A FRESH LOOK AT NETWORK SCIENCE: INTERDEPENDENT MULTIGRAPHS MODELS INSPIRED FROM STATISTICAL PHYSICS

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ABSTRACT

We consider several challenging problems in complex networks (communication, control, social, economic, biological, hybrid) as problems in cooperative multi-agent systems. We describe a general model for cooperative multi-agent systems that involves several interacting dynamic multigraphs and identify three fundamental research challenges underlying these systems from a network science perspective. We show that the framework of constrained coalitional network games captures in a fundamental way the basic tradeoff of benefits vs. cost of collaboration, in multi-agent systems, and demonstrate that it can explain network formation and the emergence or not of collaboration. Multi-metric problems in such networks are analyzed via a novel multiple partially ordered semirings approach. We investigate the interrelationship between the collaboration and communication multigraphs in cooperative swarms and the role of the communication topology, among the collaborating agents, in improving the performance of distributed task execution. Expander graphs emerge as efficient communication topologies for collaborative control. We relate these models and approaches to statistical physics.

Index Terms— network science, coevolving multigraphs, constrained coalitional games, collaboration, trust

1. INTRODUCTION

Complex networks are pervasive in several areas of science and engineering, including networked systems of humans and robots, networked unmanned autonomous vehicles, human organizations, biological networks beyond genomics (i.e. signaling and functional networks), social networks over the Web, or economic networks over the Web. In many of these networks (e.g. networked control systems, cellular networks in biology) the nodes themselves are dynamical systems. Thus these networks themselves become dynamical systems even when the node connectivity topology is static (fixed). More interestingly, in several network classes (e.g. neural networks, cellular networks in biology, networked human-machine systems) the node connectivity topology itself changes in response to feedback from the node dynamics or from the environment. Such, so-called adaptive coevolutionary networks [1], have attracted strong interest from the broad research community, especially from physicists. These collectives and their dynamics, when viewed from a dynamical systems perspective, constitute a fundamentally new class of systems, requiring rethinking the foundations of control theory and dynamical systems.

Research was partially supported by the US Air Force Office of Scientific Research under MURI grants FA9550-10-1-0573 and FA9550-09-1-0538, and by the National Science Foundation under grant CNS-1018346. Contact Email: baras@umd.edu

In these networked systems the *fundamental cycle of sense-compute/decide-actuate* occurs in a dynamic and distributed platform, where the functions of sense, compute/decide, actuate are not collocated, and are executed in a distributed and (partially) asynchronous fashion. An important ingredient of such systems, which has not been addressed intensively, is the role of *communication and information sharing* between the agents and its implications. In fact in these collectives, one needs more than one network (graph) to effectively model the agent interactions and arrive at quantitative characterizations of behavior and performance. Thus, one needs to develop a *collaboration network* (e.g. a task network) which describes how each agent collaborates with others. In addition one needs to develop a *communication network*, which describes how agents communicate with each other. Here the links describe physical communication capabilities between the agents and the associated graph can also be dynamic. In attempting to develop new foundational models for these complex networks it is important to *distinguish data from information*. Thus we find that an *information network* is necessary, which captures the information exchanged and shared between the agents; i.e. important features, models, various attributes of agents extracted from observations etc. Sensing and information extracted from sensors is part of this network. This network can also be dynamic. The current literature on complex networks is not at all clear about these distinctions, often assuming that all three networks coincide. Often there is no clear delineation about which links are physical and which are logical when describing agent interactions.

2. COEVOLVING MULTIGRAPHS

Recently we have developed and used formal analytical multi-layer multiple multigraph models for complex adaptive coevolving networks [2], inspired by earlier work in social networks [3]. This model involves three layers, each having a multigraph, which is used as an abstraction of the interactions between the agents from a collaborative decision making/action, information exchange, and communications perspective. The multigraphs are directed, with "weights" on their links and nodes. The links are relational, i.e. they represent relations between the nodes (e.g. organizational, social relations) or physical (mostly in the communication and information layer). These "weights" W_{ij}, W_i, represent the strength of a relation (or interaction) that exists between node i and$j$, or the importance of a node i. In this model, each network is modeled by a *value directed multi-graph with weighted nodes* [2] as illustrated in Figure 1. There are relations and constraints between these "weights" across the three domains, indicating the interactions between these networks. These "weights" can vary in their ana-

