Optimal Output Feedback Control Using Two Remote Sensors Over Erasure Channels

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***Abstract*—Consider a discrete-time, linear time-invariant process, two sensors and one controller. The process state is observed in the presence of noise by the sensors, which are connected to the controller via links that feature erasure. If a link transmits successfully then a finite-dimensional vector of real numbers is conveyed from the sensor to the controller. If an erasure event occurs, then any information conveyed over the link is lost. This paper addresses the problem of designing the maps that specify the processing at the controller and at the sensors to minimize a quadratic cost function. When the information is lost over the links either in an independent and identically distributed (i.i.d.) or in a (time-homogeneous) Markovian fashion, we derive necessary and sufficient conditions for the existence of maps such that the process is stabilized in the bounded second moment sense. We also solve the optimal design problem in the presence of delayed noiseless acknowledgment signals at the sensors from the controller for an arbitrary packet drop pattern. We provide explicit recursive schemes to implement our solution. We also indicate how our approach can be extended to situations when more than two sensors are available and when the sensors can cooperate. The analysis also carries over to the case when each point-to-point erasure link connecting the sensors and the controller is replaced by a network of erasure links.**

***Index Terms*—Control over communication channels, distributed estimation, linear time-invariant, networked control systems, sensor networks.**

I. Introduction

RECENTLY, significant attention has been directed towards networked control systems in which components communicate over communication channels (see, e.g., [2], [6] and the references therein). The estimation and control performance in such systems is severely affected by the properties of the channels. Communication links introduce many potentially detrimental phenomena, such as quantization error, random delays, data loss and data corruption to name a few, that may lead to performance degradation or even stability loss.

In this work, we are specifically interested in the problem of estimation and control across communication links that exhibit data loss. We consider a dynamical system evolving in time that is being observed by two sensors. The sensors need to transmit the data over communication links to a remote node, which can either be an estimator or a controller. However information transmitted over the links is erased stochastically. Preliminary work in this area has largely concentrated on the case when only one sensor is present. Within the one-sensor framework, both stability [33], [41] and performance [21], [33] have been analyzed. Approaches to compensate for the data loss to counteract the degradation in performance have also been proposed (see, e.g., [14], [21], [27], [35] and similar works). Also relevant are the works such as [3], [18], [31], [32] which look at controller structures to minimize quadratic costs for systems in which both sensor-controller and controller-actuator channels exhibit erasure. The related problem of optimal estimation across an erasure link was considered in [34] for the case of one sensor and erasures occurring in an i.i.d. fashion, and in [11] for multiple sensors and more general erasure models.

Most of the above designs aimed at designing a packet-loss compensator. The compensator accepts those packets that the link successfully transmits and propagates an estimate of the state of the process when data sent over the link is lost. If the estimator is used inside a control loop, the estimate is then used by the controller. We take a more general approach to the control of networked control systems by exploring the possibility of pre-processing (or encoding) information prior to transmission to improve the performance of a networked control system. In [13] it was shown that such pre-processing can indeed yield significant improvements in terms of stability and performance. Moreover, for a given performance level, it can also lead to a reduced amount of communication. The benefits incurred become even more apparent when the communication link is replaced by a *network* of communication links [12], [16]. This effect can also be seen in the recent works on receding horizon networked control, in which a few future control inputs are transmitted at every time step by the controller and buffered at the actuator to be used in case subsequent control updates are erased [17], [19], [24], [25].

In this work, we extend this idea to the case when multiple sensors are present. Suppose a process is observed using two sensors that transmit the data over erasure links to a controller. If the sensors can share their measurements, there is effectively only one sensor. We look at the case when cooperation between the sensors is either not permitted, or occurs over erasure links. We obtain necessary and sufficient conditions of stabilizability in terms of the state-space representation of the process and the probability of erasure over the links. We also present optimal performance achieving algorithms, under the assumption that the sensors have access to noiseless acknowledgment re-

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garding the erasure process over the links connecting them to the controller.

The problem involving the presence of multiple sensors transmitting data in an aperiodic fashion is much more complicated than the problem involving only a single sensor. The problem of finding optimal encoding algorithms for the multi-sensor case and analyzing their performance is similar to the problems of fusion of data from multiple sensors and of track-to-track fusion that have long been open. If the communication topology is fixed, with a link, if present, being perfect (no erasure), then some approaches to solve the problem are available in, e.g., [8], [9], [15], [20], [28], [36], [38]. However, in our case, information is lost randomly by the erasure links. This random loss of information reintroduces the problem of correlation between the estimation errors of various nodes [4] and renders the approaches proposed in the literature as sub-optimal [5], [7]. There are special cases for which the solution is known, e.g., when the process noise is absent [39] or when one of the sensors transmits data over a channel that does not erase information [13]. However, as stated earlier, in general, the problem is still open. Owing to a separation principle that we present, our results also carry over to this problem.

The paper is organized as follows. We begin in the next section by describing the problem set-up and our notation. In Section III, we present a summary of the stabilizability results for the case when two sensors transmit data over erasure channels. In Section IV, we present a separation principle that allows us to consider an alternative estimation problem. For the case when the sensors have access to acknowledgments from the controller, we provide a recursive algorithm which is optimal with respect to every possible realization of the erasure process in Section V-A. Stability analysis of this algorithm allows us to prove the necessity of the stabilizability conditions in Section V-B. In Section V-C, we then prove that the conditions are sufficient as well, by presenting a sub-optimal algorithm that stabilizes the system even when acknowledgments from the controller are not available. Section VI generalizes the results in various directions, including multiple sensors (Section VI-A), sensors co-operating over an erasure link (Section VI-B), and more general models of erasure events (Section VI-C).

II. Problem Formulation

sConsider the set-up of Fig. 1 denoted by process $\mathbb{P}_1$, and the following associated assumptions. The process is described by a discrete-time state-space representation of the following type:

$$\mathbf{z}(k+1) = A\mathbf{z}(k) + B\mathbf{u}(k) + \mathbf{w}(k), \ k \geq 0 \quad (1)$$

where $\mathbf{z}(k) \in \mathbb{R}^n$ is the process state, $\mathbf{u}(k) \in \mathbb{R}^l$ is the control input and $\mathbf{w}(k)$ is the process noise assumed to be white, Gaussian, zero mean with covariance $\mathbf{R}_w > 0$. The initial state $\mathbf{z}(0)$ is a Gaussian random variable with mean zero and covariance matrix $\mathbf{P}(0)$. The process state is observed using two sensors that generate measurements of the form

$$\begin{aligned} \mathbf{d}_1(k) &= C_1\mathbf{z}(k) + \mathbf{v}_1(k) \\ \mathbf{d}_2(k) &= C_2\mathbf{z}(k) + \mathbf{v}_2(k), \ k \geq 0 \end{aligned} \quad (2)$$

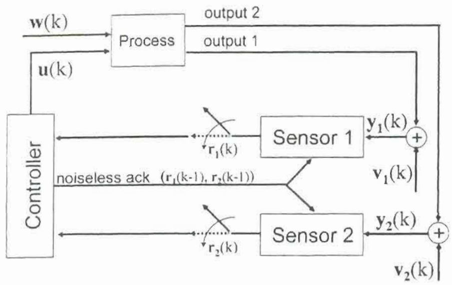

Fig. 1. Basic framework for output feedback using two remote sensors, in the presence of erasure channels. The process and measurement noises at the process are represented by $\mathbf{w}(k)$ and $\mathbf{v}_{1 \text{ or } 2}(k)$, respectively. The erasure process, in the links connecting the sensors to the controller, is governed by $\mathbf{r}_{1 \text{ or } 2}(k)$.

where $\mathbf{d}_1(k) \in \mathbb{R}^{m_1}$ and $\mathbf{d}_2(k) \in \mathbb{R}^{m_2}$. The measurement noises $\mathbf{v}_1(k)$ and $\mathbf{v}_2(k)$ are also assumed to be white, Gaussian, and zero mean with positive definite covariance matrices $R_{v,1}$ and $R_{v,2}$ respectively. For ease of notation, we adopt the concatenation $\mathbf{v}(k)^T \overset{def}{=} [\mathbf{v}_1(k)^T \mathbf{v}_2(k)^T]^T$ and denote the covariance matrix of $\mathbf{v}(k)$ by $\mathbf{R_v}$. Similarly, we define

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} \quad \mathbf{d}(k) = \begin{bmatrix} \mathbf{d}_1(k) \\ \mathbf{d}_2(k) \end{bmatrix}.$$

In addition, we adopt the following assumption for simplicity.

Assumption 1: The pairs (A, C_1) and (A, C_2) are not observable. In addition, we assume that the overall system is observable, i. e., the pair (A, C) is observable.

Assumption 1 corresponds to the more difficult scenario where the controller might have to combine the information gathered from $\mathbf{d}_1$ and $\mathbf{d}_2$. Later we show that the stability analysis for the case where (A, C_1) and (or) (A, C_2) are observable constitutes a particular case of our analysis. Thus, the assumption is without loss of generality.

Definition II.1 (Erasure Link Model): Consider that $\{\mathbf{r}_1(k)\}_{k=0}^{\infty}$ and $\{\mathbf{r}_2(k)\}_{k=0}^{\infty}$ represent Bernoulli stochastic processes taking values in the set $\{\mathbf{1}, \emptyset\}$ and characterized by a probability mass function of the following type:

$$p_{i,j} \overset{def}{=} Pr\left(\mathbf{r}(k) = (i,j)\right), \ (i,j) \in \{\mathbf{1}, \emptyset\}^2$$

where $\mathbf{r}(k) \overset{def}{=} (\mathbf{r}_1(k), \mathbf{r}_2(k))$. The process $\mathbf{r}(k)$ governs the state of the links that connect the sensors to the controller. The relationship between sensor i's output $\mathbf{s}_i(k)$ and the controller's input $\mathbf{z}_i(k)$ is described by

$$\mathbf{z}_i(k) = \begin{cases} \emptyset & \text{if } \mathbf{r}_i(k) = \emptyset \\ \mathbf{s}_i(k) & \text{if } \mathbf{r}_i(k) = \mathbf{1} \end{cases}, \ i \in \{1, 2\} \quad (3)$$

where we adopt the symbol $\emptyset$ to represent erasure, i.e., it indicates that the information sent from sensor i to the controller was lost.

Note that, in general, we do *not* assume that the erasure events in the channels are independent. However, we presuppose that the sources of randomness $\mathbf{z}(0)$, $\{\mathbf{r}(k)\}_{k=0}^{\infty}$, $\{\mathbf{v}(k)\}_{k=0}^{\infty}$ and $\{\mathbf{w}(k)\}_{k=0}^{\infty}$ are mutually independent.

We consider sensors with the following functional structure:

Definition II.2 (sensor Map Classes $\mathbb{S}_q$ and $\mathbb{S}_q^{NAK}$): For any given positive integer q, define $\mathbb{S}_q$ as the set containing all sensor maps with the following structure:

$$\mathbf{s}_i(k)=\mathcal{S}\left(k,\mathbf{d}_i(0),\ldots,\mathbf{d}_i(k),\mathbf{r}(0),\ldots,\mathbf{r}(k-1)\right),\ k\geq 0 \quad (4)$$

where i is in the set $\{1,2\}$ and $\mathbf{s}_i(k)$ takes values in $\mathbb{R}^q$. Notice that we assume that $\{\mathbf{r}(j)\}_{j=0}^{k-1}$ is made available to the sensor via noiseless acknowledgments. In addition, consider the set $\mathbb{S}_q^{NAK}$ of sensor maps with the structure

$$\mathbf{s}_i(k)=\mathcal{S}^{NAK}\left(k,\mathbf{d}_i(0),\ldots,\mathbf{d}_i(k)\right),\ k\geq 0 \quad (5)$$

where i is in the set $\{1,2\}$ and $\mathbf{s}_i(k)$ takes values in $\mathbb{R}^q$. $\mathbb{S}_q^{NAK}$ is the subset of $\mathbb{S}_q$ consisting of the sensor structures that do not rely on the knowledge of past values of the erasure process $\{\mathbf{r}(j)\}_{j=0}^{k-1}$. Equivalently, $\mathbb{S}_q^{NAK}$ can be specified as the set of sensor maps for which the sensor does not have access to acknowledgment signals. In the sequel, we will also refer to sensor maps as encoding or information processing algorithms, and to sensors as encoders.

