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DESIGNING PRODUCT FAMILIES WITH COMPETITION: A DESIGN FOR MARKET SYSTEMS APPROACH

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ABSTRACT

Manufacturing firms use product families to provide variety while maintaining economies of scale to improve manufacturing productivity. Designing a successful product family requires consideration of both customer preferences and the competition. This paper presents a design for market systems approach to product family design and solves the problem of designing a product family when the competition is simultaneously designing its product family. In particular, the problem is formulated as a two-player zero-sum game. Our analysis of this problem shows that it can be separated into multiple subproblems whose solution provides an optimal solution to the original problem. The paper presents an example to illustrate the approach.

1 INTRODUCTION

A product family is a group of related products that is derived from a product platform to satisfy a variety of market niches. Many manufacturing firms develop product platforms and design families of products based on these platforms to provide variety while maintaining economies of scale to improve manufacturing productivity. In product family design, sharing components and production processes across a platform of products enables companies to develop differentiated products efficiently by reducing design costs, to increase the flexibility and responsiveness of their manufacturing processes, and to take market share away from competitors that develop only one product at a time. Other benefits include reduced development and production costs and an improved ability to upgrade products [1]. Using a product platform enables designers and manufacturers to upgrade the whole family of products at once by changing design criteria instead of changing each product individually.

Designing product families requires aligning product, process and supply chain decisions [2]. Studies of product architecture, product variety, product positioning and product line design have increased our understanding of product family design

[3]. One aspect that has not been studied yet is product family design in a competitive market, where the competition's product family design decisions are not known when developing the product family. Shiau and Michalek [4] proposed an approach to new product design entering the market while existing products are sold by competitors.

According to [5], the study of product design has considered two types of competition in existing markets: price competition (which has the potential to affect sales in the short-term because prices can be changed quickly) and design competition (which requires more time to affect sales due to the lead time of developing a new product). Previous work has addressed the problems of designing a single product when the competitor's price decision is unknown [5], product positioning and pricing when a dominant retailer must be considered [6], and optimizing a product design while considering the likelihood of retailer acceptance [7]. Although previous work [8] proposed approaches such as Selection-Integrated Optimization (SIO) to optimize product family design with respect to the selection of platform and non-platform design variables, no previous work used game theory to consider market competition.

In this paper, we propose a product family design approach that uses game theory to construct a model that includes the competition's product design decisions. Game theory is the theory of independent and interdependent decision making. It is concerned with decision making in organizations where the outcome depends on the decisions of two or more autonomous players, one of which may be nature itself, and where no single decision maker has full control [9].

Game theory has been used previously to represent decision-making in design processes, with the players in the game representing different designers (within the same organization) who control different sets of design variables and may have different objectives [9-18]. For example, to design a pressure vessel, Lewis and Mistree [12] formulated the problem as a two player game in which the first player, who wishes to maximize

the volume, controls the diameter and length and the second player, who wishes to minimize the weight, controls the thickness. Chen and Simon [18] used game theory to model various team interactions in concurrent parametric design.

Absent from the work cited here is the problem of designing a product (or product family) when the competition is simultaneously designing its product (or product family). In this paper we consider the problem of designing a product family with the objective of maximizing market share when the competition is simultaneously designing it products. We formulate the problem as a two player zero-sum game and present an approach that finds the optimal product family design.

2 PRODUCT FAMILY DESIGN: FORMULATION

This section presents the product family design problem that we consider. There are two manufacturers (Players 1 and 2) whose product families will compete against each other in the marketplace. The marketplace comprises N distinct and independent markets, and each player has one product in each market. Player 1 designs and sells a family of products, numbered from 1 to N. Player 2's products are numbered from $N+1$ to $2N$.

