - \rho_{1i})} \quad (5)$$

where  $g_{1i} = \exp\left(-\frac{2(1 - \rho_{1i})(1 - v_{T_{1i}}^2)^2}{3\rho_{1i}(v_{T_{1i}}^2 + v_{X_{1i}}^2)}\right)$  if  $v_{T_{1i}}^2 < 1$  and  $g_{1i} = 1$  if  $v_{T_{1i}}^2 > 1$  and the variability parameter  $v_{X_{1i}}^2 = 0$  in this case.

The transfer delay is proportional to the packet size. Let the transfer rate be  $R_t$ , which depends on the switch fabric and can be measured or calculated on a real system. Because the transfer rate is typically very high, there is no queuing delay but there is a delay in addition to the packet delay. If there are  $M$  transfers in the input module, the total input transfer delay is  $ML/R_t$ , where  $L$  is the packet length.

The packet departure rate and the packet length parameters after the processor are the same as for the input to the processor, but the squared coefficient of variation of the inter-arrival time has been changed. By the QNA method, the variability parameter,  $v_{T_2}^2$ , is estimated by

$$v_{T_2}^2 \approx (1 - \rho_{1i}^2)v_{T_{1i}}^2. \quad (6)$$

The four parameters for the input to the switch are then determined.

### The Switch Module

The switch module is modeled as a non-exhaustive cyclic queue with at-most-one packet served per polling of the queue. In this part, we combine some basic methods and try to develop a better delay model. We first discuss the switch model for Poisson input traffic and then modify it for general input traffic.

#### The Conditional Cycle Times

Define  $C_i''$  to be the conditional cycle time random variable for queue  $i$ , given that a packet from queue  $i$  is served in the cycle;  $C_i'$  to be the conditional cycle time random variable for queue  $i$  given that no packet from queue  $i$  is served in the cycle.

Kuehn [10] has approximate formulas for mean cyclic times.  $c_i'' = E\{C_i''\}$  and  $c_i' = E\{C_i'\}$ . Let  $c_{\sim i}''$  and  $c_{\sim i}'$  be Kuehn's approximate values, then

$$c_{\sim i}'' = \frac{s_0 + h_i}{1 - \rho_0 + \rho_i} \quad (7)$$

$$c'_{\sim i} = \frac{s_0}{1 - \rho_0 + \rho_i} \quad (8)$$

where  $\rho_i = \lambda_{2i} h_i$  is the load contributed to the switch by the queue  $i$  traffic and  $\rho_0 = \sum_{i=1}^N \rho_i$  is the total switch load for  $N$  inputs.  $\lambda_{2i}$  is the packet arrival rate to the switch input queue  $i$ , which is equal to  $\lambda_{1i}$ , if there is no packet loss at the input module;  $h_i$  is the average packet service time, which is equal to the packet length  $E\{L_{1i}\}$  over the switch rate  $R_{w_i}$ ; and  $s_0$  is the sum of the mean polling times  $s_i$  for each queue  $i$ .

Equations (7) and (8) give extreme approximations for the conditional cycle times. If  $c_0 = s_0/(1 - \rho_0)$  is the average cycle time, the relationship among the cycle times is

$$c''_{\sim i} \geq c'_i \geq c_0 \geq c'_i \geq c'_{\sim i}. \quad (9)$$

We have found another approximation method for  $c''_i$  and  $c'_i$ . Let  $c''_{\sim i}$  and  $c'_{\sim i}$  be these approximate values where

$$c''_{\sim i} = s_0 + h_i + \sum_{j \neq i} \delta''_j h_j \quad (10)$$

$$c'_{\sim i} = s_0 + \sum_{j \neq i} \delta'_j h_j. \quad (11)$$

Here  $\delta''_j = \lambda_{2j} c''_{\sim j}$  is the probability that a packet from queue  $j$  is served in the cycle, given that a packet from queue  $j$  is served in the previous cycle;  $\delta'_j = \lambda_{2j} c'_{\sim j}$  is the probability that a packet from queue  $j$  is served in the cycle, given that no packet from queue  $j$  is served in the previous cycle.