Definition II.3 (Controller Class): Consider stochastic processes $\mathbf{z}_1(k)$ and $\mathbf{z}_2(k)$ taking values in $\mathbb{R}^q\bigcup\{\emptyset\}$. We define the controller class $\mathbb{K}$ as the set of all controllers with the following structure:

$$\mathbf{u}(k)=\mathcal{K}(k,\mathbf{z}_1(0),\mathbf{z}_2(0),\ldots,\mathbf{z}_1(k),\mathbf{z}_2(k),\mathbf{u}(0),\cdots,\mathbf{u}(\mathbf{k}-\mathbf{1})) \quad (6)$$

where $\mathbf{u}(k)$ takes values in $\mathbb{R}^l$ and l is the dimension of the control input to the process specified in (1).

Given a process and the erasure link statistics, specified by the probability mass function $p_{i,j}$, we want to minimize a quadratic cost function using controllers and sensor maps in the classes $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$ and investigate conditions for the existence of such controllers and sensor maps that stabilize the process in the following sense.

Definition II.4 (Cost Function and Stability Criterion): Consider the set-up of Fig. 1 and assume that the matrices A, B, C_1, C_2 and the erasure link statistics $p_{i,j}$ are given. For a given realization of the erasure processes, we wish to find the integer q, controller $\mathcal{K}$ and sensor maps $\mathcal{S}_1$ and $\mathcal{S}_2$ (in the specified class $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$) that minimize the familiar quadratic cost function

$$J_K=E_{\beta(k)}\left[\sum_{k=0}^{K}\left(\mathbf{z}(k)^TQ\mathbf{z}(k)+\mathbf{u}(k)^TR\mathbf{u}(k)\right)\right.$$
$$\left.+\mathbf{z}(K+1)^TP_{K+1}\mathbf{z}(K+1)\right]. \quad (7)$$

In the above equation, Q and R are positive definite matrices, $\mathbf{z}(k)$ is the state of the process and the set $\beta(k)\overset{def}{=}\{\mathbf{z}(0),\mathbf{v}(i),\mathbf{w}(i)\}_{i=0}^{k}$ is used to indicate that the expectation is taken with respect to the initial condition, the process noise and the measurement noise. For a given statistical description of the erasure processes, we consider a selection of controller $\mathcal{K}$, integer q and sensor maps $\mathcal{S}_1$ and $\mathcal{S}_2$ to be stabilizing if and only if

$$E_{\mathbf{r}_i}[J_\infty]\overset{def}{=}\lim_{K\longrightarrow\infty}E_{\mathbf{r}_i}\left[\frac{J_K}{K}\right]<\infty \quad (8)$$

where $E_{\mathbf{r}_i}[\cdot]$ indicates that the expectation is further taken with respect to the erasure events.

We wish to emphasize that the expectation in (7) is not taken with respect to the erasure processes $\mathbf{r}(i)$; the design we propose will often be optimal for any realization of the packet dropping process. Also, we make no claim about the uniqueness of either the controller or the sensor map. We will denote the minimal cost achieved in (7) by using an optimal controller for a particular encoding algorithm $\mathcal{A}$ as $J_K^{\star,\mathcal{A}}$. Finally, for a comparison between our framework and existing work on stabilizability of decentralized control under data-rate constraints, see Section III-A.

III. Conditions for Stability

In this section, we state the necessary and sufficient conditions for stabilization. The proofs of the results will be constructed in several stages, going from the proof of a separation principle in Section IV to the description of an optimal control algorithm in Section V. We will use the following result regarding the state space representation of linear processes of the form (1)–(2) that is proved in the Appendix.

Proposition III.1: Consider an n-dimensional linear and time-invariant process satisfying Assumption 1 and let $\mathbf{d}_1(k)$ and $\mathbf{d}_2(k)$, taking values in $\mathbb{R}^{m_1}$ and $\mathbb{R}^{m_2}$, constitute a bi-partition of the process output. We can always construct a state-space representation with the structure (1)–(2), where the matrices $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times l}$, $C_1\in\mathbb{R}^{m_1\times n}$ and $C_2\in\mathbb{R}^{m_2\times n}$ are written in one and only one of the following forms, which we refer to as type I and type II. The first possibility, denoted as type I, is given by:

$$\begin{aligned}A&=\begin{bmatrix}A_{1,1}&A_{1,2}\\ \mathbf{0}^{n_2\times n_1}&A_{2,2}\end{bmatrix}\\ C_1&=\begin{bmatrix}\mathbf{0}^{m_1\times n_1}&C_{1,2}\end{bmatrix}\\ C_2&=\begin{bmatrix}C_{2,1}&\mathbf{0}^{m_2\times n_2}\end{bmatrix}\end{aligned} \quad (9)$$

where $A_{i,j}\in\mathbb{R}^{n_i\times n_j}$, $C_{i,j}\in\mathbb{R}^{m_i\times n_j}$ and $n_1+n_2=n$. The following is the second possibility (type II):

$$\begin{aligned}A&=\begin{bmatrix}A_{1,1}&A_{1,2}&A_{1,3}\\ \mathbf{0}^{n_2\times n_1}&A_{2,2}&A_{2,3}\\ \mathbf{0}^{n_3\times n_1}&\mathbf{0}^{n_3\times n_2}&A_{3,3}\end{bmatrix}\\ C_1&=\begin{bmatrix}\mathbf{0}^{m_1\times n_1}&C_{1,2}&C_{1,3}\end{bmatrix}\\ C_2&=\begin{bmatrix}C_{2,1}&\mathbf{0}^{m_2\times n_2}&C_{2,3}\end{bmatrix}\end{aligned} \quad (10)$$

where $A_{i,j}\in\mathbb{R}^{n_i\times n_j}$, $C_{i,j}\in\mathbb{R}^{m_i\times n_j}$ and $n_1+n_2+n_3=n$.

In the above representations (of types I or II), $A_{1,1}$ describes the dynamics of the state subspace that is not observable from $\mathbf{d}_1(k)$, while the modes that are not observable by $\mathbf{d}_2(k)$ are dictated by the dynamics of $A_{2,2}$. If the representation is of type II, then $A_{3,3}$ specifies the dynamics of the modes that are observable by both $\mathbf{d}_1(k)$ and $\mathbf{d}_2(k)$. Such representations are particularly convenient for the purposes of this paper. Alternative modal decomposition for decentralized processes are also possible, see, for instance, [1]. Using the representation above, we can state the necessary conditions for stabilizability of the process as follows. The proof is provided in Section V-B.

Theorem III.2 (Necessary Conditions for Stabilizability): Consider the problem set-up of Fig. 1 and let $A\in\mathbb{R}^{n\times n}$,

$B \in \mathbb{R}^{n\times l}$, $C_1 \in \mathbb{R}^{m_1\times n}$ and $C_2 \in \mathbb{R}^{m_2\times n}$ be given matrices specifying the state-space representation for the process. In addition, assume that the process satisfies Assumption 1 and that the statistics of the erasure links is specified by a given probability mass function $Pr(\mathbf{r}(k) = (i,j))$, with $(i,j) \in \{\mathbf{1}, \emptyset\}^2$ that is independent of the time index k. Suppose that the state-space representation can be written as in (9) (type I). There exists a controller in the class $\mathbb{K}$, a positive integer q and sensors in the class $\mathbb{S}_q$ such that the closed loop system is stable only if the following inequalities hold:

$$\rho(A_{1,1})^2 Pr(\mathbf{r}_2(k) = \emptyset) < 1 \qquad (11)$$
$$\rho(A_{2,2})^2 Pr(\mathbf{r}_1(k) = \emptyset) < 1 \qquad (12)$$

where $\rho(A_{i,i})$ represents the spectral radius of the matrix $A_{i,i}$. If, instead, the state-space representation is of type II, i. e. of the form (10), then necessary conditions for stabilization also include the following additional inequality:

$$\rho(A_{3,3})^2 Pr\left(\mathbf{r}(k) = (\emptyset, \emptyset)\right) < 1. \qquad (13)$$

Remark III.1: The case when Assumption 1 does not hold and the process is observable using only one sensor has already been considered in the literature [13]. Our results can be applied to this case if we adopt the convention that the spectral radius of an empty matrix is 0. Thus, e.g., if the entire state is observable from $\mathbf{d}_1(k)$, then the spectral radius of $A_{1,1}$ is assumed to be 0. A similar statement can be said about the sufficiency conditions given below as well. Thus we will assume that Assumption 1 holds in our analysis from now on.

It turns out that the above conditions are also sufficient for stabilizability for sensors in the class $\mathbb{S}_q^{NAK}$ (and hence in the class $\mathbb{S}_q$). The following result is proved in Section V -C.

Theorem III.3 (Sufficient Conditions for Stabilizability): Consider the set-up of Fig. 1 and let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, $C_1 \in \mathbb{R}^{m_1\times n}$ and $C_2 \in \mathbb{R}^{m_2\times n}$ be given matrices specifying the state-space representation for the process. In addition, assume that the process is controllable and that it satisfies Assumption 1. In addition, let the statistics of the erasure link, given by the probability mass function $Pr(\mathbf{r}(k) = (i,j)), (i,j) \in \{\mathbf{1}, \emptyset\}^2$, be given. Consider that the state space representation can be written as in (9) (type I). There exists a controller of class $\mathbb{K}$, a positive integer q and sensors of class $\mathbb{S}_q^{NAK}$ such that the feedback system is stable, if the following two inequalities hold:

$$\rho(A_{1,1})^2 Pr(\mathbf{r}_2(k) = \emptyset) < 1 \qquad (14)$$
$$\rho(A_{2,2})^2 Pr(\mathbf{r}_1(k) = \emptyset) < 1 \qquad (15)$$

where $\rho(A_{i,i})$ represents the spectral radius of the matrix $A_{i,i}$. If the state-space representation is of type II, i.e., it is of the form (10), then stability is assured by requiring that the following additional inequality also holds:

$$\rho(A_{3,3})^2 Pr\left(\mathbf{r}(k) = (\emptyset, \emptyset)\right) < 1. \qquad (16)$$

Remark III.2: The inequalities in Theorems III.2 and III.3 are identical. However, Theorem III.3 states that if such inequalities hold then stabilization is achievable by using sensors of class $\mathbb{S}_q^{NAK}$, while Theorem III.2 characterizes the necessary condition for stabilization by allowing sensors of class $\mathbb{S}_q$. The fact that $\mathbb{S}_q^{NAK} \subset \mathbb{S}_q$, leads to the interesting conclusion that the use of acknowledgement signals $\{\mathbf{r}(j)\}_{j=0}^{k-1}$ at the sensors does not impact stabilizability. The use of $\{\mathbf{r}(j)\}_{j=0}^{k-1}$ is crucial, however, in an optimal control strategy that will be identified in Section V.

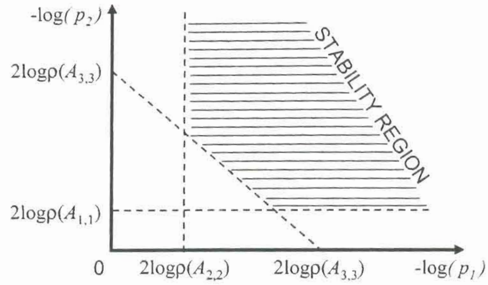

Fig. 2. Log-convex stabilizability region in terms of $p_1 = Pr(\mathbf{r}_1(k) = \emptyset)$ and $p_2 = Pr(\mathbf{r}_2(k) = \emptyset)$, under the assumption that $\mathbf{r}_1(k)$ and $\mathbf{r}_2(k)$ are independent.