Both players design simultaneously. The payoff for each player is their total sales (for all of their products). The size of each of the N markets is fixed, so any gain in sales by one manufacturer means an equivalent loss in sales by the other. Both manufacturers want to maximize their total sales across all N markets, but the sum of the total sales of the two manufacturers is constant. This problem can be modeled as a zero-sum game. We consider the problem from Player 1's perspective and assume that Player 1 has already decided to create a product family using a product platform (thus he does not design his products independently). Moreover, Player 1 has already identified the platform variables for this family. However, we make no similar assumption about Player 2's products because we assume Player 2 will produce the best design possible for each individual product. In some sense this is the worst-case for Player 1 and is, therefore, a reasonable way for Player 1 to proceed.

For modeling demand, there are various probabilistic choice models based on Discrete Choice Analysis (DCA). In this work, we use the logit model for its simplicity and capability to predict the demand of future product designs. The utility value is based on product characteristics that can be related to engineering design. The logit model is used to determine the market share for each product in each market. The market share of product j depends upon δ_j, the mean utility of product j, which measures the desirability of the product [19]. In this problem, a product's utility depends upon its customer attributes, which do not include price. As in a Cournot duopoly, we assume that the prices are externally fixed quantities [11].

Let S_j be the size of market j. Note that, in each market, there are only two products: products j and $N+j$. For product j designed and sold by Player 1, let q_j be its market share and let Q_j be its sales:

$$q_j = \frac{e^{\delta_j}}{e^{\delta_j} + e^{\delta_{N+j}}}$$

$$Q_j = q_j S_j \qquad (1)$$

Let Q be the total sales for Player 1's product family, which is the sum of the individual product sales:

$$Q = \sum_{j=1}^{N} Q_j = \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta_{N+j}}} \qquad (2)$$

Let $D = (D_1, ..., D_N)$ be Player 1's product family, let $E = (E_1, ..., E_N)$ be Player 2's products, and let F_i be the set of feasible products for Player i. Then, for $j=1, ..., N$, δ_j depends upon D_j and δ_{N+j} depends upon E_j. Player 1 needs to solve the following optimization problem:

$$\max_{D \in F_1} \min_{E \in F_2} Q \qquad (3)$$

3 PRODUCT FAMILY DESIGN: SOLUTION APPROACH

To solve this problem, we will first consider Player 2 and find a dominant strategy for Player 2. This strategy yields more sales for Player 2 than any other strategy, no matter what Player 1 does. (Note that a strategy for Player 1 is a choice of D, and a strategy for Player 2 is a choice of E.)

Because Player 2 does not necessarily use a platform, Player 2's dominant strategy is to find, for each market $j=1, ..., N$, the E_j that maximizes δ_{N+j}, which will maximize his sales in that market. Let δ_{N+j}^* be the corresponding maximum product utility.

Then, because Player 2 will follow his dominant strategy, we have the following objective function for Player 1:

$$Q = \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta_{N+j}^*}} \qquad (4)$$

Thus, Player 1's optimal strategy is to find D^* that maximizes Q. Thus, we have separated the original problem into $N+1$ subproblems that can be solved in two stages. The first stage solves N subproblems to find Player 2's products, which can be done in parallel. The second stage finds D^* to maximize Q.

4 APPLICATION: MOTOR DESIGN

To illustrate this approach to product family design, we will use the universal electric motor family developed by Simpson [20]. The product family has ten products, each designed to produce a different torque. Each motor has eight design variables. Following Simpson [21], six of these eight design variables will be used to define the product platform (as discussed above, the choice of the platform variables has been made already). These variables will have the same values for all ten products. Each product in the family will have unique values for the remaining two design variables. Tables 1 and 2 show the platform design variables and product design variables with their bounds.

Table 1. Bounds on Platform Design Variables.

Variable	Definition	Lower bound	Upper bound	units
N_c	Turns of wire (armature)	100	1500	*turns*
N_s	Turns of wire (stator), per pole	1	500	*turns*
A_{aw}	Cross sectional area of armature wire	0.01	1.0	mm^2
A_{sw}	Cross sectional area of stator wire	0.01	1.0	mm^2
r_o	Outer radius (stator)	0.01	0.1	m
t_s	Thickness (stator)	0.0005	0.01	m

Table 2. Bounds on Product Design Variables.