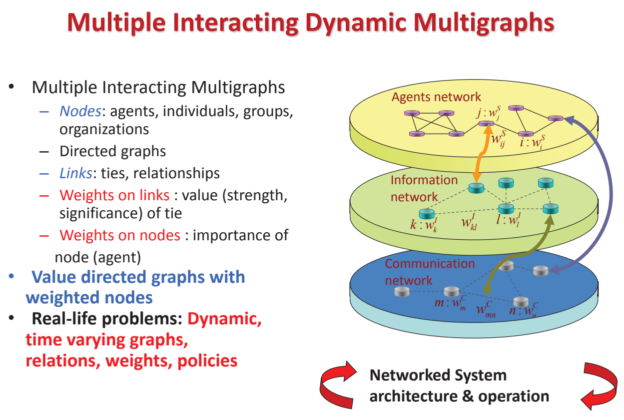

Fig. 1. Multiple multigraphs with weighted nodes and links

lytical representations from scalar, to vector and to even sets of logical rules, and can be deterministic or stochastic. As we have shown the resulting mathematical structure is quite rich, can be applied to a variety of problems, and the resulting mathematics range from multi-dimensional calculus and linear algebra to multiple partially ordered semirings [4] (e.g. max-plus semiring). It is easy to see that this model can accommodate various forms of coevolving networks through dynamic feedback interactions of the weights, across the layers. We have used this framework successfully to prove that small world communication topologies are efficient communication topologies for fast convergence of distributed consensus algorithms [5], self-organization of complex networks so as to speed-up distributed decision making algorithm execution [5], development of efficient communication topologies in distributed decision making in networked systems [6], construction of motifs d_{ij} in the communication network (small subgraphs constructed by the agents to facilitate certain task executions) inspired by a similar concept in biological networks [7], learning agent behaviors for coordination in networked systems [8]. We have shown [9] that expander graphs emerge as efficient communication network topologies for multi-agent systems. We have also utilized this framework to indicate the need for new probabilistic models in distributed multi-agent systems [10, 11], where the key idea is to build-in incompatible measurements between agents, leading to a model of probability measures on the logic of projections in a finite dimensional Hilbert space; not on the logic of subsets of a set and Cartesian products of the same. The latter is similar to the probability model used in Quantum Mechanics.

3. COLLABORATION

Collaboration in networked systems is fundamental. In such multi-agent systems trust plays an important role as a catalyst for collaboration [12]. Agents collaborate because they benefit from collaboration. Trust, intuitively enhances collaboration. Mistrust can be thought of as an obstacle (a resistance) for collaboration [13]. But trust needs to be evaluated and managed and this leads to a cost of collaboration. Our work is inspired by this fundamental tradeoff between gain and cost in the context of agent collaboration in autonomic networks [14]. Different with previous work in the literature, we study collaboration based on the notion of *coalitions* [13]. A coalition is a subset of nodes that is connected in the subgraph induced by the active links (this occurs at the top layer of Figure 1, the collaboration graph). In our work we are interested in the total productivity of the coalition formed by selfish nodes, how this is allocated among the individual nodes and the *stability* of the coalition. These notions are captured well by coalitional games.

Coalition formation has been widely studied in economics and sociology in the context of coalitional games [15, 16]. In coalitional game theory, the central concept is that of coalition formation, i.e., subsets of users that join their forces and decide to act together. Myerson [17] was the first to introduce a new game associated with communication constraints, the *constrained coalitional game*, which incorporates both the possible gains from cooperation as modeled by the coalitional game and the restrictions on communication reflected by the communication network. An important concept in coalitional games is the characteristic function v [18]. $v(G)$ is interpreted as the maximum payoff the network G can get given the network structure. A payoff allocation rule $x : G \to \mathbb{R}^N$ describes how the value $(v(G))$ associated with each network is distributed to the individual nodes. The coalitional game is usually modeled as a *two-phase process*. Players must first decide whether or not to join a coalition. This is done by iterated pairwise games, in which *both* players have to agree to activate a link between them and thus join the same coalition. In our work, this pairwise game involves, for each node, a comparison between the cost for activating the link towards the other node, and the benefit from joining the coalition that the other player is a member of. In the second step, players in one coalition negotiate the payoff allocation based on the total payoff of the coalition. The central problem is to study the convergence of the iterated pairwise game and whether the dynamics result in a stable solution.

