ENSE 623 Systems Validation and Verification

Spatial Logic and Spatio-Temporal Logic

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Part 1. Introduction

Quick Review. Hierarchies of Logic

Logic	**Purpose and Support**
Propositional	Propositional logic deals with sentences (propositions) that are either true or false. Complex sentences are constructed from simple sentences using logical connectives.
First Order	First order logic is an extension of propositional logic. Formulas are constructed from predicates (called truth functions), plus universal (for all) (A) and extensional (there exists) (E) quantifiers.
Temporal	Temporal logic describes how conditions of a system change over time – that is: **We want not only to know what is true, but when?**

Quick Review. Hierarchies of Logic

Logic	Purpose and Support
Spatial	Spatial logic is concerned with regions and their connection, allowing one to address issues of the form: **We want not only to know what is true, but where?**
Spatio-Temporal	When spatial logics are combined with temporal logic (i.e., producing a so-called spatio-temporal logic), the resulting theory allows one to address concerns of the form: **We want not only to know what is true, but when and where?**

Combining Numerical and Logical Representations

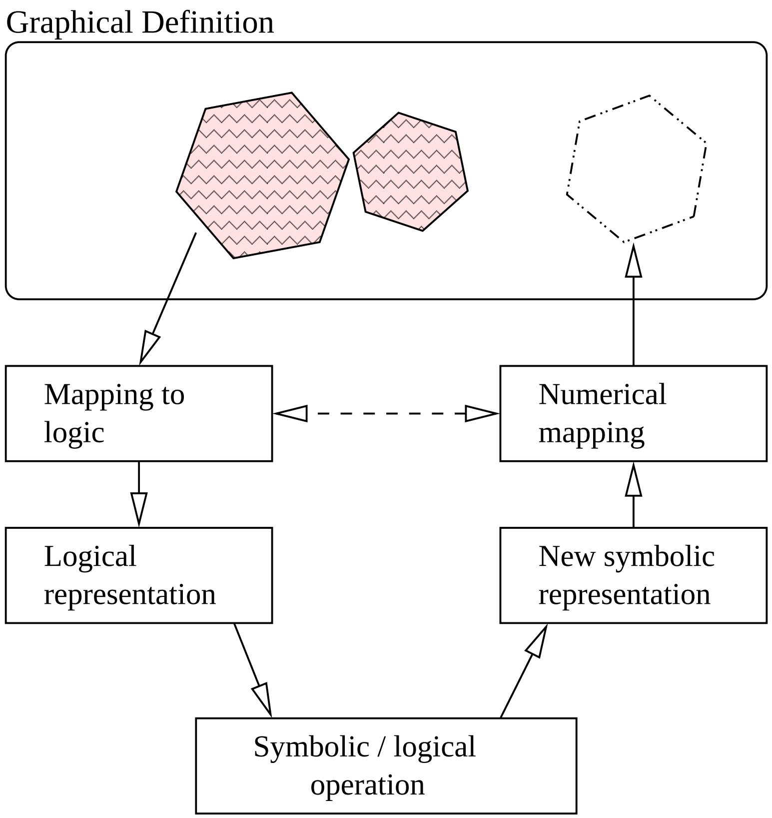
Looking Ahead

Looking ahead, we want to know:

1. What kinds of spatial entities need to be represented? Possibilities include points, line segments, polylines, lines, spaces (regions).
2. Procedures for systematically assembling complicated entities (e.g., region shapes) from basic entities.
3. What kinds of (quantitative and qualitative) reasoning need to be supported?
4. Can computers support these types of reasoning?

Combining Numerical and Logical Representations

Connectiing Graphical, Numerical, and Logical Representations of Shapes



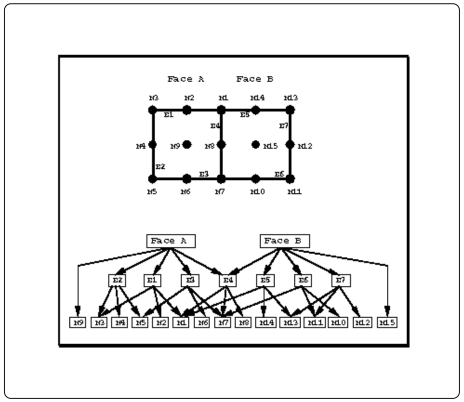
Adapted from Damski and Gero, 1996.

Part 2. Working with Spatial Logic

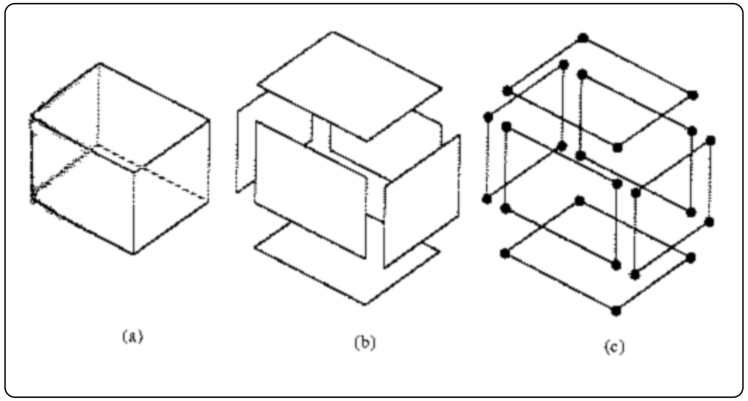
Established/Boundary Approaches to Spatial Modeling

Established approaches to spatial modeling employ boundary representations (so-called b-reps) or are based on constructions of complicated geometries from sequences of simple geometric constructions (so-called csg's).

Schematics of B-Rep Modeling



(a) Finite element model



(b) Boundary nodes and planes

Declarative Approach to Spatial Modeling

Systems Engineering Needs

Systems engineers, in contrast, are more interested in ...

... the relative positioning (i.e., qualitative positioning) and connectivity of objects.

To support this objective, two main issues need addressing:

- We need a declarative description of geometry (vs a procedural approach), and
- We need an integrated approach to spatial modeling that starts with primitive data types and operations on those data types.

Spatial Constraint Modeling

Approach ...

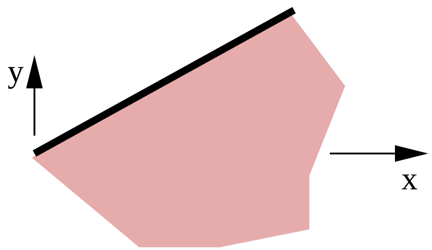
The basic idea of spatial constraint modeling:

> **Represent spatial objects finitely as infinite collections of points satisfying first-order formulas.**

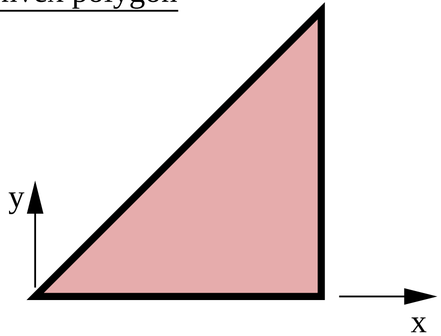
Representation of a Halfplane and Convex Polygon

A convex polygon is the intersection of a finite set of half planes:

Half plane



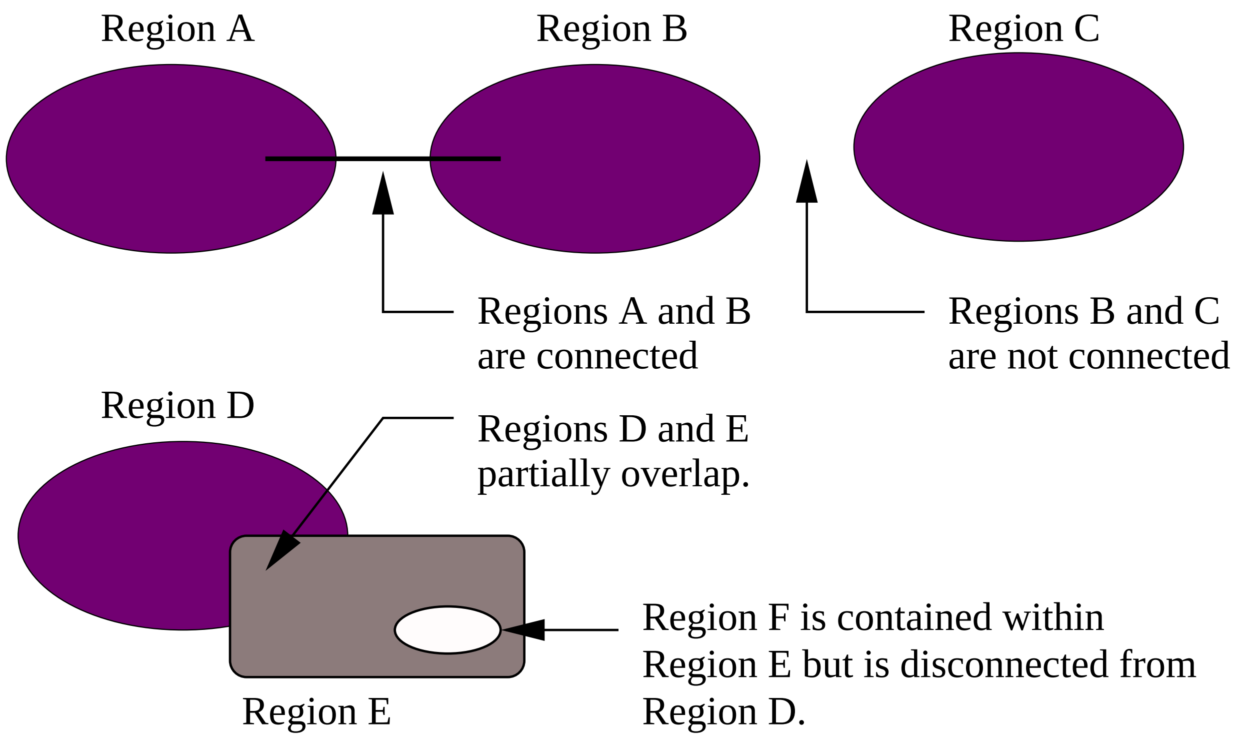
Convex polygon



Half plane region = { (x,y) | 2y − x <= 0 }   Polygon = { (x, y) | y − x < 0 ^ y > 0 ^ x < 1 }

Primitive Relations in Spatial Theory

Relationships described by Spatial Theory



Adapted from Randell et al.

Primitive Relations in Spatial Theory

Operator	Explanation
C(x,y)	C(x,y) reads as "region x connects with region y." Clearly, forall x and y, C(x,y) $\longrightarrow$ C(y,x).
DC(x,y)	DC(x,y) reads as "region x is disconnected from region y." Clearly, forall x and y, DC(x,y) $\longrightarrow$ DC(y,x).
P(x,y)	P(x,y) reads as "region x is part of region y."
PP(x,y)	PP(x,y) reads as "region x is a proper part of region y."
x = y	Reads as "region x is identical with region y."
O(x,y)	Reads as "region x overlaps with region y."
PO(x,y)	Reads as "region x partially overlaps region y."
EC(x,y)	Reads as "region x is externally connected to region y."

Primitive Relations in Spatial Theory

From Primtive Relations to Spatial Axioms

With these primitive relations in place, now we can define ...

> **... axioms for a theory from which conclusions may be drawn on spatial relationships.**

Spatial Axioms

For example, ...

1. DC(x,y) =: C(x,y)
   If points x and y are disconnected, then the cannot also be connected.
2. P(x,y) := for all z [C(z,x) $\rightarrow$ C(z,y)]
   Given that region x is part of y, then for all points z that are part of x, it follows that they are also part of region y.

Primitive Relations in Spatial Theory

Spatial Axioms Continued ...

3. 'x = y' =: $P(x,y) \wedge P(y,x)$
If regions x and y are identical, then region x is part of y and, conversely, region y and also part of region x.

4. $O(x,y)$:= there exists z [$P(z,x) \wedge P(z,y)$]
If region x overlaps region y, then there exists a region z that is part of region x and part of region y.

5. $PO(x,y) := O(x,y) \wedge P(x,y) \wedge P(y,x)$
The region x that partially overlaps region y is defined by the total overlap of x and y (i.e., the union of regions x and y), plus the part of x that isn't in y, plus the part of region y that isn't in x.

Complications

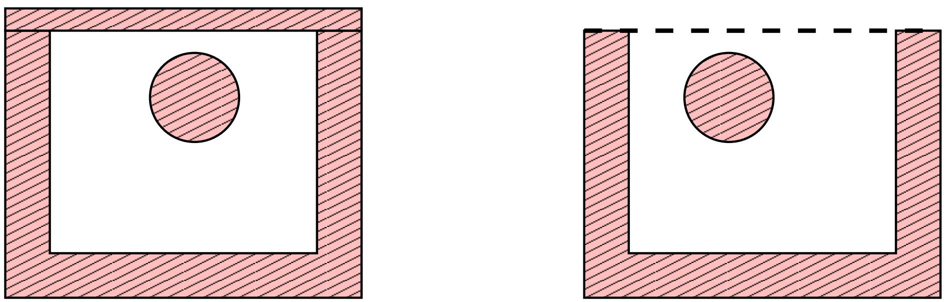
Complications ...

There are many complications that can occur in defining a theory for spaces.

For example, ...

... What does inside mean for enclosures that are partially open?

Notions of Topological and Geometric Closure



The ball is "geometrically inside" the cup and its lid.

The ball is "topologically inside" the open cup.

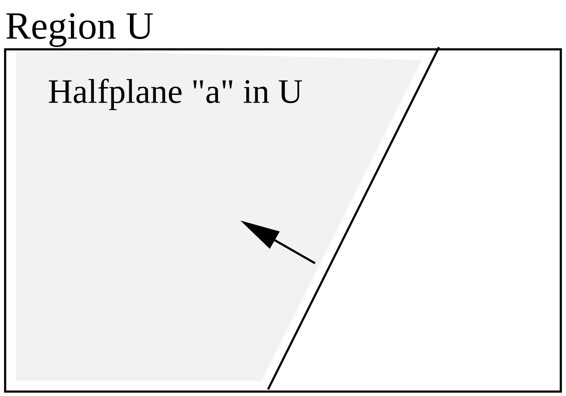
Assembly of Shapes from Halfplane Spaces

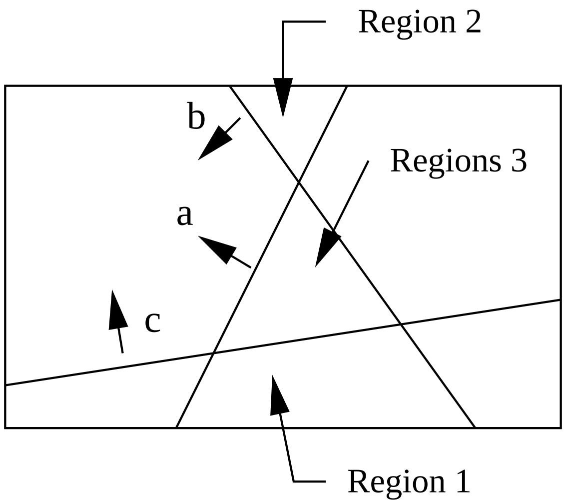
Halfplane Definition

We define a halfplane as a ...

... conceptual border that divides two sets of points.

Assembly of Shapes from Halfplanes





Assembly of Shapes from Halfplane Spaces

Formal Definition

- Let "U" be a region defined by a set of points p(x,y), U = p(x,y) .
- U can always be divided into two subsets:

$$A = \{p(x, y) : f(x, y) > 0\}\,. \tag{1}$$

$$B = \{p(x, y) : f(x, y) \leq 0\}\,. \tag{2}$$

where f(x,y) is a continuous function in U.

- A and B are non-empty sets.

Halfplanes A and B have the following properties:

- A intersection B = null set;
- A union B = U.

Assembly of Shapes from Halfplane Spaces

Halfplane First-Order Logic Formulae

Now let's define a second set C = true, false defined by the function $g : A \longrightarrow C$. The function g(x,y) is:

- The function g(x,y) is true (T) if p(x,y) belongs to A.

  Otherwise, it is false.