Remark III.3: The stabilizability conditions make intuitive sense. The quantity $\rho(A_{1,1})^2$ measures the rate of increase of the second moment of the modes that are observable using only sensor 2. To keep the estimate error covariance of these modes bounded, we need the information about these modes from sensor 2 to arrive often enough. Equation (11) formalizes this relation. Similarly, the inequality in (12) places a constraint on the probability of erasure of information from sensor 1 in terms of the rate of increase of the modes that are observable solely through sensor 1. Finally, the relation in (13) places a constraint on the probability of erasure of information from at least one of the sensors in terms of the modes that are observable from either sensor.

If the erasure processes $\mathbf{r}_1(k)$ and $\mathbf{r}_2(k)$ are independent, then the inequalities in Theorems III.2 and III.3 lead to a log-convex stabilizability region, in terms of the erasure probabilities (see Fig. 2).

A. *Comparison With Existing Results on Decentralized Stabilizability Under Data-Rate Constraints*

The works [23], [26], [37], [40] have derived necessary and sufficient conditions under a similar framework, but considering finite data-rate communication links. It is interesting to note that [37, Fig 3.] also defines a convex stabilizability region, in terms of data-rates, which is similar to our Fig. 2. However, we must stress that our result cannot be derived from the finite data-rate framework of any of these works because of the following main reasons:

1) The work of [23], [26], [37], [40] considers communication links featuring deterministic and finite data-rate constrains. Their results are derived based on information theoretic ideas and counting arguments. In contrast, our results cannot be derived using such arguments because our erasure links have infinite capacity (in the information theoretic sense), provided that the probability of erasure at the links is strictly less than one[1]. Existing work for finite data-rate (stochastic) erasure channels addresses only

[1]We assume that quantization error is not an issue since typically a communication packet will assign a large number of bits for the data transmitted by the sensors, see, e.g., [10].

a single link and it does not provide guarantees of optimality [22], [30].

2) Our work considers measurement and process noise while these works focus on autonomous asymptotic stability.
3) We also derive optimal control strategies, while the work of [37], [40] addresses solely stabilizability.

IV. A SEPARATION PRINCIPLE

We begin by presenting a separation principle that allows us to consider an equivalent estimation problem instead of the control problem formulated above. At any time k, define the time-stamp corresponding to sensor i as

$$\mathbf{t}_i(k) = \max\{j \in \{0,1,2,\ldots\} \mid j \leq k,\ \mathbf{r}_i(j) = \mathbf{1}\}.$$

The time-stamp denotes the latest time at which transmission was possible from sensor i. Using the time-stamp, we can define the maximal information set $\mathcal{I}_i^{\max}(k)$ for sensor i as

$$\mathcal{I}_i^{\max}(k) = \{\mathbf{d}_i(0), \mathbf{d}_i(1), \ldots, \mathbf{d}_i(\mathbf{t}_i(k)), \mathbf{r}_i(0), \mathbf{r}_i(1), \ldots, \mathbf{r}_i(k)\}.$$

The maximal information set is the largest set of measurements from sensor i that the controller can possibly have access to at time k. For any encoding algorithm $\mathcal{A}$, we can also define the information set corresponding to sensor i at time k as

$$\mathcal{I}_i^{\mathcal{A}}(k) = \{\mathbf{s}_i(0), \mathbf{s}_i(t_i(1)), \mathbf{s}_i(t_i(2)), \ldots, \mathbf{s}_i(t_i(k))\}$$

where $\mathbf{s}_i(k)$ is the output of the sensor i at time k, when the algorithm $\mathcal{A}$ is followed at sensor i. $\mathcal{I}_i^{\mathcal{A}}(k)$ comprises the time-samples of $\mathbf{s}_i(k)$ which can be recovered at the controller via the output of the erasure link from sensor i. For any encoding algorithm, the inclusion $\mathbb{I}_i^{\mathcal{A}}(k) \subseteq \mathbb{I}_i^{\max}(k)$ holds, where $\mathbb{I}_i^{\mathcal{A}}(k)$ and $\mathbb{I}_i^{\max}(k)$ are the smallest sigma algebras (filtrations) generated by $\mathcal{I}_i^{\mathcal{A}}(k)$ and $\mathcal{I}_i^{\max}(k)$, respectively.

Consider two encoding algorithms $\mathcal{A}_1$ and $\mathcal{A}_2$ that guarantee at every time step

$$\mathbb{I}_1^{\mathcal{A}_1}(k) \subseteq \mathbb{I}_1^{\mathcal{A}_2}(k), \quad \mathbb{I}_2^{\mathcal{A}_1}(k) \subseteq \mathbb{I}_2^{\mathcal{A}_2}(k).$$

With an optimal controller design for the two algorithms, the values of cost J_K achieved will satisfy $J_K^{\star,\mathcal{A}_1} \geq J_K^{\star,\mathcal{A}_2}$. Now consider an algorithm $\bar{\mathcal{A}}$ under which, at every time step k the encoder for sensor i transmits the set

$$S_i(k) = \{\mathbf{d}_i(0), \mathbf{d}_i(1), \ldots, \mathbf{d}_i(k), \mathbf{r}_i(0), \mathbf{r}_i(1), \ldots, \mathbf{r}_i(k)\}.$$

Note that the algorithm $\bar{\mathcal{A}}$ does not specify valid sensor maps $\mathbb{S}_q$ since the dimension of the transmitted vectors cannot be bounded by any constant q. However, if algorithm $\bar{\mathcal{A}}$ is followed, at any time step k, the decoder (and the controller) would have access to the maximal information sets $\mathcal{I}_1^{\max}(k)$ and $\mathcal{I}_2^{\max}(k)$. This implies that for any other encoding algorithm $\mathcal{A}$, the cost function $J_K^{\star,\bar{\mathcal{A}}} \leq J_K^{\star,\mathcal{A}}$.

Thus, in particular, one way to achieve the optimal value of J_K is through the combination of an encoding algorithm that makes the information sets $\mathcal{I}_i^{\max}(k)$'s available to the controller and a controller that optimally utilizes the information set. Further, one such information processing algorithm is the algorithm $\bar{\mathcal{A}}$ described above. However, this algorithm relies on the transmission of real vectors whose dimension increases linearly over time. In the sequel, we show that this difficulty can be avoided in the presence of noiseless acknowledgments. In particular, we prove that optimal performance can be achieved by using sensors of the class $\mathbb{S}_q$, where q is a finite constant quantifying the dimension of the transmitted vectors.

To this end, we begin by a statement of the familiar separation principle when algorithm $\bar{\mathcal{A}}$ is used. For a random variable $\boldsymbol{\alpha}(k)$, denote by $\hat{\boldsymbol{\alpha}}(k|\boldsymbol{\beta}(k))$ the minimum mean squared error (MMSE) estimate of $\boldsymbol{\alpha}(k)$ given the information $\boldsymbol{\beta}(k)$.

Proposition IV.1 (Separation Principle): Consider the problem defined in Section II. Suppose that the encoding algorithm $\bar{\mathcal{A}}$ as described above is followed, so that the controller has access to the maximal information sets $\mathcal{I}_i^{\max}(k)$'s at every time step k. Then, an optimal control input at time k is calculated by using the relation

$$\mathbf{u}(k) = \hat{\mathbf{u}}_{LQR}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1}\right)$$

where $\mathbf{u}_{LQR}(k)$ denotes the optimal LQR control law corresponding to the cost function (7) under full state feedback.

Proof: The proof uses standard dynamic programming arguments. We need to choose $\mathbf{u}(0), \mathbf{u}(1), \ldots, \mathbf{u}(K)$ that minimize J_K. We begin by gathering terms that depend on the choice of $\mathbf{u}(K)$ and $\mathbf{x}(K)$ and writing them as

$$\Upsilon(K) = E\big[\mathbf{x}^T(K)Q\mathbf{x}(K) + \mathbf{u}(K)R\mathbf{u}(K) + \mathbf{x}^T(K+1)P_{K+1}\mathbf{x}(K+1)\big].$$

Utilizing the process dynamics and the fact that the noise $\mathbf{w}(k)$ is white and zero mean, we can rewrite $\Upsilon(K)$ as

$$\Upsilon(K) = V(K) + O(K)$$

where

$$V(K) = E\big[\mathbf{x}^T(K)Q\mathbf{x}(K) + \mathbf{u}^T(K)R\mathbf{u}(K) + (A\mathbf{x}(K) + B\mathbf{u}(K))^T P_{K+1}(A\mathbf{x}(K) + B\mathbf{u}(K))\big]$$

and $O(K) = E[\mathbf{w^T(K)P_{K+1}w(K)}]$ depends only on the noise

$$J_K = E\left[\sum_{k=0}^{K-1}\left(\mathbf{x}^T(k)Q\mathbf{x}(k) + \mathbf{u}^T(k)R\mathbf{u}(k)\right)\right] + V(K) + O(K).$$

Thus, the only term affected by the choice of $\mathbf{u}(K)$ is $V(K)$. Completing the squares, we can write $V(K)$ as

$$V(K) = E\Big[(\mathbf{u}(K) - \mathbf{u}_{LQR}(K))^T S_K \times (\mathbf{u}(K) - \mathbf{u}_{LQR}(K)) + \mathbf{x}^T(K)P_K\mathbf{x}(K)\Big]$$

where

$$S_K = R + B^T P_{K+1} B$$
$$P_K = Q + A^T P_{K+1} A - A^T P_{K+1} B_1 S_K^{-1} B_1^T P_{K+1} A$$

and $u_{LQR}(K)$ is the LQR control law corresponding to the cost function (7) under full state feedback. If the controller had access to the entire state, it could have chosen the optimal control input as $\mathbf{u}_{LQR}(K)$. However, that is not possible now. Instead, the controller needs to calculate $\mathbf{u}(K)$ to minimize $V(K)$ using the information it has access to. The control problem, thus, reduces to an optimal (in the sense of minimum mean squared error (MMSE)) estimation problem. We can write the optimal control at time step T as

$$\mathbf{u}(K) = \hat{\mathbf{u}}_{LQR}(K|\mathcal{I}_1^{\max}(K), \mathcal{I}_2^{\max}(K), \{\mathbf{u}_1(t)\}_{t=0}^{K-1}).$$

Since all the random variables are Gaussian, the optimal estimator $\mathbf{u}(K)$ is linear. Denote the estimation error incurred due to the minimizing choice of $\mathbf{u}(K)$ by $\Omega(K)$. We have

$$V(K) = \Omega(K) + E\left[\mathbf{x}^T(K) P_K \mathbf{x}(K)\right].$$

We can thus write the cost function as

$$J_K = J_{K-1} + \Omega(K) + O(K).$$

Thus, we now need to choose control inputs for time steps 0 to $T-1$ to minimize J_K. By scanning the terms on the right hand side of the equation, we see that $O(K)$ is independent of the choice of control laws from time 0 to $K-1$. Similarly, $\Omega(K)$ is also independent of all previous control laws since these control inputs are known while calculating $\hat{\mathbf{u}}_{LQR}(K|\mathcal{I}_1^{\max}(K), \mathcal{I}_2^{\max}(K), \{\mathbf{u}_1(t)\}_{t=0}^{K-1})$. Thus, the only term that is dependent on the control inputs till time step $K-1$ is J_{K-1}. But our argument so far was independent of time index K. Thus, we can recursively apply the argument above for time steps $K-1$, $K-2$ and so on. □

There are two reasons this separation principle is useful:

1) We recognize that an optimal controller does not need to have access to the information sets $\mathcal{I}_i^{\max}(k)$'s at every time step k. The encoders and the decoder only need to ensure that the controller receives the quantity $\hat{\mathbf{u}}_{LQR}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1}\right)$, or equivalently, $\hat{\mathbf{z}}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1}\right)$.
2) If the controller has access to this quantity, the controller design part of the problem is solved. The optimal controller is the solution to the LQR control problem.

Our next result allows us to make another simplification in the problem by separating the dependence of the estimate on measurements from the effect of the control inputs. In the context of our problem, this is useful since the encoders do not have access to the control inputs[2]. Thus, the effect of the previous control inputs has to be included by the controller that has access to all previous control inputs. To this end, we state the following result.