In our work [13, 19] we have developed methods and algorithms based on constrained coalitional games in order to understand network and coalition formation under conflicting requirements and metrics, and the associated phase transitions. These methods were inspired from economics and social networks [17], statistical physics [20] and biology. The interplay between the benefit vs the cost of collaboration can be captured in pairwise iterated games that can be either of the *Prisoners Dilemma (PD)* type or of the *Coordination (Co)* type (Co) [1, 21]. We were able to obtain network formation results (including communication efficiency for small world networks [5]) as well as convergence to coalitions and the stability of the resulting coalitions [22, 14], and coalition formation based an agent learning [23].

4. TRUST AND COLLABORATION

Trust is a useful incentive for encouraging nodes to collaborate. Nodes who refrain from cooperation get lower trust value and will be eventually penalized because other nodes tend to only cooperate with highly trusted ones. Let's assume, for node i, that the loss of not cooperating with node j is a nondecreasing function of x_{ji} (where x_{ji} is the benefit node i gets from collaboration with node j) $l_{ij} = f(x_{ji})$. For simplicity, assume the characteristic function is a linear combination of the original payoff and the loss, which is shown as

$$v'(S) = \sum_{i,j \in S} x_{ij} - \sum_{i \in S, j \notin S} f(x_{ji}) \qquad (1)$$

The game with characteristic function v' is denoted as $\Gamma'(G, v')$. We then have [13]

Theorem 1 *If* $\forall i, j,\ x_{ij} + f(x_{ji}) \geq 0$, $C(\Gamma') \neq \emptyset$ *and* $x_i = \sum_{j \in V} x_{ij}$ *is a point in* $C(\Gamma')$.

Apparently, the payoff $x_i = \sum_{j \in \mathcal{N}_i} x_{ij}$ does not need any payoff negotiation. Thus we showed [13] that by introducing a *trust mechanism*, all nodes are induced to collaborate with their neighbors without any negotiation. Trust (and mistrust) is a dynamically evolving value. Collaboration is also evolving (changing). New and interesting problems arise as these two dynamics of networked systems interwine.

In our model [13], each node has a self-defined playing strategy, which is denoted by γ_i for node i. Another characteristic of each node is its trust values, which are dependent on the opinions of other nodes. Trust values of a node can be different for different players. For instance, t_{ji} and t_{ki} are the trust values of i provided by distinct agents j and k, and possibly $t_{ji} \neq t_{ki}$. The coevolution of trust values and payoffs is summarized as the following three rules: *Strategy updating rule*, *Payoff function evaluation rule*, *Trust evaluation rule*.

For simplicity, we assume the system is memoryless. All values are dependent only on parameter values at most one time step in the past. Therefore, the system can be modeled as a discrete-time system:

$$\gamma_i(t+1) = f^i(x_i(t), \gamma_i(t), \gamma_j(t), t_{ij}(t)) \quad (2)$$

$$t_{ik}(t) = g^i(t_{ij}(t), v_{jk}(t)) \quad \forall k \in N \quad (3)$$

$$x_i(t) = h^i(\gamma_i(t), \gamma_j(t)) \quad (4)$$

$$v_{ij}(t) = p^i(\gamma_j(t), t_{ji}(t)) \quad (5)$$

where j stands for all neighbors of i, and v_{ij} is the value node i votes for j. The coevolution of the collaboration (agent) network and the information (trust) network is apparent.

5. TRUST DYNAMICS

We model the trust network as a dynamic directed graph $G(k)(V(k), E(k))$, in which nodes are the entities/peers in the network and arcs represent trust relations. Based on documents and information, each node can derive *direct* trust values on target nodes. If (i, j) is a direct arc in graph G, i.e. $(i, j) \in E$, node i has direct trust d_{ij} on j, with continuous values in $[-1, 1]$. There are three types of trust we need to distinguish: real, direct and evaluated trust. The vector $T = [t_1, \ldots, t_N]$ is called the real trust vector, and $S = [s_1, \ldots, s_N]$ is the evaluated trust vector. The purpose of the global trust evaluation is to find the best estimates of t_i's, s_i's, based on all the d_{ij}'s.