We define the predicate hp(x) with the truth value,

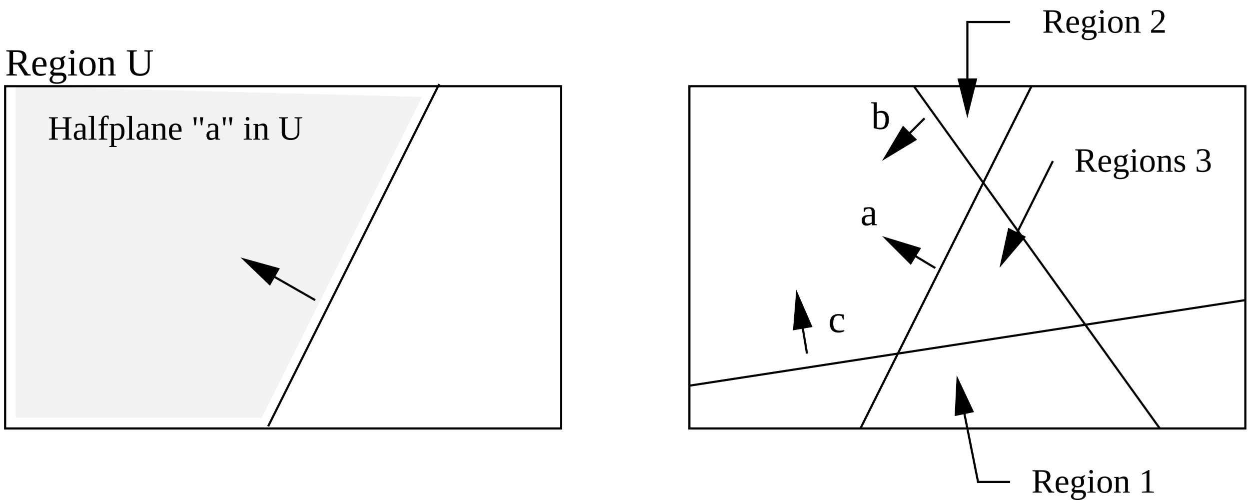
```
hp(x) is True if T is an element of C.

Otherwise, it is false.
```

Assembly of Shapes from Halfplane Spaces

Example. Declaring Shapes

A shape is a disjunctive formula of regions.



Regions 1 through 3 in the right-handside of the adjacent are described by the formula

```
Region 1 =: ~hp(a) ^  hp(b) ^ ~hp(c)
Region 2 =:  hp(a) ^ ~hp(b) ^  hp(c)
Region 3 =: ~hp(a) ^  hp(b) ^  hp(c)
```

Assembly of Shapes from Halfplane Spaces

Declaring Shapes as Combinations of Regions

For example, ...

```
Region 1 v Region 3
```

can be written

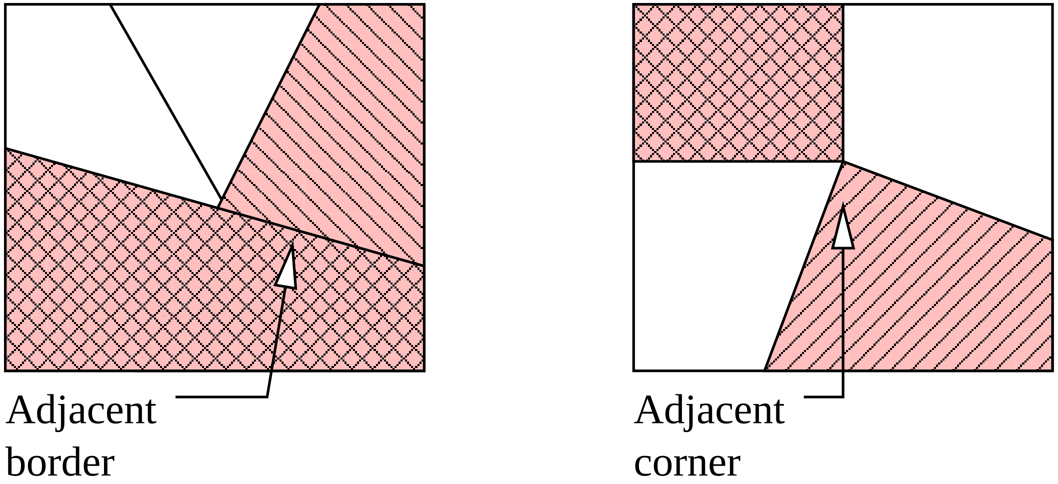
```
(~hp(a) ^  hp(b) ^ ~hp(c)) v (~hp(a) ^  hp(b) ^  hp(c))
```

and simplified to

```
~hp(a) ^ hp(b)
```

Properties of Regions

Schematics of Border and Corner Adjacency



Definition of Border Adjacent

Given a minimal description of region R_1, a second region R_2 is border adjacent to R ...

... if and only if its description differs in only one hp(x_i).

Properties of Regions

Definition of Corner Adjacency

Given a minimal description of a region R expressed by

$$hp(x_1) \wedge hp(x_2) \wedge hp(x_n), \tag{3}$$

a region R_{adj} is corner adjacent to R ...

... if and only if it differs in exactly two literals hp(x_i) and hp(x_j), where hp(x_i) and hp(x_j) in R are ¬hp(x_i) and ¬hp(x_j) in R_{adj}.

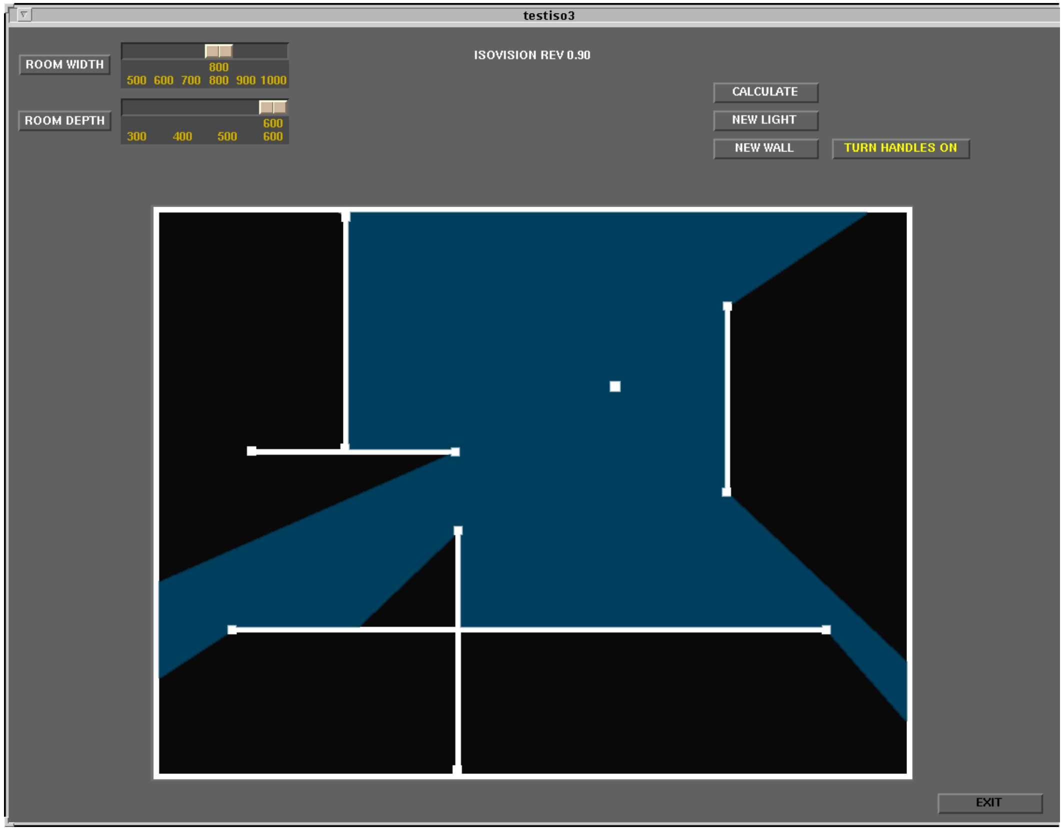
Key Limitation

- The halfspace formalism can represent two- and three-dimensional spaces – but how to represent points and lines?

For details, see Damski and Gero (1996).