The solutions to the linear equations (10) and (11) are

$$c''_{\sim i} = \frac{s_0 + h_i(1 - d_1) + d_3}{1 - \rho_0 + \rho_i(1 - d_1) + d_2} \quad (12)$$

$$c'_{\sim i} = \frac{s_0}{1 - \rho_0 + \rho_i(1 - d_1) + d_2} \quad (13)$$

where  $d_1 = \sum_{j=1}^N \frac{\rho_j}{1 + \rho_j}$ ;  $d_2 = \sum_{j=1}^N \frac{\rho_j^2}{1 + \rho_j}$  and  $d_3 = \sum_{j=1}^N \frac{h_j \rho_j}{1 + \rho_j}$ . Equations (12) and (13) also are upper and lower bound approximations for the conditional cycle times.

By combining Kuehn's approximations and our approximations, a better approximation for the conditional cycle times can be acquired. Let  $\epsilon''_{ij} = \min(\lambda_{2i} c''_{\sim j}, \lambda_{2i} c'_{\sim j}, \delta''_i, 1)$  and  $\epsilon'_{ij} = \min(\max(\lambda_{2i} c'_{\sim j}, \lambda_{2i} c''_{\sim j}, \delta'_i), 1)$ , which gives a better approximation for the conditional probabilities that a packet from queue  $j$  is served in a cycle, given that a packet from queue  $i$  is served in the cycle or given that no packet from queue  $i$  is served in the cycle. The approximations for the cycle times are

$$c''_{\underline{i}} = s_0 + h_i + \sum_{j \neq i} \epsilon''_{ij} h_j \quad (14)$$

$$c'_{\underline{i}} = s_0 + \sum_{j \neq i} \epsilon'_{ij} h_j \quad (15)$$

which give better approximations for the conditional cycle times, and can improve the queuing delay approximation.

Let  $c''_{\underline{i}(2)}$  and  $c'_{\underline{i}(2)}$  be the second moments of these conditional cycle times, then from [10], we get

$$c''_{\underline{i}(2)} = \sum_{j=1}^N (s_j^{(2)} - s_j^2) + h_i^{(2)} - h_i^2 + \sum_{j \neq i} (\epsilon''_{ij} h_j^{(2)} - \epsilon''_{ij}{}^2 h_j^2) + c''_{\underline{i}}{}^2 \quad (16)$$



$$c_{\underline{a}_i}^{(2)} = \sum_{j=1}^N (s_j^{(2)} - s_j^2) + \sum_{j \neq i} (\epsilon'_{ij} h_j^{(2)} - \epsilon_{ij}^2 h_j^2) + c_{\underline{a}_i}^{\prime 2} \quad (17)$$

where  $s_j^{(2)}$  is the second moment of the polling time and  $h_j^{(2)}$  is the second moment of the packet service time for the  $i$ th queue of the switch, which is the second moment of  $L_{1i}/R_w$ .

In the above discussion, we assume that the switch is stable. For unstable systems, an additional step must be taken to identify the unstable queues and treat them differently.

### The Mean Waiting Time of the Cyclic Queues

The mean waiting time of Poisson input traffic queuing systems depends on the mean residual time and the traffic load. A packet upon arrival to an input queue will find either that there is a head-of-line (HOL) packet or there is none. It sees different residual times in those two cases. Based on the renewal theory and M/G/1 queue theory, we can derive Kuehn's formula for the mean waiting time for queue  $i$ ,

$$E\{W_{2i}\} = \frac{c_i^{\prime(2)}}{2c_i'} + \frac{\lambda_{2i} c_i^{\prime\prime(2)}}{2(1 - \lambda_{2i} c_i^{\prime\prime})} \quad (18)$$

Kuehn used his approximations for the first and second moment of the cycle times and had a delay approximation formula.

Another method developed by Boxma and Meister [11] is based on two assumptions:

1.  $\rho_i = \lambda_{2i} c_{\sim i}^{\prime\prime}$  is the utilization observed at queue  $i$ ;
2. All arrival packets see approximately the same mean residual time  $r$ .