Proposition IV.2 (Separation of Control and Measurement Effects): Consider the problem as formulated in Section II. The quantity $\hat{\mathbf{z}}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1}\right)$ can be calculated as the sum of two quantities

$$\hat{\mathbf{z}}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1}\right) = \bar{\mathbf{z}}(k) + \boldsymbol{\psi}(k)$$

where $\bar{\mathbf{z}}(k)$ depends only on the information sets $\mathcal{I}_i^{\max}(k)$' (and not on the control inputs $\{\mathbf{u}(t)\}_{t=0}^{k-1}$) and $\psi(k)$ depend only on the control inputs (but not on the information sets).

Proof: The proof follows from the linearity of the Kalman filter. Assume, without loss of generality that $\mathbf{t}_1(k) \leq \mathbf{t}_2(k)$ The effect of the measurements can be included using the following modified Kalman filter:

Measurement Update for the modified Kalman Filter

$$\begin{aligned}
&(\mathbf{P}(m|m,m))^{-1} = (\mathbf{P}(m|m-1,m-1))^{-1} + C^T\mathbf{R}_\mathbf{v}^{-1}C\\
&(\mathbf{P}(m|m,m))^{-1}\acute{\mathbf{z}}(m|\{\mathbf{d}_1(t)\}_{t=0}^{m}, \{\mathbf{d}_2(t)\}_{t=0}^{m})\\
&= (\mathbf{P}(m|m-1,m-1))^{-1}\acute{\mathbf{z}}(m|\{\mathbf{d}_1(t)\}_{t=0}^{m-1}, \{\mathbf{d}_2(t)\}_{t=0}^{m-1})\\
&\quad + C^T\mathbf{R}_\mathbf{v}^{-1}\mathbf{d}(m).
\end{aligned}$$

Time Update for the modified Kalman Filter

$$\begin{aligned}
\mathbf{P}(m|m-1,m-1) &= A\mathbf{P}(m-1|m-1,m-1)A^T + \mathbf{R}_w\\
\acute{\mathbf{z}}(m|\{\mathbf{d}_1(t)\}_{t=0}^{m-1}, &\{\mathbf{d}_2(t)\}_{t=0}^{m-1})\\
&= A\acute{\mathbf{z}}\left(m-1|\{\mathbf{d}_1(t)\}_{t=0}^{m-1}, \{\mathbf{d}_2(t)\}_{t=0}^{m-1}\right).
\end{aligned}$$

The initial conditions are given by $\acute{\mathbf{z}}\left(0|\mathbf{d}_1(-1), \mathbf{d}_2(-1)\right) = 0$ and $\mathbf{P}(0|-1,-1) = \mathbf{P}(0)$. We calculate the quantity $\acute{\mathbf{z}}(t_2(k)|\{\mathbf{d}_1(t)\}_{t=0}^{\mathbf{t}_1(k)}, \{\mathbf{d}_2(t)\}_{t=0}^{\mathbf{t}_2(k)})$ using the measurement from both sensors from time 0 to $t_1(k)$ and only the sensor 2 from time $t_1(k+1)$ to $t_2(k)$ according to the above filter. The effect of the control inputs can be calculated through the term $\tilde{\psi}(j)$ that evolves as

$$\tilde{\psi}(m) = B\mathbf{u}(m-1) + \Gamma(m-1)\tilde{\psi}(m-1)$$

where

$$\Gamma(m) = \begin{cases} A\mathbf{P}^{-1}(m-1|m-1,m-1) & \\ \times\mathbf{P}(m-1|m-2,m-2) & m \leq \mathbf{t}_1(k)+1 \\ A\mathbf{P}^{-1}(m-1|\mathbf{t}_1(k),m-1) & \\ \times\mathbf{P}(m-1|\mathbf{t}_1(k),m-2), & \text{otherwise} \end{cases}$$

and the initial condition is $\tilde{\psi}(0) = 0$. To complete the proof, we simply identify

$$\bar{\mathbf{z}}(k) = A^{k-\mathbf{t}_2(k)}\acute{\mathbf{z}}\left(\mathbf{t}_2(k)|\{\mathbf{d}_1(t)\}_{t=0}^{\mathbf{t}_1(k)}, \{\mathbf{d}_2(t)\}_{t=0}^{\mathbf{t}_2(k)}\right)$$

$$\psi(k) = A^{k-\mathbf{t}_2(k)}\tilde{\psi}(\mathbf{t}_2(k)) + \sum_{i=0}^{k-\mathbf{t}_2(k)-1} A^i B\mathbf{u}(k-i-1).$$

For brevity of notation, in the sequel, we will denote

$$\bar{\mathbf{z}}(m|l,n) = \bar{\mathbf{z}}(m|\{\mathbf{d}_1(t)\}_{t=0}^{l}, \{\mathbf{d}_2(t)\}_{t=0}^{n}).$$

Remark IV.1: It is important to emphasize that $\bar{\mathbf{z}}(m|l,n)$ is not the estimate of $\mathbf{z}(m)$ based only on measurements from sensor 1 till time l and from sensor 2 till n. Moreover, as long as $\mathbf{z}(k)$ is stabilized (e.g., by using the algorithm described in the next section), the measurements $\mathbf{d}_i(k)$, and hence the quantity $\bar{\mathbf{z}}(m|l,n)$ will remain stable.

Consider an additional open loop process $\mathbb{P}_2$ that evolves

$$\mathbf{x}(k+1) = A\mathbf{x}(k) + \mathbf{w}(k), k \geq 0.$$

[2]Note that even the cost function may be unknown at the encoders.

The process state is observed by two sensors of the form

$$\mathbf{y}_1(k) = C_1\mathbf{x}(k) + \mathbf{v}_1(k)$$
$$\mathbf{y}_2(k) = C_2\mathbf{x}(k) + \mathbf{v}_2(k)$$

where $\mathbf{x}(0) = \mathbf{z}(0)$ and the noises have the same value at every time step as those appearing in the description of the process $\mathbb{P}_1$ (and the corresponding sensors). The encoders now have access to the measurements $\mathbf{y}_i(k)$ and transmit over analog erasure channels to an estimator that needs to calculate the MMSE estimate $\hat{\mathbf{x}}(k)$ of $\mathbf{x}(k)$ based on information it receives. The cost function is to minimize the mean squared error based cost function

$$D_k = E\left[(\mathbf{x}(k) - \hat{\mathbf{x}}(k))(\mathbf{x}(k) - \hat{\mathbf{x}}(k))^T\right]. \quad (18)$$

Either of the communication links suffers an erasure at time k if and only if the corresponding communication link in the original problem for the process $\mathbb{P}_1$ suffers an erasure. Any algorithm that ensures that the estimator in process $\mathbb{P}_2$ has access to $\bar{\mathbf{x}}(k|\mathbf{t}_1(k), \mathbf{t}_2(k))$ would ensure that the controller for $\mathbb{P}_1$ has access to $\bar{\mathbf{z}}(k|\mathbf{t}_1(k), \mathbf{t}_2(k))$. Note that for process $\mathbb{P}_2$, the quantity $\bar{\mathbf{x}}(k|m, n)$ is precisely the MMSE estimate of $\mathbf{x}(k)$ given the measurements $\mathbf{y}_1(0), \ldots, \mathbf{y}_1(m)$, $\mathbf{y}_2(0), \ldots, \mathbf{y}_2(n)$. For the process $\mathbb{P}_2$, denote by $\mathbf{e}_2(k)$ the error between the state $\mathbf{x}(k)$ and the estimate $\hat{\mathbf{x}}(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k))$. Denote for the process $\mathbb{P}_1$, the error between the state $\mathbf{z}(k)$ and the estimate at the controller $\hat{\mathbf{z}}(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k), \{\mathbf{u}(t)\}_{t=0}^{k-1})$ by $\mathbf{e}_1(k)$. By Proposition IV.2, $\mathbf{e}_1(k) = \mathbf{e}_2(k)$ at every time step k. Note that the estimates for the two processes are not the same; only the errors are. Now, because of Proposition IV.1, it is simply the error $\mathbf{e}_1(k)$ that determines the stability and performance of the process $\mathbb{P}_1$. Thus, to provide the optimal algorithm and to analyze the stability and performance for the closed-loop system, we can simply look at the open-loop process $\mathbb{P}_2$. We will denote this estimation problem as $\mathcal{P}_2$. From this point on, we will concentrate on the problem $\mathcal{P}_2$ and exploit the equivalences noted above. In particular, the stability criterion presented in Definition II.4 for the process $\mathbb{P}_1$ corresponds to the condition that

$$\sup_{k\geq 0} E\left[\mathbf{e}_2^T(k)\mathbf{e}_2(k)\right] < \infty. \quad (19)$$

V. Optimal Algorithm and Stabilizability

In this section, we will first present a recursive encoding algorithm for sensors in the class $\mathbb{S}_q$ that allows the estimator to calculate the estimate $\hat{\mathbf{x}}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k)\right)$ and is, thus, optimal. We will then analyze the stability of the algorithm which shall prove Theorem III.2. Finally, we shall prove that the conditions required for stability using an optimal algorithm are sufficient for a particular algorithm in the sensor class $\mathbb{S}_q^{NAK}$, which shall prove Theorem III.3.

A. A Recursive Algorithm For Optimal Performance

We begin with the following result.

Proposition V.1: Consider the problem $\mathcal{P}_2$. Let $\hat{\mathbf{x}}_i(k|l)$ denote the MMSE estimate of $\mathbf{x}(k)$ based on all the measurements of sensor i up to time l. Denote the corresponding error covariance by $\mathbf{P}_i(k|l)$. The estimate $\hat{\mathbf{x}}(k|l, m)$ of the state based on measurements from sensor 1 till time l and sensor 2 till time m can be calculated using a relation of the form

$$\hat{\mathbf{x}}(k|l, m) = f\left(I_{1,l,m}(k), I_{2,l,m}(k)\right)$$

where $I_{1,l,m}(k)$ does not depend on sensor 2's measurements and $I_{2,l,m}(k)$ does not depend on sensor 1's measurements.

Proof: Proof is based on the algorithm proposed in [13]. Assume, without loss of generality, that $l \leq m$. The quantity $I_{1,l,m}(k)$ is calculated using the following algorithm. For the algorithm given below, we will abuse notation a bit, and consider $C_1 = 0$ (equivalently, sensor 1 did not take any measurements) for time steps greater than l, and $C_2 = 0$ for time steps larger than m for these calculations. At each time step $j \leq k$,

1) Use a Kalman filter to obtain $\hat{\mathbf{x}}_1(j|j)$ and $\mathbf{P}_1(j|j)$.
2) Calculate

$$\lambda_1(j) = \mathbf{P}_1^{-1}(j|j)\hat{\mathbf{x}}_1(j|j) - \mathbf{P}_1^{-1}(j|j-1)\hat{\mathbf{x}}_1(j|j-1).$$

3) Calculate global error covariance matrices $\mathbf{P}(j|j, j)$ and $\mathbf{P}(j|j-1, j-1)$ using the relation

$$\begin{aligned}\mathbf{P}^{-1}(j|j, j) &= \mathbf{P}^{-1}(j|j-1, j-1) \\ &\quad + C_1^T R_{v,1}^{-1} C_1 + C_2^T R_{v,2}^{-1} C_2 \\ \mathbf{P}(j|j-1, j-1) &= A\mathbf{P}(j-1|j-1, j-1)A^T + \mathbf{R}_w.\end{aligned}$$

4) Obtain

$$\gamma(j) = \left(\mathbf{P}(j|j-1, j-1)\right)^{-1} A\mathbf{P}(j-1|j-1, j-1).$$

5) Finally calculate

$$\begin{aligned}I_{1,j,j}(j) &= \lambda_1(j) + \gamma(j)I_{1,j-1,j-1}(j-1), \ j \leq l \\ I_{1,l,j}(j) &= \lambda_1(j) + \gamma(j)I_{1,l,j-1}(j-1), \ l \leq j \leq m \\ I_{1,l,m}(j) &= \lambda_1(j) + \gamma(j)I_{1,l,m}(j-1), \ j \geq m,\end{aligned} \quad (20)$$

with $I_{1,l,m}(-1) = 0$.