Applications of Spatial Logic

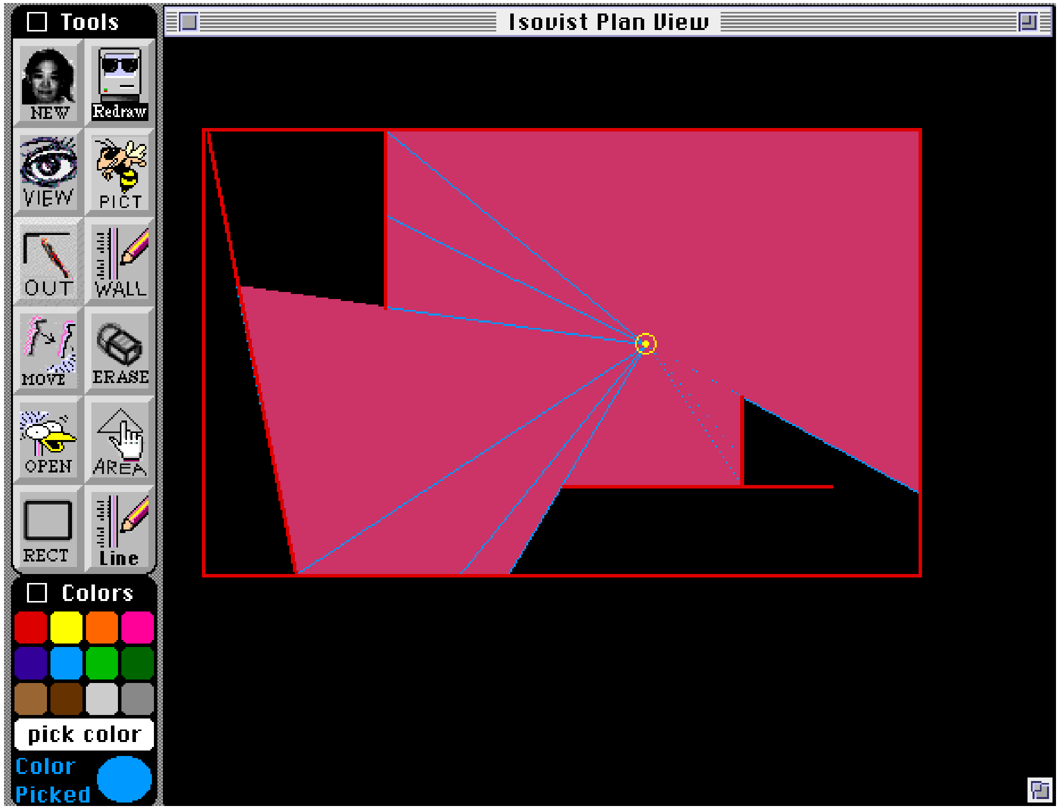
Application 1. Shadow Calculation



Source: Yi-Luen and Gross, 1997.

Applications of Spatial Logic

Application 2. Visible Area Calculation



Source: Yi-Luen and Gross, 1997.

Applications of Spatial Logic

Application 3. Binary Space Partitioning

A binary space partitioning (BSP) tree

> **...systematically divides the world into halfspace regions (i.e., it is a binary space partition).**

BSP trees play an important role in computer graphics because they ...

> **... allow polygons, as viewed from a camera, to be sorted, front to back.**

This ordering of information ...

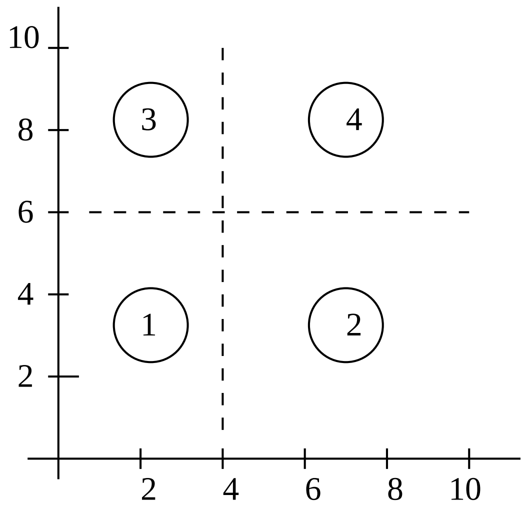
> **... computes and removes hidden (or partially obscured) surfaces from visibility.**

Applications of Spatial Logic

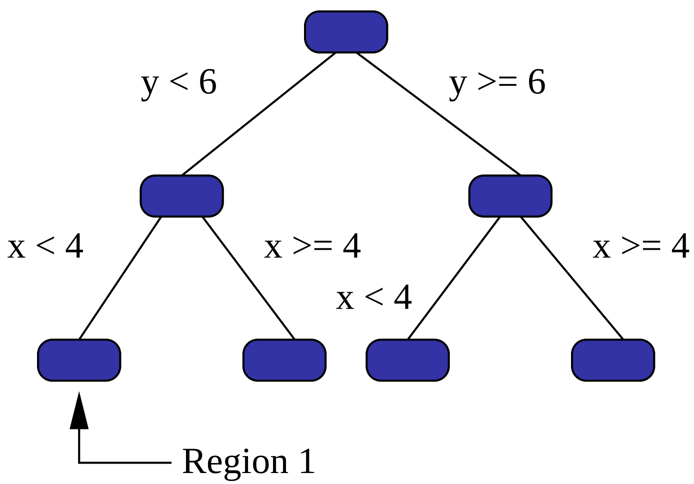
Two-step Development Procedure

1. A BSP tree of the scene (or virtual world) is constructed.
2. The viewpoint is compared to this structure to determine visibility.

Example 1. BSP for One-Level Regional Subdivision

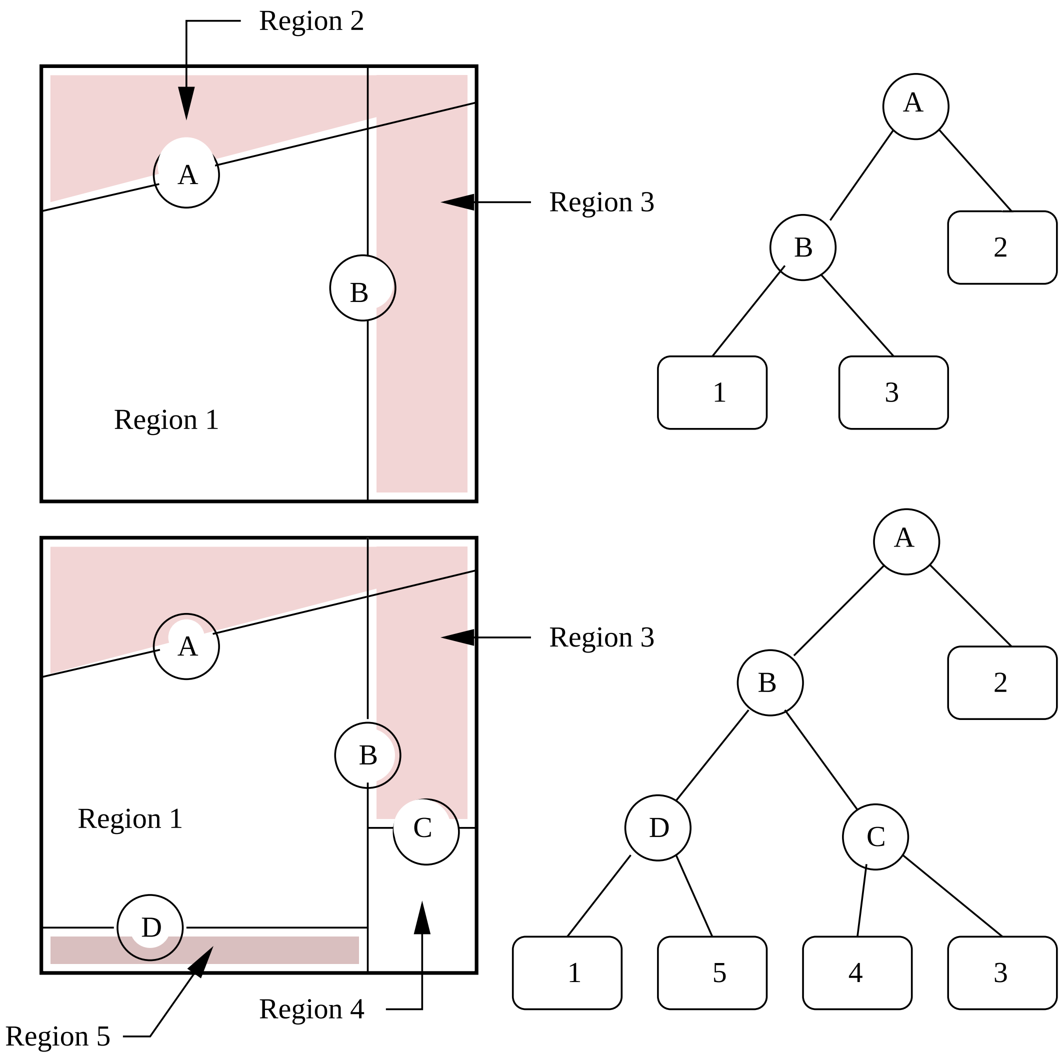


Sub−division of 10 x 10 grid.