Variable	Definition	Lower bound	Upper bound	units
I	Electric current	0.1	6	A
L	Stack length	0.01	0.2	m

Table 3. Torque Requirement Values.

Market	T_j	S_j
1	0.05	200,000
2	0.10	250,000
3	0.125	200,000
4	0.15	100,000
5	0.20	100,000
6	0.25	50,000
7	0.30	25,000
8	0.35	25,000
9	0.40	25,000
10	0.50	25,000

Because the product family has ten products, there will be ten sets of I and L, which yields twenty design variables. These plus the six platform design variables give a total 26 design variables.

The four customer attributes are the torque T (in Nm), the power P (in watts), the efficiency η, and the mass M (in kg). They are calculated from the design variables and the engineering attributes as follows:

$$
\begin{aligned}
T &= K\varphi I \\
P &= P_{in} - P_{out} \\
\eta &= P / P_{in} \\
M &= M_w + M_s + M_a
\end{aligned}
\tag{5}
$$

All of the products should have a power of 300 watts. The ten different torque requirements, which define the ten independent markets, are shown in Table 3.

Because the power and torque must meet specific requirements dictated by the marketplace and the prices are fixed, we assume that the mean utility for each product depends only upon its mass and efficiency:

$$
\delta_j = \Psi_1(M_j) + \Psi_2(\eta_j)
\tag{6}
$$

As discussed in the previous section, Player 1 can maximize his sales by solving the following optimization problem:

$$
\max \sum_{j=1}^{N} \frac{S_j e^{\delta_j}}{e^{\delta_j} + e^{\delta^*_{N+j}}}
\tag{7}
$$

subject to

$$
\begin{aligned}
T_1 &= 0.05 \\
T_2 &= 0.10 \\
T_3 &= 0.125 \\
T_4 &= 0.15 \\
T_5 &= 0.2 \\
T_6 &= 0.25 \\
T_7 &= 0.3 \\
T_8 &= 0.35 \\
T_9 &= 0.4 \\
T_{10} &= 0.5 \\
P_j &= 300 \quad j = 1,...,10
\end{aligned}
$$

For this application, we can further simplify the problem by eliminating the product design variables as follows. Given values for the platform design variables, it is possible to find values for I and L so that product j satisfies the power constraint $P = 300$ and the torque requirement T for that product j. (In the following we will drop the subscripts for convenience.)

To do this, we first determine I as a function of the torque requirement:

$$
I = \frac{T}{K\varphi}
\tag{8}
$$

Using the relationships from Appendix A, we can rewrite this and define Z and W as follows:

$$
I^2 = \frac{T\Re\pi}{N_C N_S}
$$

$$
\Re = \frac{1}{L}\left(\frac{\pi(2r_0 + t_s)}{4\mu_{steel}\mu_0 t_s} + \frac{1}{\mu_{steel}\mu_0} + \frac{7\times10^{-4}}{\mu_0(r_0 - t_s - 7\times10^{-4})} \right) = \frac{Z}{L}
$$

$$
I^2 L = \frac{T\pi Z}{N_C N_S} = W
\tag{9}
$$

The last of these equations can be rewritten to express L in terms of I:

$$
L = \frac{W}{I^2}
\tag{10}
$$

Now we consider power, which is the difference between input power and output power:

$$
P = P_{in} - P_{out}
\tag{11}
$$

Using the relationships for P_{in} and P_{out} from Appendix A, we can derive the following expression:

$$P = 115I - \left(I^2(R_a + R_S) + 2I\right) \qquad (12)$$

After including the definitions of R_a and R_S from Appendix A and defining the quantities C_1, C_2, C_3, and C_4 (all of which depend only upon the platform design variables), we have an expression for power in terms of I and L:

$$C_1 = 1.69 \times 10^{-8} \times 10^6 = 1.69 \times 10^{-2}$$

$$C_2 = \frac{2N_c}{A_{aw}} + \frac{4N_s}{A_{sw}}$$

$$C_3 = 4(r_0 - t - 7 \times 10^{-4})\frac{N_C}{A_{aw}} \qquad (13)$$

$$C_4 = 8(r_0 - t)\frac{N_S}{A_{sw}}$$

$$P = 113I - C_1 I^2 \left(C_2 L + C_3 + C_4\right) \qquad (14)$$

Because $P = 300$, substituting Equation (13) into Equation (17) yields a quadratic function of I:

$$C_1(C_3 + C_4)I^2 - 113I + C_1 C_2 W + P = 0 \qquad (15)$$

Solving this expression will give us two values for I. We use the smaller value because the larger value exceeds the upper bound on I. From this value of I, we can determine L using Equation 13. This procedure can be used to determine the design variables for each of the ten products in the product family. Therefore, it is not necessary to include the product design variables in the search to solve the optimization problem, which can be reformulated as a problem with just the six platform design variables.

Although Player 2 uses the same type of universal motor technology, we don't make any assumptions about Player 2's platform. The best that Player 2 can do is to design independent products, one for each torque requirement, with no platform design variables. Thus, to find Player 2's dominant strategy, we separate the problem into ten parallel problems, one for each of the ten products. The objective of each problem is to maximize the mean utility $\delta_j = \Psi_1(M_j) + \Psi_2(\eta_j)$ subject to the power and torque constraints.

5 RESULTS

First we considered Player 2's problem. For each torque requirement we found values for all eight design variables to maximize the product utility (cf. [22]). Table B.2 lists the values of the design variables and customer attributes for Player 2's products. (Again, the power equals 300 watts for all of the designs.)

Then we considered Player 1's product family design problem. In this problem we propose two approaches. In the first approach we optimize Q (Equation 5) using the markets shown in Table 3. The optimization problem was solved ten times using ten different initial points for the platform design variables (listed in Appendix B). All ten runs yielded the same values for the platform design variables (these are shown in Table 4).

From these values, the ten torque requirements, and the power constraint, we determined values for the product design variables using the procedure presented in the previous section. Table 5 shows the values of the product design variables and the customer attributes for the ten products. (Recall that the power equals 300 watts for all of the designs.) We will call this set of products Family A.

Given these solutions and a market size of 1,000,000, we determined the sales of each player (shown in Table 6). As expected, Player 2's sales are slightly higher. However, it is interesting to note that the difference is less than 4,000 units, which shows that, by adopting a product platform, Player 1 does not give up much in terms of sales.

In the second approach we solve a different subproblem to find the product platform. We replace the original objective function (total sales) with the following quantity:

Table 4. Player 1's Optimal Product Platform.

Variable	Optimum Value	units
N_c	1170.6427	*turns*
N_s	500	*turns*
A_{aw}	0.2890	mm^2
A_{sw}	0.2892	mm^2
r_o	0.01	m
t_s	0.0045	m

Table 5. Player 1's Product Designs and Customer Attribute Values for each Torque Value for Family A.

Torque	I	L	M	η
0.05	2.9234	0.0037	0.1661	0.8923
0.10	3.0076	0.0071	0.2118	0.8674
0.125	3.0498	0.0087	0.2327	0.8553
0.15	3.0922	0.0101	0.2524	0.8436
0.2	3.1771	0.0128	0.2886	0.8211
0.25	3.2625	0.0152	0.3208	0.7996
0.3	3.3482	0.0152	0.3495	0.7791
0.35	3.4344	0.0192	0.3751	0.7596
0.4	3.5209	0.0209	0.3979	0.7409
0.5	3.6953	0.0237	0.4360	0.7059

Table 6. Total Sales for Each Player (Player 1 uses Family A).