The second assumption is good if the traffic load is light or the queues are not very unbalanced, otherwise a large error exists. Based on those assumptions, the mean waiting time is approximated by

$$E_{BM}\{W_{2i}\} = \frac{r}{1 - \lambda_{2i} c_{\sim i}^{\prime\prime}} \quad (19)$$

where  $r$  is a parameter to be determined. Boxma and Meister used the conservation law, which was first developed by Watson [12], to evaluate  $r$ . This gives  $r$  and the waiting time approximation for queue  $i$  as

$$r \approx \frac{1 - \rho_0}{(1 - \rho_0)\rho_0 + \sum_{i=1}^N \rho_i^2} C_{NE} \quad (20)$$

$$E_{BM}\{W_{2i}\} \approx \frac{1 - \rho_0 + \rho_i}{1 - \rho_0 - \lambda_{2i} s_0} r \quad (21)$$

where

$$C_{NE} = \rho_0 \frac{\sum_{i=1}^N \lambda_{2i} h_i^{(2)}}{2(1 - \rho_0)} + \rho_0 \frac{s_0^{(2)}}{2s_0} + \frac{s_0}{2(1 - \rho_0)} (\rho_0 + \sum_{i=1}^N \rho_i^2).$$

Equation (21) gives a closed form formula for the queue waiting time approximation which, for moderate switch loads and slightly unbalanced queues, gives more accurate results than the approximation for (18) found by Kuehn's method. Later, some numerical results will show that when the switch load is high and queues are very unbalanced (the load of the maximally loaded queue is more than twice the load of the minimally load queue), large errors can exist, which are even worse than Kuehn's approximation. The errors produced are due to the two assumptions. If either one could be improved, the errors would be reduced.

The first method we introduced is to use  $c_{\sim i}^{\prime\prime}$  as the approximation for the conditional cycle time  $c_i^{\prime\prime}$  instead of  $c_{\sim i}^{\prime\prime}$  in (19), then the residual time and the mean waiting time are approximated by

$$r_0 \approx \frac{(1 - \rho_0)C_{NE}}{\sum_{i=1}^N \frac{\rho_i(1 - \rho_0 - \lambda_{2i}s_0)}{1 - \lambda_{2i}c''_{\pm i}}} \quad (22)$$

$$E_1\{W_{2i}\} \approx \frac{r_0}{1 - \lambda_{2i}c''_{\pm i}}. \quad (23)$$

Equation (23) gives a better approximation than (21), especially when the switch load is high and queues are highly unbalanced, due to a more accurate approximation for the mean cycle time  $c''_i$ .

Table 1. Waiting Times for the First Queue

Load	Simulation	Method 1	Method 2	BM	Kuehn
0.2	.4047(.012)	.4077(.74)	.4135(2.1)	.4073(.64)	.3860(-4.6)
0.4	.9959(.029)	.9975(.16)	1.026(3.0)	.9932(-.27)	.8683(-13.)
0.6	2.548(.068)	2.479(-3.)	2.562(.54)	2.436(-4.4)	1.941(-3.1)
0.8	11.13 (.25)	10.40 (-3.)	10.62(-2.)	9.602(-10.)	6.917(-35.)
0.85	22.18 (.65)	21.62(-2.5)	21.94(-.1)	18.82(-15.)	13.43(-39)
0.9	139.9(23)	163.2 (17.)	164.4(18.)	95.7(-32)	91.7(-34)

Table 2. Waiting Times for the Second Queue

Load	Simulation	Method 1	Method 2	BM	Kuehn
0.2	.3814(.013)	3.800(-.5)	.3766(-1.3)	.3801(-.34)	.3576(-6.2)
0.4	.8225(.031)	.8383(1.9)	.8219(-.07)	.8411 (2.3)	.7282(-11)
0.6	1.651(.044)	1.735(5.1)	1.692(2.48)	1.768(6.7)	1.421(-16.)
0.8	3.469(.086)	4.325(25.)	4.272(23.5)	4.935(42.)	2.800 (8.7)
0.85	4.255(.038)	5.959(40.)	5.931(39.0)	7.790(83.1)	5.856(37.6)
0.9	5.335(.056)	9.006(69.)	9.069(70.)	22.83(328.)	11.88(123.)