In [22, 24] we analyzed stochastic evolution of trust in the case where we have both bad and good nodes in the network. The model developed in [22, 24] utilizes various stochastic models for the conditional probability of d_{ij}, denoted as $P(d_{ij}|t_i, t_j)$, for good nodes, and a uniform conditional probability of d_{ij} when node i is bad; because it achieves maximum entropy (we do not know typically the policies of bad nodes). To evaluate a particular node in the network, say node i, all the neighbors of i having a direct trust on i should be taken into account. The natural approach is to aggregate all neighbors' direct trust values, using a *voting rule*. The whole trust evaluation evolves as the voting rule iterates throughout the network and can be written as

$$s_i(k+1) = f\left(d_{ji}(k) s_j(k) \mathbf{1}(s_j(k) > 0) | j \in \mathcal{N}_i\right). \quad (6)$$

We studied the evolution of the evaluated trust vector S and its values when the process converges. The motivation of trust evaluation is to be able to detect bad nodes and trust good nodes. It is important to investigate whether S can correctly estimate the trust vector T at the steady state. In the stochastic model, the trust evaluation problem is interpreted as an estimation problem. T is the vector to be estimated and S is the estimate. We have shown [22, 24] that the distributions of S form a Markov random field (MRF) [25]. We have shown [22, 24] the following.

Theorem 2 *The probability of the estimated trust value for a certain node i,* s_i*, given the estimated trust values of all the other nodes in the network, is the same as the probability of* s_i*, given only the estimated trust values of the neighbors of* i;$\Pr[s_i|S/\{s_i\}] = \Pr[s_i|s_j, \forall j \in \hat{\mathcal{N}_i}]$.

The well-known Hammersley-Clifford theorem [25] proves the equivalence between a MRF on a graph and the Gibbs distribution. Inspired by this representation, we introduced in [13, 22, 24] a stochastic voting rule which provides the exact optimal solution.

$$\Pr\left[s_i(k+1)|s_j(k), j \in \hat{\mathcal{N}_i}\right] = \quad (7)$$

$$\frac{\prod_{j \in \hat{\mathcal{N}_i}} \Pr[d_{ij}, d_{ji}|s_i(k+1), s_j(k)] \Pr[s_i(k+1)]}{Z_i(k)} \quad (8)$$

where $Z_i(k)$ is the standrad normalization factor. Our trust evaluation rule is essentially an updating rule. In the autonomic environment we considered random asynchronous updates. This updating, with the stochastic voting rule can be considered as a Markov chain, where the states are composed of all possible configurations of the vector S. We showed that the Markov chain converges and gave the explicit formula for the stationary distribution [22, 24].

Theorem 3 *For the stochastic voting rule and using random asynchronous updates, the stochastic trust evaluation converges to the steady state with the unique stationary distribution* $\pi_S = M(S)/Z$.

5.1. Ising Model and Spin Glasses

It is straightforward to link the stochastic voting rule (eqn. (8)) with the Ising model [26] in statistical physics. The Ising model describes interaction of magnetic moments or "spins" of particles, where some particles seek to align with one another (ferromagnetism), while others try to anti-align (antiferromagnetism). In the Ising model, s_i is the orientation of the spin at particle i. $s_i = 1$ or -1 indicates the spin at i is "up" or "down" respectively. A Hamiltonian for a configuration S is

$$H(S) = -\sum_{(i,j)} J_{ij} s_i s_j - mH \sum_i s_i. \quad (9)$$

The first term represents the interaction between spins. The second term represents the effect of the external (applied) magnetic field. Then the probability of configuration S is

$$\Pr[S] = \frac{e^{-\frac{1}{kT} H(S)}}{Z}, \quad (10)$$

where T is the temperature and k is the Boltzmann constant. In the Ising model, the local interaction "strengths" J_{ij}'s are all equal to a constant J, which is either 1 or -1. The Ising model has been extensively studied since 1920. In recent years, an extension of the Ising model called the Edwards-Anderson model of spin glasses is used to study local interactions with independently random J_{ij} [20], which corresponds to $d_{ij} + d_{ji}$ in our voting rule.

We investigated in [22, 24] properties of the steady state distribution in networks where there exist adversaries. We discovered and analyzed phase transition phenomena, and the performance of the voting rule as a function of the number and type of adversaries.

6. CONCLUSIONS

We described several methods for the analysis of complex networked systems. We introduced a three layer coevolving multigraph model and illustrated that constrained coalitional games provide a powerful methodology and algorithms. We illustrated the methods in an example involving stochastic evaluation of trust and collaboration. We described how these methods are inspired by statistical physics.

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