Player	*Sales*
1	498,698
2	501,302

$$X = \sum_{j=1}^{N} e^{\delta_j} \qquad (16)$$

This optimization problem was solved ten times using ten different initial points for the platform design variables (listed in Appendix B). This generated ten different solutions, but seven of them had the same objective function value. Table 7 shows one of the best solutions found.

Given this product platform, for each torque, we calculated the appropriate values for I and L. Table 8 lists the values of I and L and the customer attributes for each product. (Recall that the power equals 300 watts for all of the designs.) We will call this set of products Family B.

Because Family B is different from Family A, the total sales for Player 1 (across all ten markets) is different (and smaller). Player 2 still uses the product family shown in Table 4. Table 9 shows the total sales for each player.

Table 10 compares the sales in each market for each family. As shown, the products in Family A have larger market shares (compared to those of Family B) in the larger markets (those with a torque requirement of 0.05, 0.10, and 0.125). Although Family B has more sales in some smaller markets, Family A has greater total sales. Interestingly, the difference in total sales is small, indicating that using a simpler objective function to determine the product platform variables is a good heuristic.

Figure 1 shows the mass and efficiency of each product in each family. Player 1's high-torque products have higher mass and slightly lower efficiency than Player 2's high-torque products. The mass and efficiency of the players' medium-torque products are very close. Player 1's low-torque products have slightly lower mass but lower efficiency than Player 2's low-torque products.

Table 7. Player 1's Product Platform (Family B).

N_C	N_S	A_{aw}	A_{sw}	r_0	t_s
970.1875	500.0000	0.2850	0.2854	0.0100	0.0032

Table 8. Player 1's Product Designs and Customer Attribute Values for each Torque Value for Family B.

Torque	I	L	M	η
0.05	2.9525	0.0036	0.1745	0.8835
0.10	3.0293	0.0068	0.2142	0.8612
0.125	3.0678	0.0083	0.2325	0.8503
0.15	3.1065	0.0097	0.2499	0.8398
0.2	3.1840	0.0123	0.2821	0.8193
0.25	3.2620	0.0147	0.3110	0.7997
0.3	3.3403	0.0168	0.3371	0.7810
0.35	3.4190	0.0187	0.3606	0.7630
0.4	3.4982	0.0204	0.3817	0.7457
0.5	3.6577	0.0233	0.4177	0.7132

Table 9. Total Sales when Player 1 uses Family B.

Product Family	Sales
Player1	497,684
Player2	502,316

Table 10. Sales in Each Market.

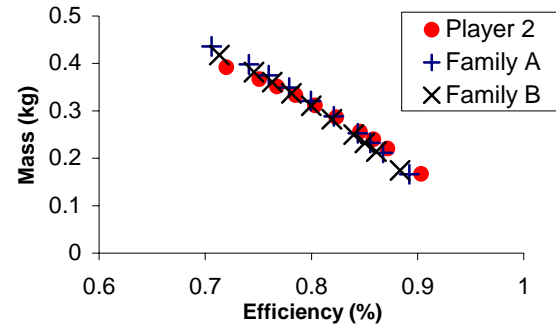| Market	Product Family A	Product Family B
1	99,468	98,818
2	125,000	124,538
3	100,022	99,778
4	49,993	49,929
5	49,915	49,951
6	24,895	24,958
7	12,412	12,462
8	12,376	12,443
9	12,340	12,420
10	12,276	12,378
Total	498,698	497,684

Figure 1. The Mass and Efficiency of Each Player's Products.

6 DISCUSSION: PRODUCT FAMILY DESIGN PROCESS

The analysis of the product family design problem and the results from the motor design example show that a design optimization problem that initially appears to be quite complex can be solved by replacing it with an appropriate set of subproblems, which we call a separation [22]. Unlike decomposition, which requires multiple iterations to converge to a solution, each subproblem in the separation is solved only once. In this paper we propose two approaches to solve the problem. In the first approach we try to maximize total sales for our products based on competitor's best design.