Applications of Spatial Logic

Example 2. Assemby of BSP for 2D Virtual World



Applications of Spatial Logic

Example 2. Key Points ...

- In the upper diagram.

  Region 1 is defined by halfplanes "A" and "B." Halfplane A (or wall A) partitions the space into two parts – regions 1 and 2.

- Then halfplane B,

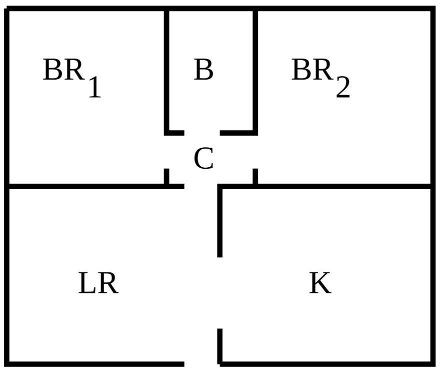
  Partitions region 1 into two parts, a refined description of region 1 and a new region 3. Notice that region 3 is bound by wall B and the partition formed by wall A.

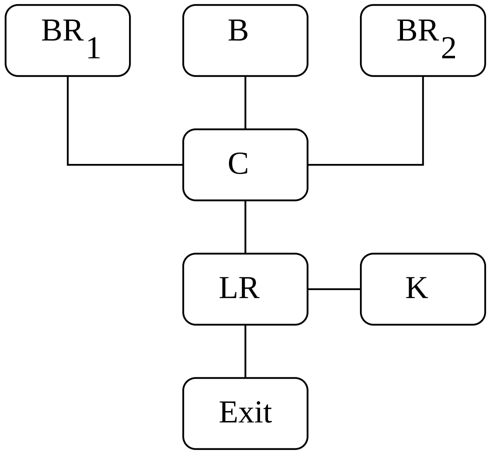
- The lower diagram is generated by adding two more walls, C and D. Wall C is added to region 1; Wall D to region 3.

Go to java demo!!!!

Applications of Spatial Logic

Topological Connectivity of Regions in a Simple Floorplan





- The building must have at least one exit point (i.e., a means to get outside).
- All rooms must have access to the exit point.
- Bedrooms should be adjacent to bathrooms.
- The kitchen should be adjacent to the living room.

Applications of Spatial Logic

Logical Analysis of Building Architectures

We can say things like:

```
P( Kitchen , House );
```

That is, ...

... the kitchen is ”part of” the house.

Although the rooms do not overlap, ...

... they are connected via doorways through walls.

Graph of Region Connectivity

From the connectivity graph we can state several facts: e.g.,

```
C( Kitchen , Living Room );
C( Living room, Exit );
```

Applications of Spatial Logic

Formal Analysis of Evacuation Support

Suppose that we have a connectivity rule:

$$C(x,z) := C(x,y) \wedge C(y,z); \tag{4}$$

and that

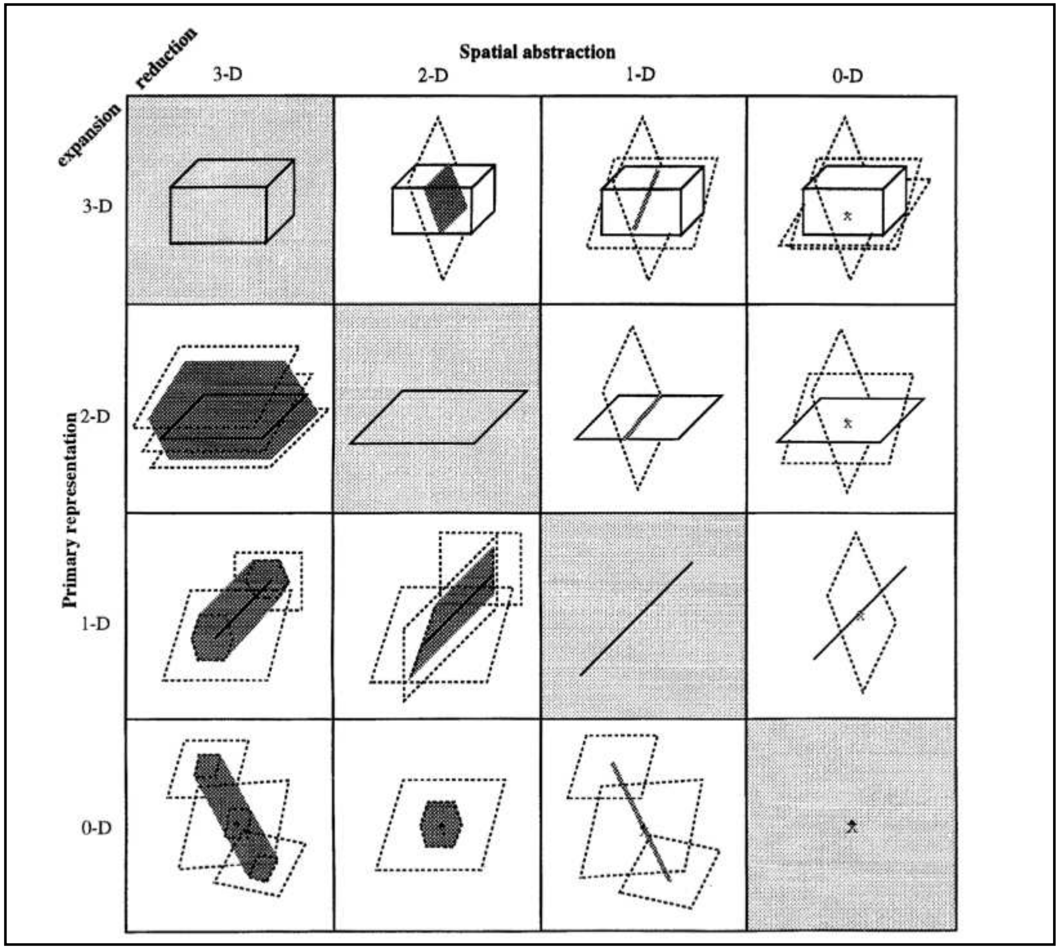
$$x \in \{BR_1, BR_2, B, C, LR, K, Exit\} \tag{5}$$

The house will have adequate evacuation support if,

$$\forall(x)C(x, Exit) \tag{6}$$

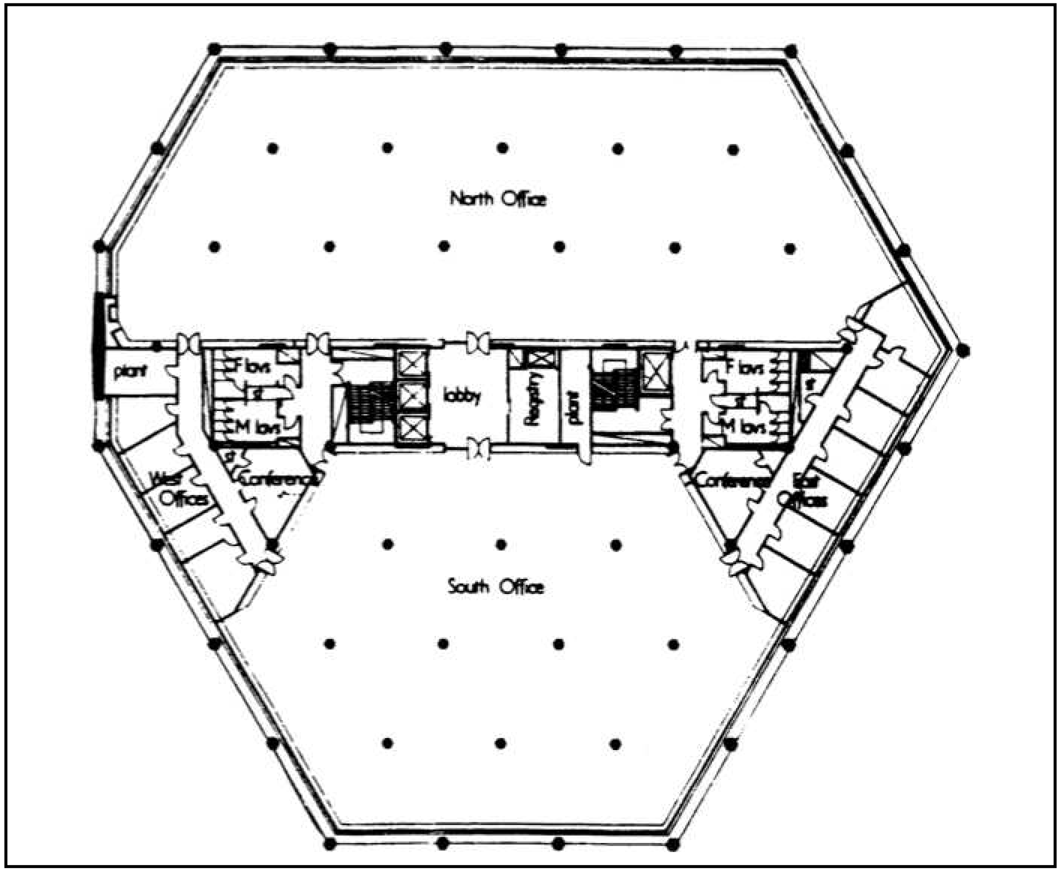
Architectural Design of Buildings at CMU