The quantity $I_{2,l,m}(k)$ is calculated by a similar algorithm except using the local estimates $\hat{\mathbf{x}}_2(j|j)$ and covariance $\mathbf{P}_2(j|j)$. Finally, the estimate $\hat{\mathbf{x}}(k|l, m)$ is calculated using the relation

$$\left(\mathbf{P}(k|k, k)\right)^{-1}\hat{\mathbf{x}}(k|l, m) = I_{1,l,m}(k) + I_{2,l,m}(k) \quad (21)$$

where $\mathbf{P}(k|k, k)$ is calculated as above. That $\hat{\mathbf{x}}(k|l, m)$ is indeed the MMSE estimate given all the measurements from sensor 1 till time l and from sensor 2 till time m can be proved by utilizing the block diagonal structure of the matrix $\mathbf{R}_\mathbf{v}$ as in the proof of Theorem 2 in [13]. □

The above result identifies quantities that need to be transmitted by the two sensors to calculate the MMSE estimate of $\mathbf{x}(k)$. The quantities depend only on local measurements at the sensors; however, an implicit assumption is that each sensor is informed about the times l and m.

Definition V.1 (Algorithm $\mathcal{A}_{ack}$): We now provide an optimal encoding algorithm, denoted for future reference as the algorithm $\mathcal{A}_{ack}$, for the sensor class $\mathbb{S}_q$. Let $\mathbf{P}(k|l, m)$ denote the error covariance of the MMSE estimate of the state $\mathbf{x}(k)$ calculated using measurements from sensor 1 till time l and from sensor 2 till time m. At each time step k

- Encoder for Sensor 1: Because of the noiseless acknowledgments, sensor 1 can calculate the time stamp $\mathbf{t}_2(k-1)$. Encoder 1 calculates and transmits two quantities: $I_{1,k,k}(k)$ and $I_{1,k,\mathbf{t}_2(k-1)}(k)$. Note that in both cases, measurements only from sensor 1 till time k are used.
- Encoder for Sensor 2: Sensor 2 calculates and transmits $I_{2,k,k}(k)$ and $I_{2,\mathbf{t}_1(k-1),k}(k)$.
- Decoder at the Estimator: The estimator maintains three quantities.
  — the estimate $\hat{\mathbf{x}}^{dec}(k)$ with the initial value $\hat{\mathbf{x}}^{dec}(-1) = 0$,
  — a vector $I_1^{dec}(k)$ for the contribution from sensor 1 with the initial value $I_1^{dec}(-1) = 0$.
  — a vector $I_2^{dec}(k)$ for the contribution from sensor 2 with the initial value $I_2^{dec}(-1) = 0$.

  At every time step k, the decoder faces one of four situations.

1) $\mathbf{r}_1(k) = \mathbf{r}_2(k) = \emptyset$: The decoder calculates

$$\begin{aligned} I_1^{dec}(k) &= \mathbf{P}^{-1}(k|\mathbf{t}_1(k), k)A \\ &\quad\times \mathbf{P}(k-1|\mathbf{t}_1(k-1), k-1)I_1^{dec}(k-1) \\ I_2^{dec}(k) &= \mathbf{P}^{-1}(k|k, \mathbf{t}_2(k))A \\ &\quad\times \mathbf{P}(k-1|k-1, \mathbf{t}_2(k-1))I_2^{dec}(k-1) \\ \hat{\mathbf{x}}^{dec}(k) &= A\hat{\mathbf{x}}^{dec}(k-1). \end{aligned}$$

2) $\mathbf{r}_1(k) = \emptyset, \mathbf{r}_2(k) = \mathbf{1}$: The decoder calculates

$$\begin{aligned} I_1^{dec}(k) &= \mathbf{P}^{-1}(k|\mathbf{t}_1(k), k)A \\ &\quad\times \mathbf{P}(k-1|\mathbf{t}_1(k-1), k-1)I_1^{dec}(k-1) \\ I_2^{dec}(k) &= I_{2,k,k}(k) \\ \hat{\mathbf{x}}^{dec}(k) &= \mathbf{P}(k|\mathbf{t}_1(k), k)(I_1^{dec}(k) + I_{2,\mathbf{t}_1(k),k}(k)). \end{aligned}$$

3) $\mathbf{r}_1(k) = \mathbf{1}, \mathbf{r}_2(k) = \emptyset$: The decoder calculates

$$\begin{aligned} I_1^{dec}(k) &= I_{1,k,k}(k) \\ I_2^{dec}(k) &= \mathbf{P}^{-1}(k|k, \mathbf{t}_2(k))A \\ &\quad\times \mathbf{P}(k-1|k-1, \mathbf{t}_2(k-1))I_2^{dec}(k-1) \\ \hat{\mathbf{x}}^{dec}(k) &= \mathbf{P}(k|k, \mathbf{t}_2(k))(I_{1,k,\mathbf{t}_2(k)}(k) + I_2^{dec}(k)). \end{aligned}$$

4) $\mathbf{r}_1(k) = \mathbf{r}_2(k) = \mathbf{1}$: The decoder calculates

$$\begin{aligned} I_1^{dec}(k) &= I_{1,k,k}(k) \\ I_2^{dec}(k) &= I_{2,k,k}(k) \\ \hat{\mathbf{x}}^{dec}(k) &= \mathbf{P}(k|k, k)\left(I_{1,k,k}(k) + I_{2,k,k}(k)\right). \end{aligned}$$

We can state the following result.

Theorem V.2: In the algorithm $\mathcal{A}_{ack}$

$$\hat{\mathbf{x}}^{dec}(k) = \hat{x}\left(k|\mathcal{I}_1^{\max}(k), \mathcal{I}_2^{\max}(k)\right)$$

where $\mathcal{I}_i^{\max}(k)$'s are the maximal information sets defined earlier.

Proof: The proof is straight-forward given Proposition V.1. At any time step k, the term $I_1^{dec}(k)$ equals $I_{1,\mathbf{t}_1(k),k}(k)$ and $I_2^{dec}(k)$ equals $I_{2,k,\mathbf{t}_2(k)}(k)$. For any of the four possi ities of channel outputs, it can be verified that the estimat calculated according to (21).

Note that the algorithm is optimal, yet involves a const amount of transmission and processing. Each sensor can cal late the terms it transmits using a recursive algorithm of the fo outlined in (20).

Remark V.1 (Optimality for Any Drop Sequence and 'Washing Away' Effect): So far, we have made no assumpti on the realization of the erasure process nor on the knowle of the statistics of the erasure events at any of the nodes. algorithm provides the optimal estimate for an arbitrary reali tion of the erasure process, irrespective of whether the eras process can be modeled as i.i.d. or as a more sophistica model like a Markov chain. The algorithm results in the opti estimate at every time step for any realization of the eras process, not merely in the optimal average performance. A note that if data is received from sensor i at any time step the effect of all previous erasures from that sensor is 'wash away'. The estimate at the receiving node becomes identical the case when all measurements $\mathbf{y}_i(0), \mathbf{y}_i(1), \ldots, \mathbf{y}_i(k)$ w available, irrespective of which previous data had been erase

Remark V.2: If the closed loop process $\mathbb{P}_1$ is stab $\bar{\mathbf{z}}(m|l,n)$, and hence the quantities $\mathcal{I}_{(.,.,.)}(.)$ transmitted by sensors, remain stable. If the state $\mathbf{z}(k)$ becomes unstable, course, these quantities will also become unbounded. Howev the optimality of the algorithm implies that, in such a ca no other quantity transmitted by the sensors would stabil the process. Note that the quantities being transmitted for problem $\mathcal{P}_2$ may not be stable. However, this problem has o been posed for technical convenience. Of course, problem may be of individual interest. Estimating an open-loop unsta process has been studied by many researchers recently analog erasure channels under a variety of settings. It may noted that the quantity transmitted in all these works (e.g., measurements) is unstable.

B. Necessary Conditions for Stabilizability

By analyzing the stability of the optimal algorithm $\mathcal{A}_{ack}$, can obtain necessary conditions for stability for any encod algorithm in the class $\mathbb{S}_q$ (and, in turn, $\mathbb{S}_q^{NAK}$). We shall n the following result.

Proposition V.3: Consider the process in (17) being obser by a sensor of the form

$$\bar{\mathbf{y}}(k) = \bar{C}\mathbf{x}(k) + \bar{\mathbf{v}}(k)$$

where $\bar{\mathbf{v}}(k)$ is white Gaussian noise with zero mean and cov ance $\mathbf{R}$. Let $f(X)$ denote the Riccati recursion correspond to this sensor as applied on the matrix X, thus

$$f(X) = AXA^T + \mathbf{R}_w - AX\bar{C}^T\left(\bar{C}X\bar{C}^T + \mathbf{R}\right)^{-1}\bar{C}XA \quad (\ldots$$

Further, let $f^m(X)$ denote the above Riccati recursion appl m times on the matrix X, i.e.

$$f^m(X) = \underbrace{f\left(f\left(\cdots f\left(X\right)\cdots\right)\right)}_{f \text{ applied } m \text{ times}}. \quad (\ldots$$

Finally, let p be a scalar. Then, the sum

$$S = X + pf(X) + p^2 f^2(X) + \cdots + p^m f^m(X) \qquad (24)$$

is bounded as $m \longrightarrow \infty$ if and only if

$$p\left|\rho(\bar{A})\right|^2 < 1$$

where $\rho(\bar{A})$ is the spectral radius of the unobservable part of A when the pair $(A, \bar{C})$ is put in the observer canonical form. In particular, if $\bar{C} = 0$, so that $f(X)$ is given by the Lyapunov recursion

$$f(X) = AXA^T + \mathbf{R}_w$$

then the sum (24) converges if and only if $p\left|\rho(A)\right|^2 < 1$, where $\rho(A)$ is the spectral radius of matrix A.

Proof: The proof follows along the lines of Theorem 4 in [11] by considering the evolution of estimate error covariance for the modes that are unobservable from $\bar{C}$. Details are omitted for space constraints. □

Proof of Theorem III.2: As discussed above, the stability conditions for process $\mathbb{P}_1$ are identical to those for $\mathbb{P}_2$. Define the Riccati operators $f_1(.)$, $f_2(.)$ and $f_\emptyset(.)$ in a fashion similar to (21) when sensor 1, sensor 2 and no sensor is used, respectively. Also define $f_1^m(.)$, $f_2^m(.)$ and $f_\emptyset^m(.)$ analogously. Finally, define $M(k)$ to be the error covariance of the MMSE estimate of $\mathbf{x}(k+1)$ when all the measurements from sensors 1 and 2 till time step k are available. Because of the assumption on observability of (A, C), $M(k)$ converges exponentially to a steady-state value denoted by $M^\star$.

