CS2351
Data Structures

Lecture 11:
Graph and Tree Traversals II

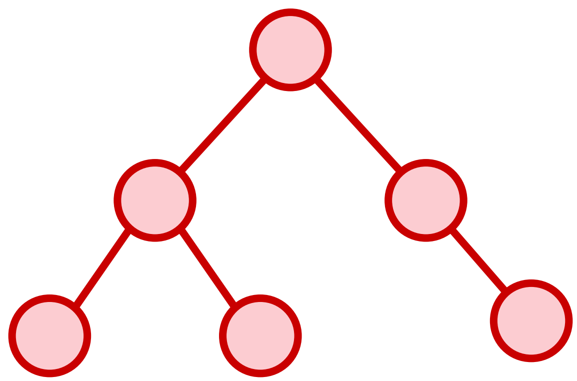
About this lecture

- We introduce some popular algorithms to traverse a rooted ordered binary tree
  1. Level Order (similar to BFS)
  2. Pre-order, Post-order, In-order (similar to DFS)
- Then, we will discuss a related topic called expression tree

Level Order Traversal

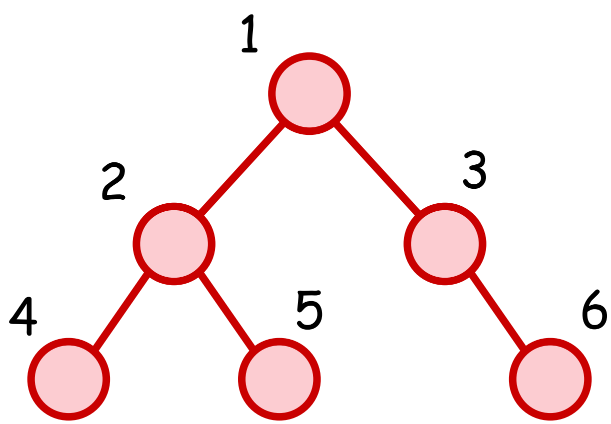
Level Order

- Imagine we have a rooted binary tree, and we apply the BFS algorithm on the root (as the source)
- What will happen ?



Level Order

- The nodes of the tree will be visited in the following order :



- This is called the level order traversal

Implementation

- To implement level order traversal, we just run BFS on the root
- Since each node (except root) in a rooted tree has exactly one parent, it can only be discovered once during BFS
  - No need to have an extra array to remember if a node is marked or not, and we need only a queue
- Running time : O(|V|)

Preorder/Postorder/Inorder Traversal

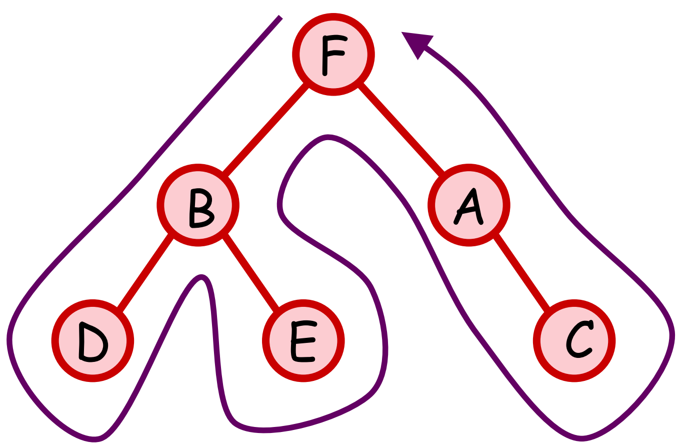
DFS Traversal on a Tree

- We now describe 3 popular algorithms to traverse a tree
  - Preorder, Postorder, Inorder
  - They are all based on DFS
- The only difference is:

  "During the traversal, what time they will output the content of a node"

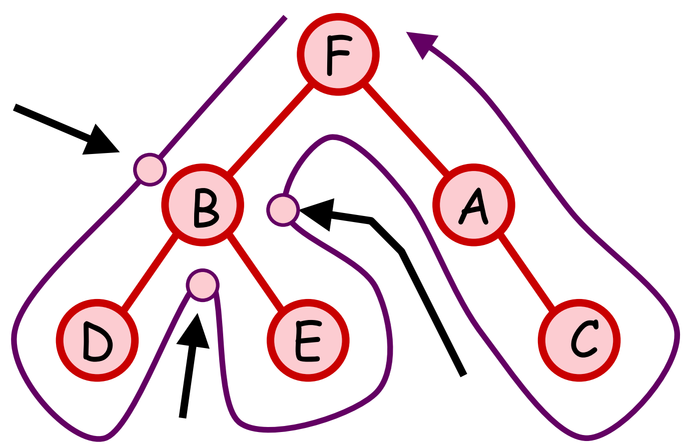
DFS on a Tree

- When we apply DFS on a tree, when it visits a node :
  - it calls DFS recursively on left child
  - then DFS recursively on right child