Table 3. Waiting Times for the Third and Fourth Queues

Load	Simulation	Method 1	Method 2	BM	Kuehn
0.2	.3780(.013)	.3665(-3.0)	.3584(-5.2)	.3671(-2.9)	.3413(-9.7)
0.4	.7487(.024)	.7655(-2.2)	.7277(-2.8)	.7709(-2.9)	.6478(-13.)
0.6	1.307(.023)	1.441(10.3)	1.336(-2.2)	1.484(13.5)	1.140(-13.)
0.8	2.259(.022)	2.906(28.6)	2.655(17.5)	3.422(54.8)	2.567(13.6)
0.85	2.569(.024)	3.621(40.9)	3.314(29.)	4.911(91.2)	3.653(42.2)
0.9	2.910(.027)	4.723(62.)	4.351(50.)	12.31(323.)	6.270(115.)

A second method we derived is to use equation (18) as a preliminary approximation for the mean waiting time by substituting  $c''_i$ ,  $c'_i$ ,  $c''_i(2)$  and  $c''_i(2)$  with  $c''_{\pm i}$ ,  $c'_{\pm i}$ ,  $c''_{\pm i}(2)$  and  $c''_{\pm i}(2)$ . Let  $E_K\{W_{2i}\}$  be this preliminary approximation for the waiting time of the  $i$ th input line. We then scale  $E_K\{W_{2i}\}$  by a factor  $b$  so that Watson's conservation law is satisfied, to obtain  $E_2\{W_{2i}\}$ , the corresponding approximation of the mean waiting time.

$$E_2\{W_{2i}\} = b \cdot E_K\{W_{2i}\} \quad (24)$$

where  $b$  is given below,

$$b = \frac{C_{NE}}{\sum_{i=1}^N \rho_i(1 - \lambda_{2i}c_0)E_K\{W_{2i}\}}. \quad (25)$$

When  $b$  is found, the mean waiting time  $E_2\{W_{2i}\}$  is given by equation (24). In most situations, the second method gives better approximations than the other methods.

Tables 1, 2, 3 and 4 show some numerical results for delay in a 4 queue system with constant polling times  $s_i = 0.05$  and exponential packet service times with means  $h_i = 1$ . The arrivals to the queues are Poisson with average rates  $\lambda_1 = 2\lambda_2 = 4\lambda_3 = 4\lambda_4$ . These values were chosen to yield an unbalanced system for which the load at queue 1 is twice that at queue 2 and four times the load at queues 3 and 4. Tables 1, 2 and 3 give the waiting time of the first queue, the second queue, and the third and fourth queues respectively. The first column is the total switch load for  $N$  inputs. Simulation results from a special purpose simulation program are given in the second column, with the 95th percentile confidence interval range shown in parentheses. The mean waiting time delay for each method and their percentage errors (compared with the simulation results) are shown in the remaining columns. The results show that improved methods one and two give better waiting time approximations than both Boxma-Meister's method and Kuehn's method. The difference between method 1 and 2 is not large.

In the above discussion, the mean waiting times of the cyclic server queues are for Poisson arrival traffic. If the input traffic is general with two parameters,  $\lambda_{2i}$  and  $v_{T_{2i}}^2$ , the formula for the waiting time must be modified. According to the GI/G/1 theorem<sup>1</sup>, the mean waiting time,  $E_G\{W_{2i}\}$ , for general input traffic can be approximated by

$$E_G\{W_{2i}\} = g_{2i} \left( \frac{c_{2i}'' \rho_{2i} (v_{T_{2i}}^2 - 1)}{2(1 - \rho_{2i})} + E_2\{W_{2i}\} \right) \quad (26)$$

where  $g_{2i} = \exp\left(-\frac{2(1 - \rho_{2i})(1 - v_{T_{2i}}^2)^2}{3\rho_{2i}(v_{T_{2i}}^2 + v_{L_{2i}}^2)}\right)$  if  $v_{T_{2i}}^2 < 1$  and  $g_{2i} = 1$  if  $v_{T_{2i}}^2 > 1$ ;  $\rho_{2i} = \lambda_{2i} c_{2i}''$  is the utilization observed at queue  $i$ . Equation (26) gives us the waiting time for general input traffic.