Let $E[\mathbf{P}(k)]$ denote the expected error covariance of the estimate of the state $\mathbf{x}(k+1)$ as calculated at time k. The conditions required for stabilizability of the error covariance $E[\mathbf{P}(k)]$ are identical to those required for stabilizability of the expected error covariance of the estimate of the state $\mathbf{x}(k)$ calculated at time k. Define the events $\mathcal{E}_{mn}(k)$ as follows. The event $\mathcal{E}_{mn}(k)$ denotes that at time k, the last transmission was successfully received from sensor 1 at time m and from sensor 2 at time n. Moreover, we allow the indices to attain the value -1. For $m = -1$ or $n = -1$, the event $\mathcal{E}_{mn}(k)$ denotes the event when transmission from the corresponding sensor was never possible till time k. Thus $-1 \leq m, n \leq k$. Denote the error covariance conditioned on the event $\mathcal{E}_{mn}(k)$ happening by $\mathbf{P}_{mn}(k)$. Due to Theorem V.2, $\mathbf{P}_{mn}(k)$ is the error covariance for the MMSE estimate of $\mathbf{x}(k+1)$ based on measurements $\mathbf{y}_1(0)$, $\mathbf{y}_1(1), \ldots, \mathbf{y}_1(m)$ from sensor 1 and $\mathbf{y}_2(0)$, $\mathbf{y}_2(1), \ldots, \mathbf{y}_2(n)$ from sensor 2. Let p_{mn} be the (time-invariant) probability of the event $\mathcal{E}_{mn}(k)$ occurring. We can thus write

$$E[\mathbf{P}(k)] = \sum_{m=-1}^{k} \sum_{n=-1}^{k} p_{mn}\mathbf{P}_{mn}(k).$$

Since each term in the summation is positive semi-definite, a necessary condition for the sum to be bounded is that any sub-sequence in the sum is bounded. We will consider three particular sub-sequences and show that the conditions in (11)–(13) are necessary for stabilizability. First consider the sequence

$$\begin{aligned} S_1(k) &= \sum_{m=0}^{k} p_{mk}\mathbf{P}_{mn}(k) \\ &= Pr(\mathbf{r}_1(k) = \mathbf{1})Pr(\mathbf{r}_2(k) = \mathbf{1})\Big(M(k) \\ &\quad + Pr(\mathbf{r}_1(k) = \emptyset)f_2\left(M(k-1)\right) \\ &\quad + \cdots + \left(Pr(\mathbf{r}_1(k) = \emptyset)\right)^k f_2^k\left(M(0)\right)\Big). \end{aligned}$$

Since $M(k)$ converges exponentially to $M^\star$ as $k \rightarrow \infty$, we can substitute $M^\star$ for the conditional error covariances to study the convergence. Thus, we obtain

$$\lim_{k \longrightarrow \infty} S_1(k) = Pr(\mathbf{r}_1(k) = \mathbf{1})Pr(\mathbf{r}_2(k) = \mathbf{1}) \sum_{m=0}^{\infty} \left(Pr(\mathbf{r}_1(k) = \emptyset)\right)^m f_2^m\left(M^\star\right).$$

Thus, using Proposition V.3, we can prove that this sum converges only if (11) holds In a similar fashion, we can prove that the condition in (12) is necessary by considering the sub-sequence

$$S_2(k) = \sum_{n=0}^{k} p_{kn}\mathbf{P}_{kn}.$$

Finally, the sub-sequence

$$S_3(k) = \sum_{n=0}^{k} p_{nn}\mathbf{P}_{nn}$$

yields the necessary condition

$$\rho(A)^2 Pr\left(\mathbf{r}(k) = (\emptyset, \emptyset)\right) < 1. \qquad (25)$$

Since $\rho(A) = \max\{\rho(A_{i,i})\}$, there are two cases.

1) If $\rho(A) = \rho(A_{3,3})$, (25) reduces to (13) and the proof is complete.
2) If either $\rho(A) = \rho(A_{1,1})$ or $\rho(A) = \rho(A_{2,2})$, (25) is subsumed by either (12) or (11). Moreover, (25) implies (13). Thus, the proof is complete in this case as well.

□

C. Sufficient Conditions for Stabilizability

We now present the proof of Theorem III.3 by considering a particular algorithm in the class $\mathbb{S}_q^{NAK}$. Even though Proposition IV.1 considers only the case when the controller can estimate the state given the maximal information sets $\mathcal{I}_i^{\max}(k)$'s, we note that:

1) For any sensor map and estimator that guarantees that the state $\mathbf{z}(k)$ of the process evolving as in (1) can be estimated with bounded error covariance, a controller of the form $\mathbf{u}(k) = F\hat{\mathbf{z}}(k)$ with $\rho(A+F) < 1$, will guarantee stability of the closed loop system.
2) Since all the previous control inputs are known to the controller, the encoding algorithm only needs to ensure that

the state $\mathbf{x}(k)$ of the process $\mathbb{P}_2$ can be estimated with a bounded error covariance using sensors of the form $\mathbb{S}_q^{NAK}$.

We now propose such an algorithm, denoted by $\mathcal{A}_{nack}$. Due to Proposition III.1, we can consider the process to be either of type I or of type II. We can also partition the state space $\mathbf{x}(k)$ of the process in one of two ways.

1) If the process is of type I, denote

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k)^{n_1\times 1} \\ \mathbf{x}_2(k)^{n_2\times 1} \end{bmatrix}. \tag{26}$$

2) If the process is of type II, denote

$$\mathbf{x}(k) = \begin{bmatrix} \mathbf{x}_1(k)^{n_1\times 1} \\ \mathbf{x}_2(k)^{n_2\times 1} \\ \mathbf{x}_3(k)^{n_3\times 1} \end{bmatrix}. \tag{27}$$

Now consider the following algorithm. At each time step k

- Encoder for Sensor 1:
  - If the process is of type I, sensor 1 calculates and transmits the estimate $\hat{\mathbf{x}}_2^{loc,1}(k)$ of the modes $\mathbf{x}_2(k)$ of the process using its local measurements $\mathbf{y}_1(0), \mathbf{y}_1(1), \ldots, \mathbf{y}_1(k)$.
  - If the process is of type II, sensor 1 calculates and transmits the estimate $\hat{\mathbf{x}}_2^{loc,1}(k)$ and $\hat{\mathbf{x}}_3^{loc,1}(k)$ of the modes $\mathbf{x}_2(k)$ and $\mathbf{x}_3(k)$ of the process using its local measurements $\mathbf{y}_1(0), \mathbf{y}_1(1), \ldots, \mathbf{y}_1(k)$.
- Encoder for Sensor 2:
  - If the process is of type I, sensor 2 calculates and transmits the estimate $\hat{\mathbf{x}}_1^{loc,2}(k)$ of the modes $\mathbf{x}_1(k)$ of the process using its local measurements $\mathbf{y}_2(0), \mathbf{y}_2(1), \ldots, \mathbf{y}_2(k)$.
  - If the process is of type II, sensor 2 calculates and transmits the estimate $\hat{\mathbf{x}}_1^{loc,2}(k)$ and $\hat{\mathbf{x}}_3^{loc,2}(k)$ of the modes $\mathbf{x}_1(k)$ and $\mathbf{x}_3(k)$ of the process using its local measurements $\mathbf{y}_2(0), \mathbf{y}_2(1), \ldots, \mathbf{y}_2(k)$.
- Decoder:
  - If the process is of type I, the decoder maintains an estimate $\hat{\mathbf{x}}_1(k)$ of the modes $\mathbf{x}_1(k)$ and $\hat{\mathbf{x}}_2(k)$ of the modes $\mathbf{x}_2(k)$. At every time step k, the decoder takes the following actions.
    1) If $\mathbf{r}_2(k) = \emptyset$, $\hat{\mathbf{x}}_1(k) = A\hat{\mathbf{x}}_1(k-1)$, else $\hat{\mathbf{x}}_1(k) = \hat{\mathbf{x}}_1^{loc,2}(k)$.
    2) If $\mathbf{r}_1(k) = \emptyset$, $\hat{\mathbf{x}}_2(k) = A\hat{\mathbf{x}}_2(k-1)$, else $\hat{\mathbf{x}}_2(k) = \hat{\mathbf{x}}_2^{loc,1}(k)$.

    It then constructs the estimate $\hat{\mathbf{x}}(k)$ by stacking the estimates $\hat{\mathbf{x}}_1(k)$ and $\hat{\mathbf{x}}_2(k)$.
  - If the process is of type II, the decoder maintains estimates $\hat{\mathbf{x}}_1(k)$, $\hat{\mathbf{x}}_2(k)$ and $\hat{\mathbf{x}}_3(k)$ of the modes $\mathbf{x}_1(k)$, $\mathbf{x}_2(k)$ and $\mathbf{x}_3(k)$ respectively. At every time step k, the decoder takes one of the following actions:
    1) If $(\mathbf{r}_1(k), \mathbf{r}_2(k)) = (\mathbf{1}, \mathbf{1})$

$$\begin{aligned}\hat{\mathbf{x}}_1(k) &= \hat{\mathbf{x}}_2^{loc,1}(k)\\ \hat{\mathbf{x}}_2(k) &= \hat{\mathbf{x}}_1^{loc,2}(k)\\ \hat{\mathbf{x}}_3(k) &= \hat{\mathbf{x}}_3^{loc,1}(k).\end{aligned}$$

    2) If $(\mathbf{r}_1(k), \mathbf{r}_2(k)) = (\emptyset, \mathbf{1})$

$$\begin{aligned}\hat{\mathbf{x}}_1(k) &= \hat{\mathbf{x}}_1^{loc,2}(k)\\ \hat{\mathbf{x}}_2(k) &= A\hat{\mathbf{x}}_2(k-1)\\ \hat{\mathbf{x}}_3(k) &= \hat{\mathbf{x}}_3^{loc,2}(k).\end{aligned}$$

    3) If $(\mathbf{r}_1(k), \mathbf{r}_2(k)) = (\mathbf{1}, \emptyset)$

$$\begin{aligned}\hat{\mathbf{x}}_1(k) &= A\hat{\mathbf{x}}_1(k-1)\\ \hat{\mathbf{x}}_2(k) &= \hat{\mathbf{x}}_2^{loc,1}(k)\\ \hat{\mathbf{x}}_3(k) &= \hat{\mathbf{x}}_3^{loc,1}(k).\end{aligned}$$

    4) If $(\mathbf{r}_1(k), \mathbf{r}_2(k)) = (\emptyset, \emptyset)$

$$\begin{aligned}\hat{\mathbf{x}}_1(k) &= A\hat{\mathbf{x}}_1(k-1)\\ \hat{\mathbf{x}}_2(k) &= A\hat{\mathbf{x}}_2(k-1)\\ \hat{\mathbf{x}}_3(k) &= A\hat{\mathbf{x}}_3(k-1).\end{aligned}$$

    It then constructs the estimate $\hat{\mathbf{x}}(k)$ by stacking estimates $\hat{\mathbf{x}}_1(k)$, $\hat{\mathbf{x}}_2(k)$ and $\hat{\mathbf{x}}_3(k)$.

We shall now prove that under the conditions (14)–(16), estimate $\hat{\mathbf{x}}(k)$ of the state $\mathbf{x}(k)$ is stable in the sense of (19)

Proof of Theorem III.3: We give the proof if the proc is of type II. The proof for type I is similar. By constructi the estimates $\hat{\mathbf{x}}_2^{loc,1}(k)$, $\hat{\mathbf{x}}_1^{loc,2}(k)$, $\hat{\mathbf{x}}_3^{loc,1}(k)$ and $\hat{\mathbf{x}}_3^{loc,2}(k)$ stable. Denote the corresponding error covariance matrices $K_1(k)$, $K_2(k)$, $K_3(k)$ and $K_4(k)$ respectively.

1) For the modes $\mathbf{x}_3(k)$, the error covariance evolves as

$$\mathbf{P}_3(k) = \begin{cases} K_3(k), & \text{w.p. } Pr(\mathbf{r}_1(k) = \mathbf{1}) \\ K_4(k), & \text{w.p. } Pr(\mathbf{r}(k) = (\emptyset, \mathbf{1})) \\ A_{3,3}\mathbf{P}_3(k-1)A_{3,3}^T + \mathbf{R}_{w,3}, & \text{w.p. } Pr(\mathbf{r}(k) = (\emptyset, \emptyset)) \end{cases}$$

where $\mathbf{R}_{w,3}$ is covariance matrix of the process noise tering the evolution of the modes $\mathbf{x}_3(k)$. Thus if (16) hol the error for the modes $\mathbf{x}_3(k)$ will remain stable.

2) For the modes $\mathbf{x}_2(k)$, the error covariance in estimating modes $\mathbf{x}_3(k)$ can thus be considered as additive noise w bounded covariance. The error covariance for these mo evolves as

$$\mathbf{P}_2(k) = \begin{cases} K_1(k), & \text{w.p. } Pr(\mathbf{r}_1(k) = \mathbf{1} \\ A_{2,2}\mathbf{P}_2(k)A_{2,2}^T + \mathbf{R}_{w,2}, & \text{w.p. } Pr(\mathbf{r}_1(k) = \emptyset \end{cases}$$

where $\mathbf{R}_{w,2}$ is the covariance of noise and error incur through the estimation of modes $\mathbf{x}_3(k)$. Thus if (15) hol the error for the modes $\mathbf{x}_2(k)$ will be stable.