In the second approach however instead of maximizing total sales (Q), we maximize a simpler intermediate quantity (the quantity X) that is sufficient to identify a dominant strategy. This leads to a suboptimal solution, but the separation requires less effort because it ignores Player 2. In the motor design example, we can eliminate the product design variables from the problem for Player 1 and solve to find just the platform design variables, which reduces the size of the search space. Finally, for Player 2, we separate his problem into ten subproblems, one for each of his products. Using this separation, we get an exact optimal solution to the original problem. This is not always the case, unfortunately.

7 SUMMARY AND CONCLUSIONS

This paper introduces an important and novel problem in the area of design for market systems. The problem is one of designing a product family to maximize sales in the presence of a competitor whose product family design is simultaneous. We formulated the problem as a two-player zero-sum game and find dominant strategies for each player. We used a well-known motor family design problem to illustrate the approach.

The approach taken in this paper is an example of the use of separation to solve a complex design optimization problem by replacing it with a set of subproblems and solving each subproblem once [22]. The separation exploits the special structure of this two-player game; in particular, the game has dominating strategies, so the separation focuses on finding those dominating strategies. The first approach is an exact separation that gives the optimal product family. The second approach is a simpler, approximate separation that provides a near optimal product family. The next step in this work is to consider the objective of maximizing profit. Because the two manufacturers may have different cost structures, it will be necessary to formulate the problem as a two-player mixed-motive game. Work in this direction is ongoing.

8 REFERENCES

[1] Simpson, T.W., Siddique, Z., and Jiao, J., eds., Product Platform and Product Family Design: Methods and Applications, Springer, New York, 2006.

[2] Jiao, R., Simpson, T., and Siddique, Z., "Product family design and platform-based product development: a state-of-the-art review," Journal of Intelligent Manufacturing, Volume 18, Number 1, pp. 5-29, 2007.

[3] Yu, J.S., Gonzalez-Zugasti, J.P., and Otto, K.N., "Product Architecture Definition Based Upon Customer Demands," Journal of Mechanical Design, Vol. 121, Issue 3, pp. 329-335, 1999.

[4] Shiau, C.-S., and Michalek, J.J., "Optimal Product Design Under Price Competition," Journal of Mechanical Design, Vol. 131, Issue 1, January 2009.

[5] Shiau, C.-S., and Michalek, J.J., "Optimal Product Design Under Price Competition," Proceedings of the 2008 ASME International Design Engineering Technical Conference, DETC2008-49176, New York, August 3-6, 2008.

[6] Luo, L., Kannan, P.K., and Ratchford, B.T., "New Product Development Under Channel Acceptance," Journal of Marketing Science, Vol. 26, No. 2, pp. 149-163, 2007.

[7] Williams, N., Azarm, S., and Kannan, P.K., "Engineering Product Design Optimization for Retail Channel Acceptance," Journal of Mechanical Design, Vol. 130, No. 6, pp. 061402-1-10, 2008.

[8] Khire, Ristesh A., Messac, Achille, Timothy Simpson W., "Optimal Design of Product Families Using Selection-Integrated Optimization (SIO) Methodology," Multidisciplinary Analysis and Optimization Conference, 6 - 8 September 2006, Portsmouth, Virginia

[9] Vincent, T.L., "Game Theory as a Design Tool," Journal of Mechanisms, Transmissions and Automation in Design, Vol. 105, pp. 165-170, 1983.

[10] Rao, S.S., and Hati, S.K., "Game Theory Approach in Multicriteria Optimization of Function Generating Mechanisms," Journal of Mechanical Design, Vol. 101, pp. 398-406, 1979.

[11] Kelly, A., Decision Making using Game Theory, Cambridge University Press, Cambridge, 2003.

[12] Lewis, K., and Mistree, F., "Collaborative, Sequential, and Isolated Decisions in Design," Journal of Mechanical Design, Vol. 120, Issue 4, pp. 643-652, 1998.