Table 4 shows the results for the same system as Tables 1 to 3 except that the arrivals are not Poisson. The inter-arrival times have a squared coefficient of variation equal to 2. For simulation, a hyper-exponential distribution was used to generate the inter-arrival times. No comparison can be made to the Kuehn and Boxma-Meister results since these are not applicable to non-Poisson traffic. The results are not as accurate as for the Poisson arrival case, but are adequate for engineering purposes.

Table 4. Waiting Times for Non-Poisson Arrivals

Load	Q1 Sim	Method 1	Method 2	Q2 Sim	Method 1	Method 2	Q3,4 Sim	Method 1	Method 2
0.2	.560(.015)	.510(-8.9)	.516(-7.9)	.446(.016)	.433(-2.9)	.430(-3.7)	.407(.024)	.393(-3.4)	.385(-5.4)
0.4	1.66(.067)	1.32(-21)	1.35(-19)	1.11(.023)	1.01(-9.0)	.990(-11)	.887(.016)	.850(-4.1)	.812(-8.4)
0.6	5.74(.226)	3.39(-41)	3.47(-40)	2.74(.084)	2.19(-20)	2.14(-22)	1.71(.032)	1.66(-2.9)	1.55(-9.1)
0.8	29.7(6.17)	14.4(-52)	14.6(-51)	6.98(.628)	5.73(-18)	5.67(-19)	3.24(.094)	3.47(7.1)	3.22(-62)
0.85	61.3(8.31)	29.8(-51)	30.1(-51)	9.48(.451)	7.96(-16)	7.93(-16)	3.80(.089)	4.36(15)	4.05(6.6)

## Model of the Output Module

Packets coming to the output module go through two tandem queues for service. One is for packet processing and the other is at the output line. Packet processors perform the same function as in the input queue module. The output line is a T1 line, which may be partitioned into channels. Therefore, there are three possible transmission rates, 56 kbps, 384 kbps or 1.344 Mbps for the output transmission. The queuing model is simple. The only problem is to calculate the four incoming traffic parameters to each output module. We will estimate the traffic parameters to the output queues in the following subsections.

Let:

- $\lambda_{3j}$  be the packet arrival rate to the packet processor queue of the output module  $j$ ;
- $E\{L_{3j}\}$  be the average packet length of the traffic to the queue;
- $v_{L_{3j}}^2$  be the squared coefficient of variation of the packet length;
- $v_{T_{3j}}^2$  be the square coefficient of variation of the packet inter-arrival time to the queue.

Still assuming that all packets destined to output queue  $j$  arrive there without loss, then from [1], we get

$$\lambda_{3j} = \sum_{PVC_k \in \text{output } j} \alpha_k \quad (27)$$

$$E\{L_{3j}\} = \sum_{PVC_k \in \text{output } j} E\{L_k\} \alpha_k / \lambda_{3j} \quad (28)$$

$$v_{X_{3j}}^2 = v_{L_{3j}}^2 = \sum_{PVC_k \in \text{output } j} \frac{\alpha_k E^2\{L_k\} (1 + v_{L_k}^2)}{\lambda_{3j} E\{L_{3j}\}^2} - 1 \quad (29)$$

These three parameters are calculated independently of the previous queue information, but  $v_{T_{3j}}^2$  depends on that information and requires more calculation. It is derived in the following steps.