Representation/Manipulation of 1, 2, and 3 Dimensional Entities



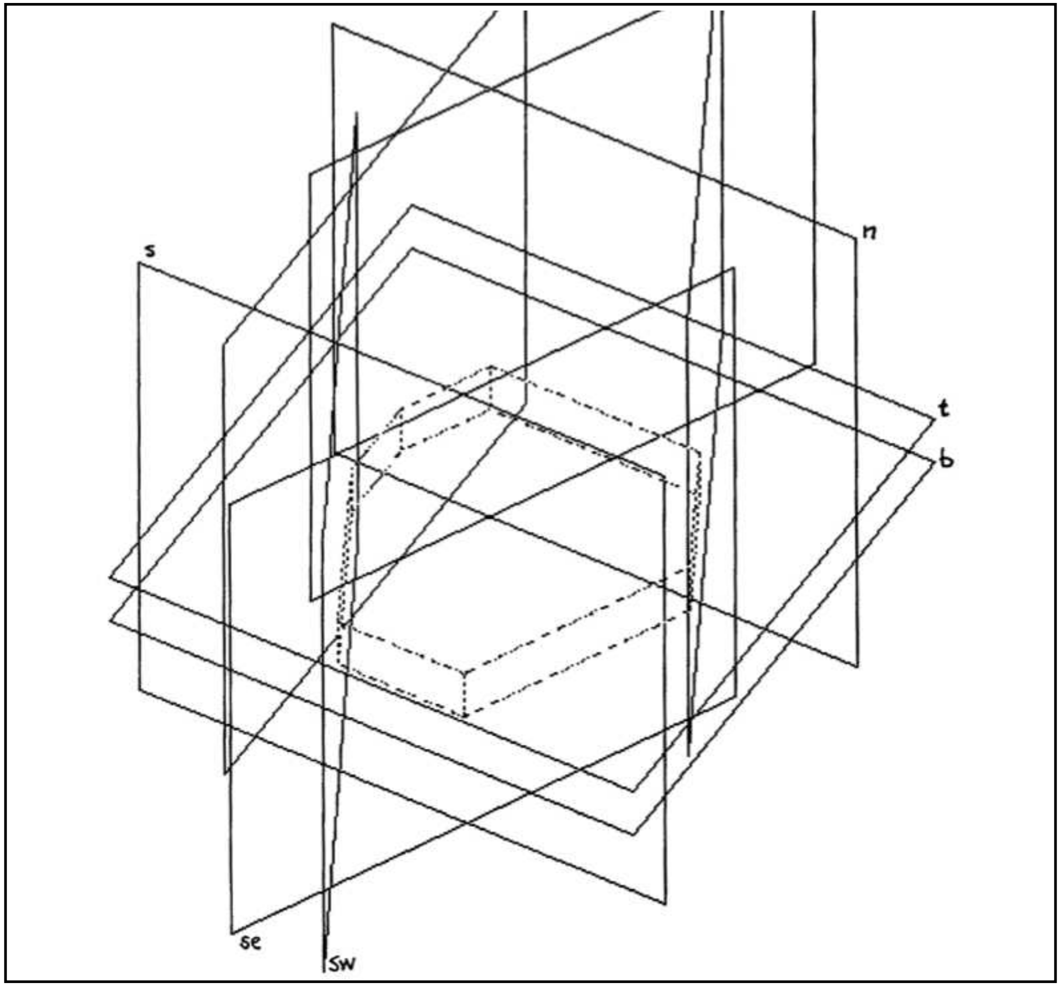
Architectural Design of Buildings at CMU

Floorplan of an Office Building



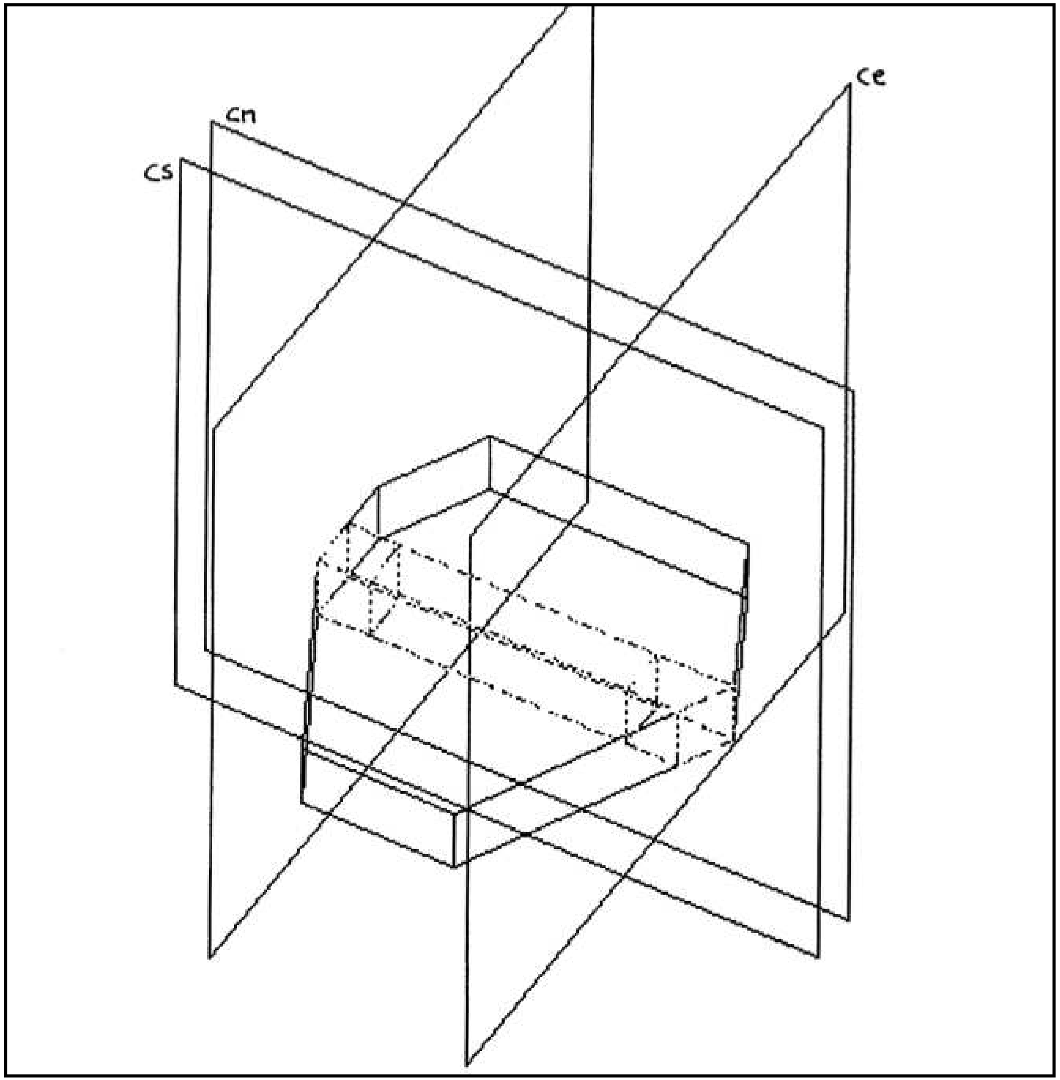
Architectural Design of Buildings at CMU