[13] Rao, J.R.J., Badhrinath, K., Pakala, R., and Mistree, F., "A Study of Optimal Design under Conflict Using Models of Multi-Player Games," Engineering Optimization, Vol. 28, pp. 63-94, 1997.

[14] Lewis, K., and Mistree, F., "Modeling Interactions in Multidisciplinary Design: A Game Theoretic Approach," Journal of Aircraft, Vol. 35, pp. 1378-1392, 1996.

[15] Chanron, V., and Lewis, K., "A study of convergence in decentralized design processes," Journal of Research in Engineering Design, Vol. 16, pp. 133-145, 2005.

[16] Hernandez, G., Seepersad, C.C., and Mistree, F., "Designing for Maintenance: A Game Theoretic Approach," Journal of Engineering Optimization, Vol. 34, pp. 561-577, 2002.

[17] Marston, M., and Mistree, F., "Game-Based Design: A Game Theoretic Extension to Decision-Based Design," Proceedings of the ASME International DETC/CIE, DTM-14578, Atlanta, Georgia, September 10-13, 2000.

[18] Chen, L., and Li, S., "Concurrent Parametric Design Using a Multifunctional Team Approach," Proceedings of 2001 ASME International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, DAC-21038, Pittsburgh, Pennsylvania, September 9-12, 2001.

[19] Berry, S.T., "Estimating Discrete-Choice Models of Product Differentiation," RAND Journal of Economics, Vol. 25, No. 2, pp. 242-262, 1994.

[20] Simpson, T.W., "A Concept Exploration Method for Product Family Design," Ph.D. thesis, Georgia Institute of Technology, Atlanta, Georgia, 1998.

[21] Simpson, T.W., "Methods for Optimizing Product Platform and Product Families: Overview and Classification," in Product Platform and Product Family Design: Methods and Applications, Simpson, T.W., Siddique, Z., and Jiao, J., eds., Birkhäuser, New York, 2006.

[22] Karimian, P., and Herrmann, J.W., "Separating Design Optimization Problems to Form Decision-Based Design Processes," Journal of Mechanical Design, Vol. 131, Issue 1, January 2009.

APPENDIX A. ENGINEERING PARAMETERS AND ATTRIBUTES

ENGINEERING PARAMETERS

Length of air gap $l_g = 7.0 \times 10\text{-}4 \ m$

Terminal voltage $V_t = 115 \ V$

Resistivity of copper $\rho = 1.69 \times 10\text{-}8 \ Ohms \bullet m$

Permeability of free space $\mu_o = 4\pi \times 10\text{-}7 \ H/m$

Number of stator poles $p_{st} = 2$

Cost of copper $C_c = 2.2051 \ \$/kg$

Cost of steel $C_s = 0.882 \ \$/kg$

Density of copper $\delta_c = 8,960 \ kg/m^3$

Density of steel $\delta_s = 7,861.09 \ kg/m^3$

Engineering Attributes

Magnetizing intensity [Ampere turns/m]
$$H = N_c I /(l_c + l_r + 2l_g)$$

Mean path length within the stator [m]
$$l_c = \pi(2r_o + t_s)/2$$

Diameter of armature [m] $\quad l_r = 2(r_o - t_s - l_g)$

Input power [W] $\quad P_{in} = V_t I$

Power losses due to copper and brushes [W]
$$P_{out} = I^2(R_a + R_s) + 2I$$

Armature wire length [m] $\quad l_{aw} = (2L + 2l_r)N_c$

Stator wire length [m] $\quad l_{sw} = p_{st}(2L + 4(r_o - t_s))N_s$

Armature wire resistance [Ohm] $\quad R_a = \rho l_{aw}/A_{aw} \times 10^6$

Stator wire resistance [Ohm] $\quad R_s = \rho l_{sw}/A_{sw} \times 10^6$

Mass windings [kg] $\quad M_w = (l_{aw}A_{aw} + l_{sw}A_{sw})\delta_c \times 10^{-6}$