a) Let  $v_{SW_i}^2$  be the squared coefficient of variation of the input queue  $i$  packet inter-departure time at the output of the switch module. Then,

$$v_{SW_i}^2 = \rho_{2i}^2 v_{X_{2i}}^2 + (1 - \rho_{2i}^2) v_{T_{2i}}^2. \quad (30)$$

b) Traffic from input queue  $i$  will split among the output queues. The probability,  $\gamma_{ij}$ , that a packet from queue  $i$  goes to output queue  $j$  is, approximately,

$$\gamma_{ij} = \frac{\sum_{PVC_k \in (i,j)} \alpha_k}{\lambda_{2i}} \quad (31)$$

where  $(i, j)$  is defined to be a path from input queue  $i$  to output queue  $j$ . Thus, the variability parameter,  $v_{SW_{i,j}}^2$ , of the packet inter-arrival time from input queue  $i$  to output queue  $j$  is,

$$v_{SW_{i,j}}^2 = 1 - \gamma_{ij} + \gamma_{ij} v_{SW_i}^2. \quad (32)$$

c) The traffic from input queues to the output queue  $j$  is combined and forms the traffic to the output module. If we assume that the traffic from different queues are independent, we can use the QNA's traffic-merge method to combine the traffic.

Let  $\theta_{ij}$  be the fraction of traffic from input queue  $i$  to output queue  $j$ , that is

$$\theta_{ij} = \frac{\sum_{PVC_k \in (i,j)} \alpha_k}{\lambda_{3i}}. \quad (33)$$

Hence, the variability parameter of inter-arrival to the output module  $j$  is

$$v_{T_{3j}}^2 = (1 - w_{3j}) + w_{3j} \sum_{i=1}^N (\theta_{ij} v_{SW_{i,j}}^2) \quad (34)$$

where  $w_{3j} = 1/[1 + 4(1 - \rho_{3j})^2((\sum_{i=1}^N \theta_{ij}^2)^{-1} - 1)]$  and  $\rho_{3j} = \lambda_{3j} E\{X_{3j}\} = \lambda_{3j} E\{X_{1j}\}$  is the load of the output processor  $j$ . The input traffic parameters to the output packet processor are then completely specified by formulas (27) through (34).

## Delay of the Output Module

There are five sources of delay for an output module: packet processing delay, waiting time delay in the receive cache, packet transfer delay, waiting time delay for multiplexing and transmission and the transmission delay.

The first queue in the output module is the packet processor queue, which performs the same function as in the input queue. Changing the subscript  $1i$  of (5) into  $3j$ , gives the formula for the mean waiting time,  $E\{W_{3j}\}$ , of the output packet processor. The transfer delay is the same as that of the input module. The packet inter-departure variability parameter,  $v_{T_{4j}}^2$ , after the packet processor can be calculated, which is an input parameter to the transmission queues. Due to the lossless assumption, the other parameters for transmission queue  $j$  (i.e.,  $\lambda_{4j}$ ,  $E\{L_{4j}\}$  and  $v_{L_{4j}}^2$ ) are the same as those for the input to the output packet processor (i.e., the values given by (27) to (29)).  $E\{W_{4j}\}$  is then calculated using (5) with these parameter values.

The last queue of the frame relay switch is at the output transmission line. The transmission line can be a full T1 line or channelized. In the former case, the arrival traffic is known, it is a simple GI/G/1 queue; in the later case, the traffic to output module  $j$  is split onto the channels. We consider that it is randomly split. The QNA method can be applied to calculate the parameters and delays of the traffic streams.

### Switch Delay of PVC Packets

The PVC packet delay through the switch is defined to be the interval from the time a PVC packet arrives to the input module of the switch till it leaves the output module of the switch. If  $PVC_k$  goes through the switch via input module  $i$  and the output module  $j$ , the average switch delay,  $E\{D_k\}$ , of the  $PVC_k$  packets is the sum of the waiting time delay and the service time delay of the queues it passes through in the switch. The average waiting time delay is the same for any PVC going through the same path; but the service time delay is different for PVCs whose traffic has different average packet lengths. Let  $E\{W_{ij}\}$  be the waiting time delay of the path from input module  $i$  to output module  $j$  and  $E\{S_k\}$  be the average service time delay for  $PVC_k$  packets going through the switch. Then, if  $PVC_k$  is via path  $(i, j)$ , the  $PVC_k$  packet delay is,

$$\begin{aligned} E\{D_k\} &= E\{W_{ij}\} + E\{S_k\} \\ &= E\{W_{1i}\} + E_G\{W_{2i}\} + E\{W_{3j}\} + E\{W_{4j}\} \\ &\quad + E\{X_{1i}\} + E\{X_{3j}\} + E\{L_k\} \left( \frac{2M}{R_t} + \frac{1}{R_w} + \frac{1}{R_{4j}} \right) \end{aligned} \quad (35)$$