3) A similar argument shows that if (14) is satisfied, the er for the modes $\mathbf{x}_1(k)$ will be stable.

Remark V.3 (Presence of Delays): We can consider the e mation problem $\mathcal{P}_2$ when the communication channels imp a stochastic delay. The above stability conditions are not alte by the imposition of such delays, as long as they are finite. T can be seen from the following three facts:

1) Stability conditions for the case of no delay are necess for stability for the case when delay is present.

2) For the case when a stochastic delay upper bounded by a finite value $d_{\max}$ is present, sufficient conditions for stability can be obtained by replacing the time-varying delay by a constant delay of value equal to $d_{\max}$.
3) If the algorithm $\mathcal{A}_{nack}$ is used when the communication channel introduces a constant delay, the stability conditions are not altered. This is because our analysis will carry over directly to the case when $x(k-d_{\max})$ is being estimated at time k. Moreover, if an algorithm yields a stable estimate for $x(k-d_{\max})$, it will yield a stable estimate for $x(k)$ simply through a time update.

Note that the above argument works even if data packets from the sensors are re-arranged because of the delay. Also, note that infinite delays are equivalent to erasures and can, thus, be treated in our framework.

VI. Extensions and Generalizations

A. Case of Multiple Sensors

It is fairly obvious that Theorems III.2 and III.3 can be generalized to the case when N sensors are present. We present the following stability result while omitting the proof.

Proposition VI.1: Consider the process in (1) being observed by N sensors, such that the i-th sensor generates measurements according to the model

$$\mathbf{y}_i(k) = C_i\mathbf{x}(k) + \mathbf{v}_i(k), \quad 1 \le i \le N.$$

The sensors transmit data over erasure channels, with the event of erasure in the i-th channel being denoted by $\mathbf{r}_i = \emptyset$. Consider the 2^N possible ways of choosing l out of the N sensors, for all values of l between 0 and N. For the j-th such way, let the sensors chosen be denoted by $n_1, n_2, \ldots, n_l$ and sensors not chosen by $m_1, m_2, \ldots, m_t$. Denote by $\mathcal{C}^j$ the matrix formed by stacking the matrices $C_{m_1}, C_{m_2}, \ldots, C_{m_t}$. Finally, denote by ρ^j the spectral radius of the unobservable part of A when the pair $(A, \mathcal{C}^j)$ is put in the observer canonical form. A necessary and sufficient condition for the existence of a positive integer q, an encoding algorithm of either the type $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$ and a controller that stabilize the process is that the following 2^N inequalities for $1 \le j \le 2^N$ be satisfied:

$$Pr\left(\mathbf{r}_{n_1} = \emptyset, \mathbf{r}_{n_2} = \emptyset, \ldots, \mathbf{r}_{n_l} = \emptyset\right)\left|\rho^j\right|^2 < 1.$$

B. Communication Over Networks of Erasure Channels

We can also consider the case when sensors transmit information not over erasure channels, but over networks, in which each link is modeled using the erasure model described above. It is fairly obvious that the algorithms used for proving the necessity of the stabilizability conditions in Theorem III.2 and for proving the sufficiency in Theorem III.3 can be generalized to this case, provided there is a provision for time-stamping the transmitted vectors. As an example, consider the algorithm used to prove sufficiency.

- If the networks connecting the two sensors to the controller are disjoint, each link in the two networks carries two quantities as above.
    1) Sensor 1 calculates and transmits the estimates $\hat{\mathbf{x}}_2^{loc,1}(k)$ and $\hat{\mathbf{x}}_3^{loc,1}(k)$ at every time step. Similarly, sensor 2 calculates and transmits the estimates $\hat{\mathbf{x}}_1^{loc,2}(k)$ and $\hat{\mathbf{x}}_3^{loc,2}(k)$ at every time step. The time-stamps correspond to the latest measurements used in calculating these estimates.
    2) Every node in the network checks the time-stamp of data received over the incoming edges and the estimate in its memory. It chooses the data with the latest time-stamp, transmits it along outgoing links and stores it in the memory for the next time step.
    3) The controller constructs the estimate in the same way as in the two-channel case.
- If the networks share links, however, each link carries four quantities. While the sensors calculate and transmit local estimates, each node in the network transmits data corresponding to the latest received values of all the four estimates: $\hat{\mathbf{x}}_2^{loc,1}(.)$, $\hat{\mathbf{x}}_3^{loc,1}(.)$, $\hat{\mathbf{x}}_1^{loc,2}(.)$ and $\hat{\mathbf{x}}_3^{loc,2}(.)$. Using this data, the controller can calculate the estimate.

Note that the intermediate nodes in the network do not need acknowledgments from the controller.

We can also use the techniques from [12] for the case when only one sensor is present and extend the stability conditions to this case. We state the following result without proof.

Proposition VI.2: Consider the set-up of Fig. 1 with the erasure links being replaced by networks in which each link is modeled as a erasure link with given probability of erasure. Let $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times l}$, $C_1 \in \mathbb{R}^{m_1\times n}$ and $C_2 \in \mathbb{R}^{m_2\times n}$ be given matrices specifying the state-space representation for the process. In addition, assume that the process is observable and controllable and that its state-space representation is of type I or type II. If the state space representation is of type I, then there exists a controller of class $\mathbb{K}$, a positive integer q and sensors of class $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$ such that the feedback system is stable if and only if the following inequalities hold:

$$\rho(A_{1,1})^2 p_{maxcut,2} < 1 \tag{28}$$

$$\rho(A_{2,2})^2 p_{maxcut,1} < 1 \tag{29}$$

where $\rho(A_{i,i})$ represents the spectral radius of the matrix $A_{i,i}$. If the state-space representation is of type II then the necessary and sufficient conditions for stabilizability include the following additional inequality:

$$\rho(A_{3,3})^2 p_{maxcut,12} < 1. \tag{30}$$

In the above inequalities, the terms $p_{maxcut,i}$ denote the max-cut probabilities of the network. For the case when the erasure over distinct links are independent events, they can be calculated as follows:

1) To calculate $p_{maxcut,1}$, form a cut by partitioning the node set of the network connecting sensor 1 and the network into two sets: the source set containing the sensor 1 and the sink set containing the controller. For this cut, calculate the cut-probability by multiplying the erasure probabilities for the edges going from the source set to the sink set. The maximum such cut-probability over all possible cuts is $p_{maxcut,1}$.

2) To calculate $p_{maxcut,2}$, proceed as above. However, the source set now contains sensor 2 instead of sensor 1.
3) To calculate $p_{maxcut,12}$, proceed as above. However, the source set now contains both sensor 1 and sensor 2.

A special case of the network arises when each sensor transmits data over a single link to the controllers. However, in addition, the sensors can cooperate by communicating with each other over a link. If the link does not exhibit erasure, then the two sensors, in effect, form one sensor and the results of [13] apply. However, if this link also exhibits erasure, then we obtain the following stability conditions:

Corollary VI.3 (Sensors Cooperating Over a Erasure Link): Consider the set-up of Fig. 1 with an additional bidirectional link connecting the two sensors. Let the event of erasure over the link connecting the two sensors by $\mathbf{r}_3(k) = \emptyset$. Denote

$$q_1 = \max\big(Pr(\mathbf{r}_1(k) = \emptyset), Pr(\mathbf{r}_2(k) = \emptyset, \mathbf{r}_3(k) = \emptyset)\big)$$
$$q_2 = \max\big(Pr(\mathbf{r}_2(k) = \emptyset), Pr(\mathbf{r}_1(k) = \emptyset, \mathbf{r}_3(k) = \emptyset)\big).$$

If the state space representation is of type I, then there exists a controller of class $\mathbb{K}$, a positive integer q and sensors of class $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$ such that the feedback system is stable if and only if the following inequalities hold

$$q_1\rho(A_{2,2})^2 < 1, \quad q_2\rho(A_{1,1})^2 < 1$$

where $\rho(A_{i,i})$ represents the spectral radius of the matrix $A_{i,i}$. If the state-space representation is of type II then the necessary and sufficient conditions for stabilizability include the following additional inequality:

$$\rho(A_{3,3})^2 Pr(\mathbf{r}_1(k) = \emptyset, \mathbf{r}_2(k) = \emptyset) < 1. \tag{31}$$

C. Markov Drops

While the algorithm $\mathcal{A}_{ack}$ was optimal for arbitrary realizations of the erasure process, the stability analysis so far assumed that erasure events were i.i.d.. This condition can be relaxed. A popular model for the bursty nature of packet drops in a wireless channel is according to a Markov chain. The simplest such model is the classical Gilbert-Elliot channel model. In this model, the channel is assumed to exist in one of two possible modes: state 0 corresponding to a packet drop and state 1 corresponding to no packet drop. The channel transitions between the two states according to a Markov chain. We have the following result.

Proposition VI.4 (Necessary and Sufficient Conditions for Stabilizability for Markovian Packet Drops): Consider the set-up of Fig. 1. Let the statistics of the erasure links 1 and 2 be described by Markov chains. For the link i, let $q_{00,i}$ denoting the conditional probability of an erasure at time $k+1$ given an erasure at time k. Finally, let the erasures over the two channels be independent. If the state space representation is of type I, then there exists a controller of class $\mathbb{K}$, a positive integer q and sensors of class $\mathbb{S}_q$ or $\mathbb{S}_q^{NAK}$ such that the feedback system is stable if and only if the following inequalities hold:

$$\rho(A_{1,1})^2 q_{00,2} < 1 \tag{32}$$

$$\rho(A_{2,2})^2 q_{00,1} < 1$$

where $\rho(A_{i,i})$ represents the spectral radius of the matr
If the state-space representation is of type II then stat
assured if and only if the following inequality also hold

$$\rho(A_{3,3})^2 q_{00,1} q_{00,2} < 1.$$

VII. Conclusion

In this paper, we considered the problem of cont
a process using measurements from multiple sensor
information from the sensors to the controller is trans
over links where erasure (data loss) is governed by a stoc
process. We identified necessary and sufficient conditic
the stabilizability of a linear and time-invariant process,
second moment sense. The allowable stabilization poli
the sensors are constrained to place vectors of constant c
sion for possible transmission over the erasure links. Unc
assumption that the controller is able to transmit acknov
ments back to the sensors, we identified an encoding alg
that minimizes a quadratic cost. We also considered v
extensions to the basic set-up.

Appendix
Proof of Proposition III.1

Consider that we are given a linear and time-invariant p
with the properties specified in the statement of the Propo
and whose state-space representation is specified by ma
$A^{s0} \in \mathbb{R}^{n\times n}$, $B^{s0} \in \mathbb{R}^{n\times l}$, $C_1^{s0} \in \mathbb{R}^{m_1\times n}$ and $C_2^{s0} \in \mathbb{R}$
Here C_1^{s0} and C_2^{s0} represent a bipartition of the output a
stands for *stage zero* . Below we outline a procedure, comp
three stages, that will lead to an equivalent state-space of
or type II, as defined in the statement of the Proposition.

First Stage: Since, from Assumption 1, the proc
not observable from $y_1(t)$ alone, or equivalently the
$(A^{(s0)}, C_1^{(s0)})$ is not observable, we can use the c
ical structure theorem [29, page 340, eq. (22)] to con
that there exists a transformation $P^{0\longrightarrow 1} \in \mathbb{R}^{n\times n}$
that the matrices $A^{(s1)} \stackrel{def}{=} P^{0\longrightarrow 1}A^{(s0)}\left(P^{0\longrightarrow 1}\right)^{-}$
$C_1^{(s1)} \stackrel{def}{=} C_1^{(s0)}\left(P^{0\longrightarrow 1}\right)^{-1}$ have the following structure

$$A^{(s1)} = \begin{bmatrix} A_{1,1}^{(s1)} & A_{1,2}^{(s1)} \\ 0^{n_1'\times n_1} & A_{2,2}^{(s1)} \end{bmatrix}$$
$$C_1^{(s1)} = \begin{bmatrix} 0^{m_1\times n_1} & C_{1,2}^{(s1)} \end{bmatrix}$$

where $n_1 + n_1' = n$, $A_{2,2}^{(s1)} \in \mathbb{R}^{n_1'\times n_1'}$ and $C_{1,2}^{(s1)} \in \mathbb{R}^{m}$
Notice that n_1 is a strictly positive integer because the
$(A^{(s0)}, C_1^{(s0)})$ is not observable. The remaining ma
defining the new state-space representation are give
$C_2^{(s1)} \stackrel{def}{=} C_2^{(s0)}\left(P^{0\longrightarrow 1}\right)^{-1}$ and $B^{(s1)} \stackrel{def}{=} P^{0\longrightarrow 1}B^{(s0)}$.