Mass of stator [kg] $\quad M_s = \pi L(r_o^2 - (r_o - t_s)^2)\delta_s$

Mass of armature [kg] $\quad M_a = \pi L(r_o - t_s - l_g)^2 \delta_s$

Motor constant [dimensionless] $\quad K = N_c/\pi$

Magneto magnetic force [A turns] $\quad \Im = N_s I$

Magnetic flux [Wb] $\quad \varphi = \Im/\Re$

Total reluctance [A turns/Wb] $\quad \Re = \Re_s + \Re_a + 2\Re_g$

Stator reluctance [A turns/Wb] $\quad \Re_s = l_c/(2\mu_{steel}\mu_o A_s)$

Armature reluctance [A turns/Wb]
$$\Re_a = l_r/(\mu_{steel}\mu_o A_a)$$

Reluctance of one air gap [A turns/Wb]
$$\Re_g = l_g/(\mu_o A_g)$$

Cross sectional area of stator [m^2] $\quad A_s = t_s L$

Cross sectional area of armature [m^2] $\quad A_a = l_r L$

Cross sectional area of air gap [m^2] $\quad A_g = l_r L$

Relative permeability of steel [dimensionless]
$$\mu_{steel} = -0.2279H^2 + 52.411H + 3115.8 \quad H \leq 220$$
$$\mu_{steel} = 11633.5 - 1486.33\ln(H) \quad 220 < H \leq 1000$$
$$\mu_{steel} = 1000 \quad\quad\quad\quad\quad\quad\quad\quad H > 1000$$

APPENDIX B. INITIAL DESIGNS.

Table B.1: Initial designs for Separations S1 and S2.

N_c	N_s	A_{aw}	A_{sw}	r_o	t_s
622.4461	10.2789	0.1798	0.0855	0.0296	0.0087
971.5237	41.0603	0.1669	0.2469	0.0299	0.0036
622.4460	10.2836	0.2004	0.1590	0.0294	0.0051
971.4546	52.0387	0.2522	0.9947	0.0113	0.0021
373.0639	21.7458	0.2813	1	0.0159	0.0008
971.5356	48.7206	0.2976	0.9760	0.0142	0.0025
383.8721	33.1335	0.2371	1	0.0103	0.0005
971.5287	56.2628	0.2115	0.2162	0.0123	0.0013
483.5892	223.6510	0.0644	1	0.0243	0.0005
970.7074	128.5235	0.2849	0.2776	0.0181	0.0098

Table B.2: Player 2's Best Products and Customer Attribute Values for each Torque Value.

Torque	N_C	N_S	A_{aw}	A_{sw}	r_0	t_s	I	L	M	η
0.05	624.4289	331.9046	0.2936	0.2936	0.0100	0.0040	2.8878	0.0100	0.167	0.9034
0.10	970.0895	441.3717	0.2955	0.2955	0.0100	0.0046	2.9927	0.0100	0.2203	0.8717
0.125	1045.8406	488.0866	0.2951	0.2951	0.0100	0.0044	0.0389	0.0100	0.2399	0.8584
0.15	1142.0968	500	0.2937	0.2937	0.0100	0.0042	3.0842	0.0100	0.2562	0.8458
0.2	1207.8453	500	0.2921	0.2921	0.0100	0.0037	3.1682	0.0109	0.2863	0.8234
0.25	1240.4244	500	0.2909	0.2909	0.0100	0.0034	3.2466	0.0119	0.3118	0.8035
0.3	1273.7696	500	0.2896	0.2897	0.0100	0.0031	3.3230	0.0128	0.3331	0.785
0.35	1306.6778	500	0.2884	0.2884	0.0100	0.0029	3.3984	0.0136	0.3513	0.7676
0.4	1338.0306	500	0.2870	0.2870	0.0100	0.0028	3.4735	0.0142	0.3669	0.751
0.5	1394.6589	500	0.2845	0.2845	0.0100	0.0026	3.6239	0.0153	0.392	0.7198