The first four terms are the waiting time delay; the remaining terms are the service time delays.  $E\{X_{1i}\}$  and  $E\{X_{3j}\}$  are due to the input and the output packet processors;  $R_t$  is the packet transfer rate of the input or output modules;  $R_w$  is the service rate of the cyclic queue switch and  $R_{4j}$  is the service rate of the transmission line.

## CONCLUSIONS

This paper has described an approach being used to develop a tool to enable network operators to better manage large communication networks. This tool is based on the ability to model the network switches for delay as well as customer traffic for both its average demand and burstiness. These models are then used in a general routing procedure, described in [13], to obtain optimal paths along which to route traffic through the network. This routing

algorithm takes into account delay to route the traffic. A graphical interface, not reported on here, is also being developed in conjunction with the models which enables the network operator to see the network and its performance and to accept or override the actions proposed by the tool. We are currently working on extending the applicability of the model to new services and switches.

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## REFERENCES

- [1] W. Whitt "The Queueing Network Analyzer" The Bell System Technical Journal. Vol.62, No.9, November 1983.
- [2] W. Whitt "Performance of the Queueing Network Analyzer" The Bell System Technical Journal. Vol.62, No.9, November 1983.
- [3] R. Guerin, H. Ahmadi and M. Naghshineh. "Equivalent Capacity and Its Application to Bandwidth Allocation in High-Speed Networks.", IEEE J. Select. Areas Commun., Vol. 9., No. 7., pp. 968-981, Sept. 1991.
- [4] K.M. Khalil, K.Q. Luc and D.V. Wilson. "LAN Traffic Analysis and Workload Characterization," Proc. 15th Conf. on Local Comp. Netw., pp. 112-122, Oct 1990.
- [5] H.J. Fowler and W.E. Leland, "Local Area Traffic Characteristics, with Implications for Broadband Network Congestion Management." IEEE J. Select. Areas Commun. Vol. 9, No. 7, pp. 1139-1149, September 1991.
- [6] W.E. Leland and D.V. Wilson. "High Time-Resolution Measurement and Analysis of LAN Traffic: Implications for LAN Interconnection." Proc. INFOCOM '91, pp. 1360-1366, April 1991.
- [7] D. S. Lee, B. Melamed, A. Reibman and B. Sengupta. "Analysis of a Video Multiplexer using TES as a Modeling Methodology," IEEE GLOBECOM'91, vol. 1, pp. 16-20.
- [8] J. Beran, R. Sherman, M. S.Taqqu, and W. Willinger. "Variable Bit-Rate Video Traffic and Long-Range Dependence," Pre-publication Copy.
- [9] K.T. Marshall, "Some Inequalities in Queueing," Oper. Res., Vol. 16, No. 3, pp. 651-665, May-June 1968.
- [10] P.J. Kuehn. "Multiqueue Systems with Nonexhaustive Cyclic Service," The Bell System Technical Journal, Vol. 58, No. 3, pp. 671-698, March 1979.
- [11] O.J. Boxma and B. Meister, "Waiting-Time Approximations for Cyclic-Service Systems with Switch-over Times." Performance Eval. Rev. vol.14 pp. 254-262, 1986.
- [12] K.S. Watson. "Performance Evaluation of Cyclic Service Strategies - a Survey," Performance'84, ed. E. Gelenbe, pp. 521-533, North-Holland, Amsterdam, 1984.
- [13] Z. Flikop, " Routing Optimization in Packet Switching Communication Networks." European J. of Oper. Res., Vol. 19, pp. 262-267, 1985.