Second Stage: We start by partitioning $C_2^{(s1)}$ in th
lowing way:

$$C_2^{(s1)} = \begin{bmatrix} C_{1,1}^{(s1)} & C_{1,2}^{(s1)} \end{bmatrix}$$

where $C_{1,2}^{(s1)} \in \mathbb{R}^{m_2 \times n_1'}$. We can now apply, once again, the *canonical structure theorem* [29, page 340, eq. (22)] to show the existence of a transformation $P^{1 \longrightarrow 2} \in \mathbb{R}^{n_1' \times n_1'}$ such that the matrix $C_{2,2}^{(s2)} \overset{def}{=} C_{2,2}^{(s1)} \left(P^{1 \longrightarrow 2}\right)^{-1}$ features one of the following structures:

$$C_{2,2}^{(s2)} = \begin{cases} \begin{bmatrix} 0^{m_2 \times n_2} & C_{2,3}^{(s3)} \end{bmatrix}, & \text{if } C_{2,2}^{(s2)} \text{ is not zero (type II)} \\ 0^{m_2 \times n_1'} & \text{otherwise (type I)} \end{cases} \tag{38}$$

where $C_{2,3}^{(s3)} \in \mathbb{R}^{m_2 \times n_3}$ and $n_2 + n_3 = n_1'$. Similarly, for the matrix $A_{2,2}^{(s2)} \overset{def}{=} P^{0 \longrightarrow 1} A^{(s1)} \left(P^{0 \longrightarrow 1}\right)^{-1}$ the following holds:

$$(\text{process is type II}) \Longrightarrow A_{2,2}^{(s2)} = \begin{bmatrix} A_{2,2}^{(s3)} & A_{2,3}^{(s3)} \\ 0^{n_3 \times n_2} & A_{3,3}^{(s3)} \end{bmatrix} \tag{39}$$

where $A_{2,2}^{(s3)} \in \mathbb{R}^{n_2 \times n_2}$ and $A_{3,3}^{(s3)} \in \mathbb{R}^{n_3 \times n_3}$. Recall that according to Assumption 1 the process is not observable from $\mathbf{y}_2$ alone, implying that n_2 is a nonzero positive integer.

Third Stage: Consider the following state transformation:

$$P \overset{def}{=} \begin{bmatrix} I & 0^{n_1 \times n_1'} \\ 0^{n_1' \times n_1} & P^{1 \longrightarrow 2} \end{bmatrix} P^{0 \longrightarrow 1}. \tag{40}$$

We can now use the previous analysis to show, by inspection, that the state-space representation given by the matrices $A^{(s3)} \overset{def}{=} PA^{(s0)}P^{-1}, B^{(s3)} \overset{def}{=} PB^{(s0)}, C_1^{(s3)} \overset{def}{=} C_1^{(s0)}P^{-1}$ and $C_2^{(s3)} \overset{def}{=} C_2^{(s0)}P^{-1}$ is in the form specified by (9) if the process is type I and that otherwise the matrices will have the structure (10). □

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REFERENCES

[1] B. D. O. Anderson and J. B. Moore, "Time-varying feedback laws for decentralized control," *IEEE Trans. Automat. Control*, vol. AC-26, no. 5, pp. 1133–1139, Oct. 1981.

[2] P. Antsaklis and J. Baillieul, "Special issue on networked control systems," *IEEE Trans. Automat. Control*, vol. 49, no. 9, pp. 1421–1423, Sep. 2004.

[3] B. Azimi-Sadjadi, "Stability of networked control systems in the presence of packet losses," in *Proc. IEEE Conf. Decision Control*, Dec. 2003, vol. 1, pp. 676–681.

[4] Y. Bar-Shalom, "On the track-to-track correlation problem," *IEEE Trans. Automat. Control*, vol. AC-26, no. 2, pp. 571–572, Apr. 1981.

[5] Y. Bar-Shalom and L. Campo, "The effect of the common process noise on the two-sensor fused-track covariance," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-22, no. 6, pp. 803–805, Nov. 1986.

[6] L. Bushnell, "Special issue on networks and control," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 22–23, Feb. 2001.

[7] K. C. Chang, R. K. Saha, and Y. Bar-Shalom, "On optimal track-to-track fusion," *IEEE Trans. Aerosp. Electron. Syst.*, vol. AES-33, no. 4, pp. 1271–1276, Oct. 1997.

[8] C. Y. Chong, "Hierarchical estimation," in *Proc. 2nd MIT/ONR C^3 Workshop*, 1979, [CD ROM].

[9] C. Y. Chong, S. Mori, and K. C. Chang, "Information fusion in distributed sensor networks," in *Proc. Amer. Control Conf.*, 1985, pp. 830–835.

[10] D. Georgiev and D. M. Tilbury, "Packet-based control: The H_2-optimal solution," *Automatica*, vol. 42, no. 1, pp. 137–144, Jan. 2006.

[11] V. Gupta, T. Chung, B. Hassibi, and R. M. Murray, "On a stochastic sensor selection algorithm with applications in sensor scheduling and dynamic sensor coverage," *Automatica*, vol. 42, no. 2, pp. 251–260, Feb. 2006.

[12] V. Gupta, A. F. Dana, J. P. Hespanha, R. M. Murray, and B. Hassibi, "Data transmission over networks for estimation and control," *IEEE Trans. Automat. Control*, submitted for publication.

[13] V. Gupta, B. Hassibi, and R. M. Murray, "Optimal LQG control across a packet-dropping link," *Syst. Controls Lett.*, vol. 56, no. 6, pp. 439–446, Jun. 2007.

[14] C. N. Hadjicostis and R. Touri, "Feedback control utilizing packet dropping network links," in *Proc. IEEE Conf. Decision Control*, 2002, pp. 1205–1210.

[15] H. R. Hashemipour, S. Roy, and A. J. Laub, "Decentralized structures for parallel Kalman filtering," *IEEE Trans. Automat. Control*, vol. AC-33, no. 1, pp. 88–94, Jan. 1988.

[16] A. Hassibi, S. P. Boyd, and J. P. How, "Control of asynchronous dynamical systems with rate constraints on events," in *Proc. IEEE Conf. Decision Control*, Dec. 1999, pp. 1345–1351.

[17] J. P. Hespanha, P. Naghshtabrizi, and Y. Xu, "Networked control systems: Analysis and design," *Proc. IEEE*, vol. 95, no. 1, pp. 138–162, Jan. 2007.

[18] O. C. Imer, S. Yuksel, and T. Basar, "Optimal control of LTI systems over communication networks," *Automatica*, vol. 42, no. 9, pp. 1429–1440, Sep. 2006.

[19] P. A. Kawka and A. G. Alleyne, "Stability and performance of packet-based feedback control over a Markov channel," in *Proc. Amer. Control Conf.*, 2006, pp. 2807–2812.

[20] B. C. Levy, D. A. Castanon, G. C. Verghese, and A. S. Willsky, "A scattering framework for decentralized estimation problem," *Automatica*, vol. 19, no. 4, pp. 373–384, 1983.

[21] Q. Ling and M. D. Lemmon, "Power spectral analysis of netwroked control systems with data droputs," *IEEE Trans. Automat. Control*, vol. 49, no. 6, pp. 955–960, Jun. 2004.

[22] N. C. Martins, M. A. Dahleh, and N. Elia, "Feedback stabilization of uncertain systems in the presence of a direct link," *IEEE Trans. Automat. Control*, vol. 51, no. 3, pp. 438–447, Mar. 2006.

[23] A. S. Matveev and A. V. Savkin, "Decentralized stabilization of linear systems via limited capacity communication networks," in *Proc. IEEE Conf. Decision Control*, 2005, pp. 1155–1161.

[24] L. A. Montestruque and P. Antsaklis, "On the model-based control of networked systems," *Automatica*, vol. 39, no. 10, pp. 1837–1843, Oct. 2003.

[25] P. Naghshtabrizi and J. Hespanha, "Anticipative and non-anticipative controller design for network control systems," in *Networked Embedded Sensing and Control, Volume 331 of Lecture Notes in Control and Information Science*, P. J. Antsaklis and P. Tabuada, Eds. New York: Springer Verlag, 2006, pp. 203–218.

[26] G. N. Nair, R. J. Evans, and P. E. Caines, "Stabilising decentralized linear systems under data rate constraints," in *Proc. IEEE Conf. Decision Control*, 2004, pp. 3992–3997.

[27] J. Nilsson, "Real-Time Control Systems With Delays," Ph.D. dissertation, Department of Automatic Control, Lund Institute of Technology, Lund, Sweden, 1998.

[28] B. S. Rao and H. F. Durrant-Whyte, "Fully decentralized algorithm for multi-sensor Kalman filtering," *Proc. Inst. Elect. Eng.*, vol. 138, no. 5, pp. 413–420, 1991.

[29] W. J. Rugh, *Linear System Theory*, 2nd ed. Englewood Cliffs, NJ: Prentice Hall Information and Systems Sciences Series, 1996.

[30] A. Sahai and S. Mitter, "The necessity and sufficiency of anytime capacity for stabilization of a linear system over a noisy communication linkpart I: Scalar systems," *IEEE Trans. Inform. Theory*, vol. 52, no. 5, pp. 3369–3395, Aug. 2006.

[31] V. Saligrama and D. A. Castanon, "Reliable distributed estimation with intermittent communications," in *Proc. IEEE Conf. Decision Control (CDC)*, 2006, pp. 6763–6768.

[32] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry, "Foundations of control and estimation over lossy networks," *Proc. IEEE*, vol. 95, no. 1, pp. 163–187, Jan. 2007.

[33] P. J. Seiler, "Coordinated Control of Unmanned Aerial Vehicles," Ph.D. dissertation, University of California, Berkeley, CA, 2002.

[34] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. Jordan, and S. Sastry, "Kalman filtering with intermittent observations," *IEEE Trans. Automat. Control*, vol. 49, no. 9, pp. 1453–1464, Sep. 2004.

[35] S. C. Smith and P. Seiler, "Estimation with lossy measurements: Jump estimators for jump systems," *IEEE Trans. Automat. Control*, vol. 48, no. 12, pp. 1453–1464, Dec. 2003.

[36] J. L. Speyer, "Computation and transmission requirements for a decentralized linear-quadratic Gaussian control problem," *IEEE Trans. Automat. Control*, vol. AC-24, no. 2, pp. 266–269, Apr. 1979.

[37] S. Tatikonda, "Some scaling properties of larger distributed control systems," in *Proc. IEEE Conf. Decision Control*, 2003, pp. 3142–3147.

[38] D. Willner, C. B. Chang, and K. P. Dunn, "Kalman filter algorithms for a multisensor system," in *Proc. IEEE Conf. Decision Control*, 1976, pp. 570–574.

[39] A. Willsky, M. Bello, D. Castanon, B. Levy, and G. Verghese, "Combining and updating of local estimates and regional maps along sets of one-dimensional tracks," *IEEE Trans. Automat. Control*, vol. 27, no. 8, pp. 799–813, Aug. 1982.

[40] S. Yuksel and T. Basar, "Optimal signaling policies for decentralized multi-controller stabilizability over communication channels," *IEEE Trans. Automat. Control*, vol. 52, no. 10, pp. 1969–1974, Oct. 2007.

[41] W. Zhang, M. S. Branicky, and S. M. Philips, "Stability of networked control systems," *IEEE Control Syst. Mag.*, vol. 21, no. 1, pp. 84–89, Feb. 2001.

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