Swarms in Three Dimensions

E.W. Justh¹, P.S. Krishnaprasad¹,²
¹Institute for Systems Research, ²ECE Department

Abstract

- Modeling and analysis of novel control laws for vehicles moving at constant speed.
- Practical motivation: coordinating the flight of meter-scale UAVs (unmanned aerial vehicles). Possible implications for UUV, UGV, or USV swarms, or biological swarming/schooling systems.
- Objective: UAV formation demo in collaboration with NRL.

Rectilinear Control Law

\[ u_j = \frac{1}{n} \sum_{k \neq j} \eta \left( \frac{r_{jk}}{r_{jk}^2} \right) \left( x_k - x_j \right) + f\left( \frac{r_{jk}}{r_{jk}^2} \right) \left( y_k - y_j \right) + \mu x_j \cdot y_j \]

Gyroscopic Interaction Laws

- In our formation control model, particles (i.e., vehicles) interact through gyroscopic forces.
- Gyroscopic forces do no mechanical work: the kinetic energy (and hence the speed) of each particle remains constant.

Circling Control Law

\[ u_j = \frac{1}{n} \sum_{k \neq j} \eta \left( \frac{r_{jk}}{r_{jk}^2} \right) \left( x_k - x_j \right) + f\left( \frac{r_{jk}}{r_{jk}^2} \right) \left( y_k - y_j \right) + \mu x_j \cdot y_j \]

Future Research: Boundary-Following

Idea: control inputs for the moving vehicle are determined by the trajectory of the closest point on the obstacle surface.

References


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 ISR collaborator: Fumin Zhang

Equilibrium Shapes

- The control laws are assumed to be invariant under rigid motions in three-dimensional space.
- Therefore, we can define appropriate shape variables (which capture relative distances and angles between vehicles).
- Shape equilibria correspond to steady-state formations.

Naval Research Lab (NRL) collaborators:
Jeff Heyer, Larry Schuette, David Tremper


Goals: boundary following and non-collision.


(rectilinear formation, circling formation, helical formation) are controls.

Global convergence result for \( n = 2 \) (Justh, Krishnaprasad 2004) using the Lyapunov function:

\[ V = -\ln\left(1 + x_1 \cdot x_1\right) + h(r_2 - r_1) \]

penalizes heading-direction differences

penalizes inter-vehicle distances which are too large or small

Global convergence result for \( n = 2 \) (Justh, Krishnaprasad 2004).