Planes defining Boundaries of the Building Exterior



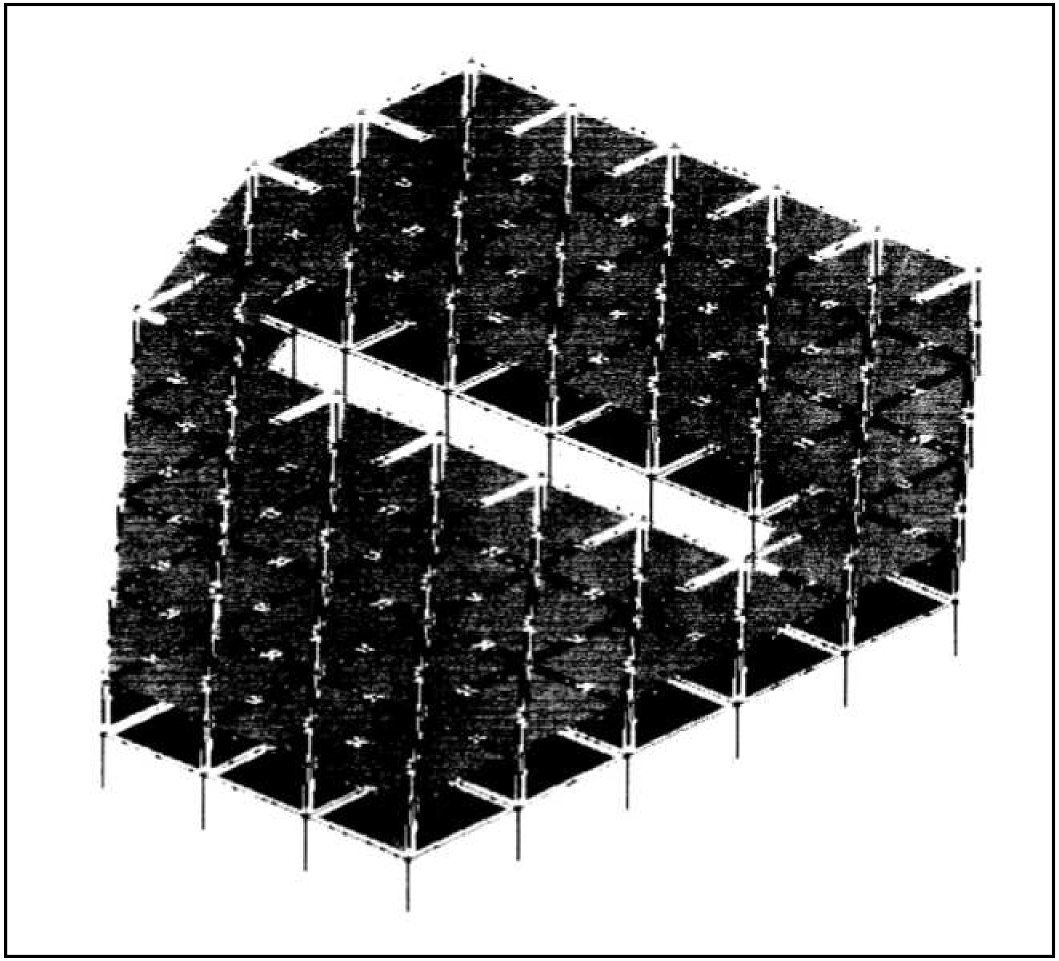
Architectural Design of Buildings at CMU

Planes defining 3 Zones and the Building Core



Architectural Design of Buildings at CMU

One, Two, and Three-Dimensional Entities in the Building



Part 3. Working with Spatio-Temporal Logic

Spatio-Temporal Logic

Objectives

When spatial logics are combined with temporal logic (i.e., producing a so-called spatio-temporal logic), the resulting theory allows one to address concerns of the form:

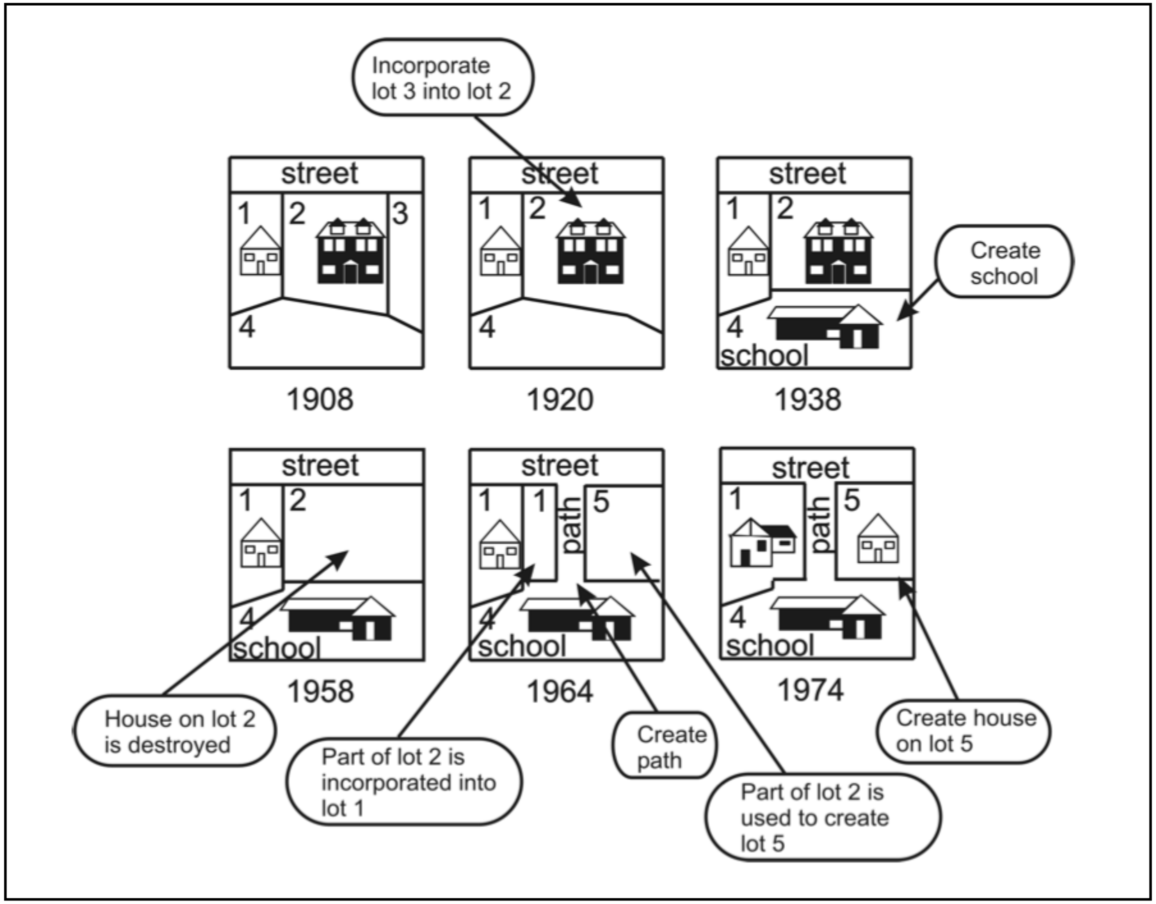
We want not only to know what is true, but when and where?

Common Applications

- An entity moves from one space to another over time (e.g., migration of animals).
- The purpose and boundaries of a space change over time.

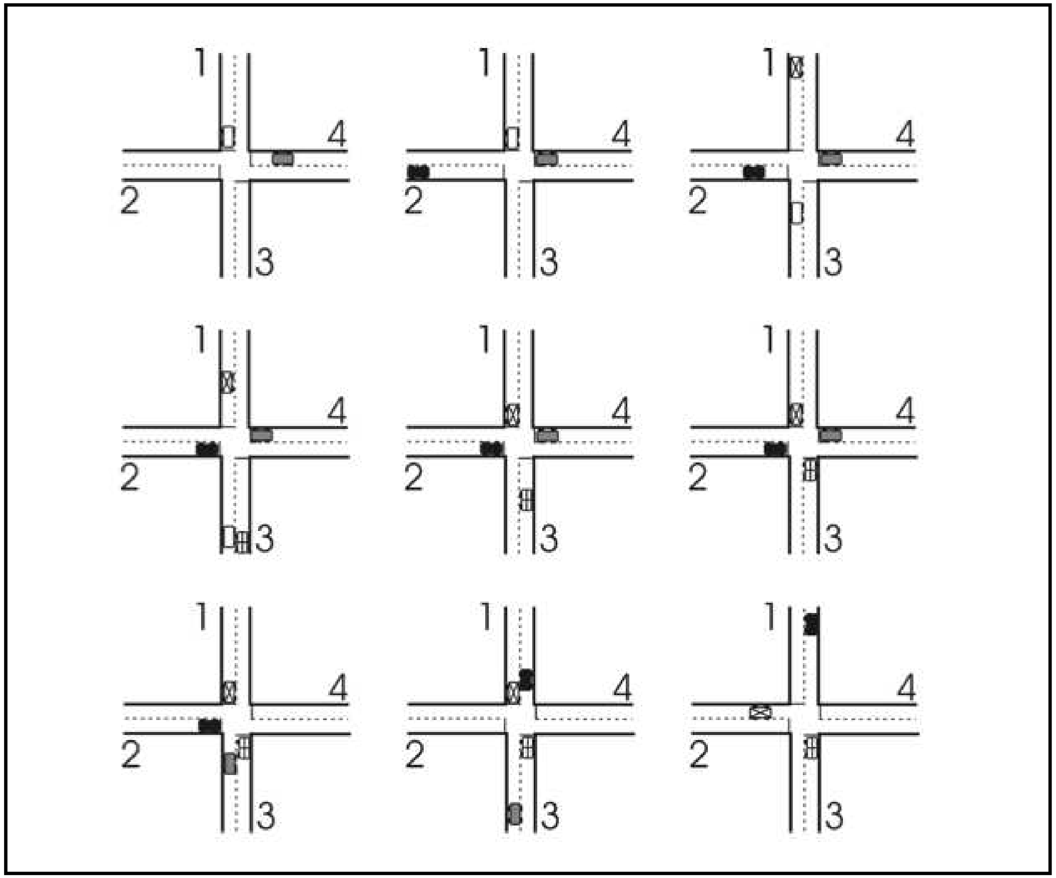
Motivating Applications

Spatio-Temporal Modeling in the GeoInformation Sciences



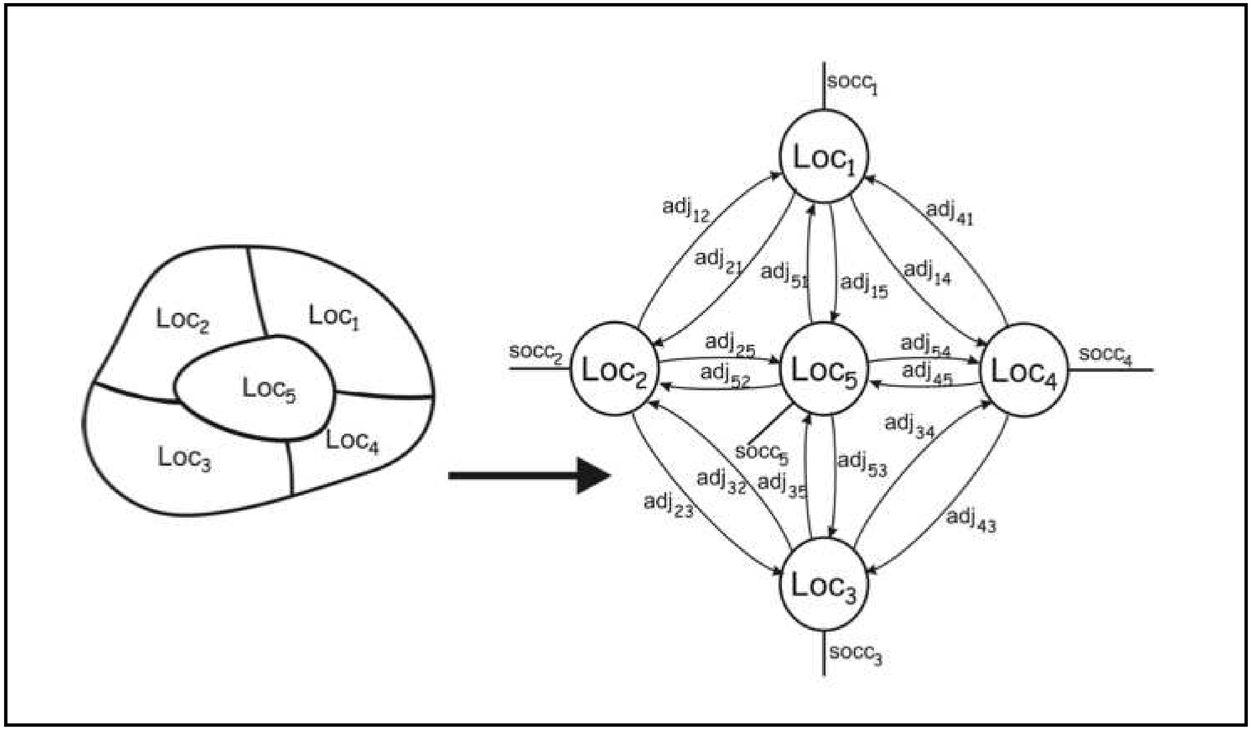
Motivating Applications

Spatio-Temporal Modeling for a Traffic Intersection



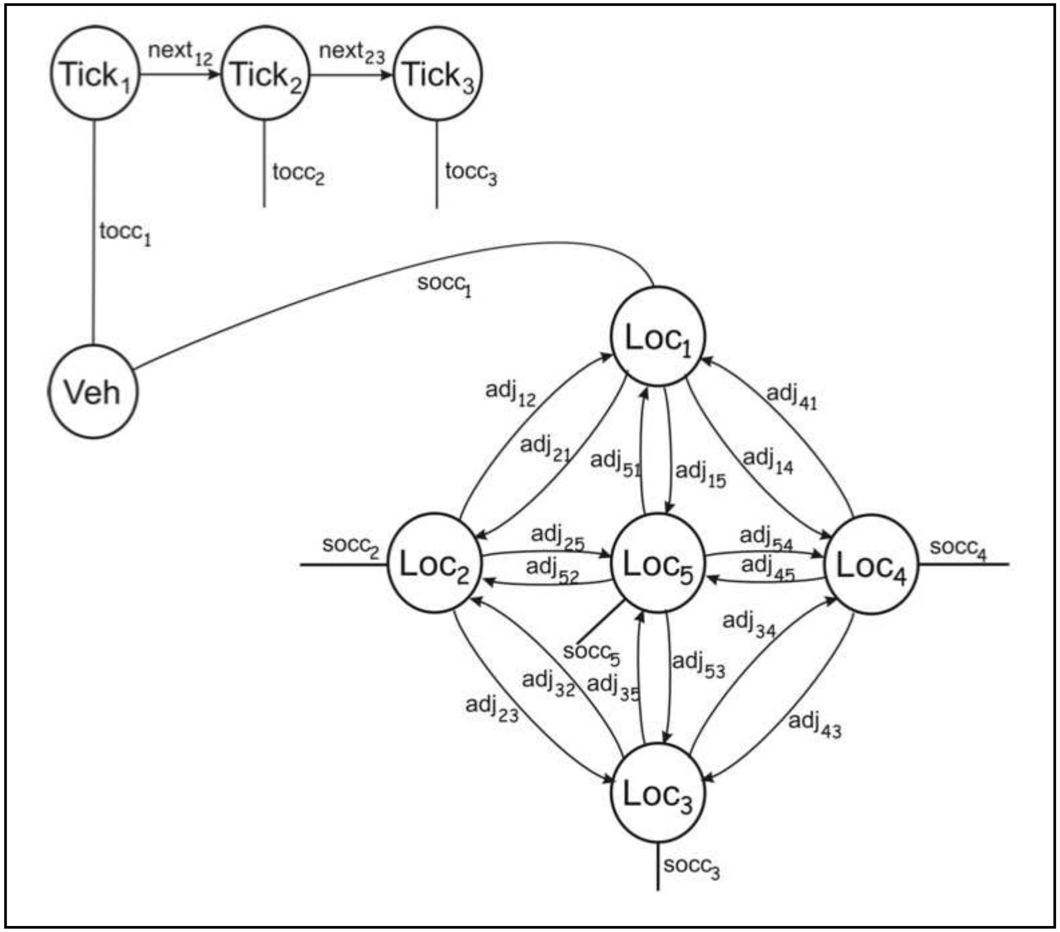
Methods of Analysis

Partitioning of Regions into Manageable Spaces



Methods of Analysis

Linking Spatial and Temporal Processes



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