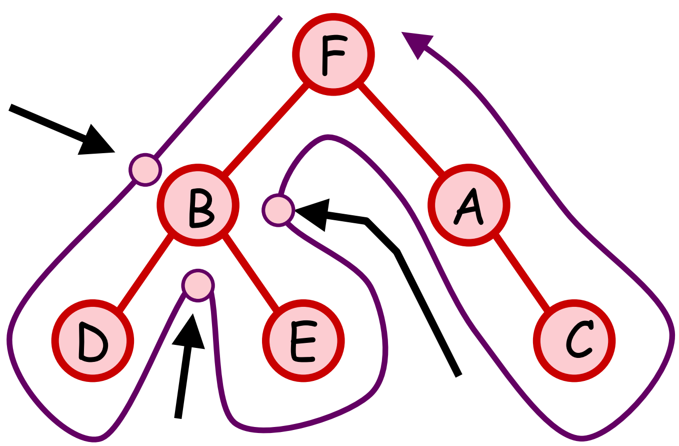
DFS on a Tree

- A node is actually visited a few times
  - Exactly 3 times for binary tree
- They include: the time before the first DFS, and the times after each DFS



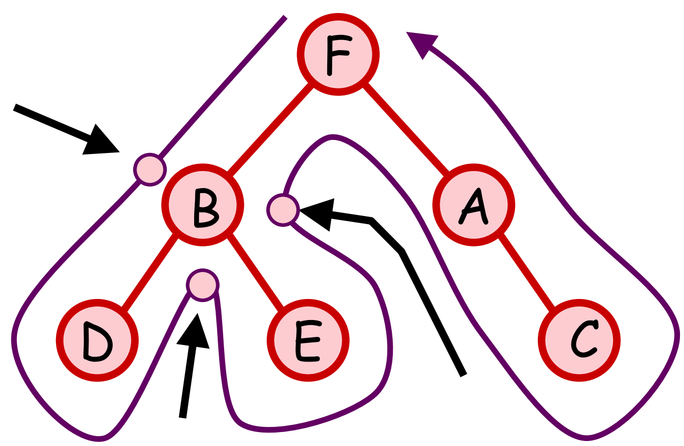
Preorder Traversal

- The preorder traversal prints the content of a node when it is first visited
- In our example, we print : FBDEAC



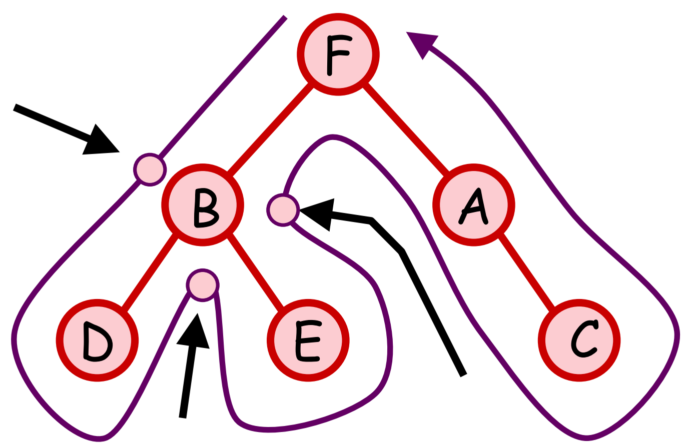
Postorder Traversal

- The postorder traversal prints the content of a node when it is last visited
- In our example, we print : DEBCAF



Inorder Traversal

- The inorder traversal prints the content of a node just before we visit right child
- In our example, we print : DBEFAC



Implementation

- To implement the above traversal algorithms, we first see that DFS on a binary tree can be done as follows :

```
DFS (u)  {
        1.  Call DFS (u.left) ;
        2.  Call DFS (u.right) ;
}
```

At the main program, we call DFS (root)

Implementation

- Then the preorder traversal is implemented as follows :

```
Preorder (u)  {
        1.  Print content of u ;
        2.  Call Preorder (u.left) ;
        3.  Call Preorder (u.right) ;
 }
```

At the main program, we call Preorder (root)

Implementation

- Similarly, the postorder traversal is implemented as follows :

```
Postorder (u) {
    1. Call Postorder (u.left) ;
    2. Call Postorder (u.right) ;
    3. Print content of u ;
}
```

At the main program, we call Postorder (root)

Implementation

- And the inorder traversal is implemented as follows :

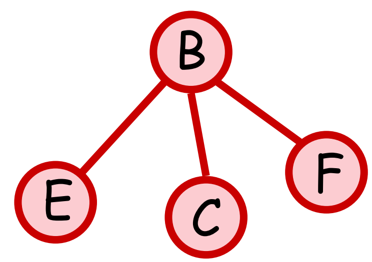
```
Inorder (u) {
        1. Call Inorder (u.left) ;
        2. Print content of u ;
        3. Call Inorder (u.right) ;
}
```

At the main program, we call Inorder (root)

Remarks

- Running time : O(|V|) time
- The preorder and postorder traversals are well-defined for non-binary trees
  - For inorder, to visit a node with degree more than 2, there are 2 common ways: One prints the content after the first DFS, and one prints after every DFS except the last

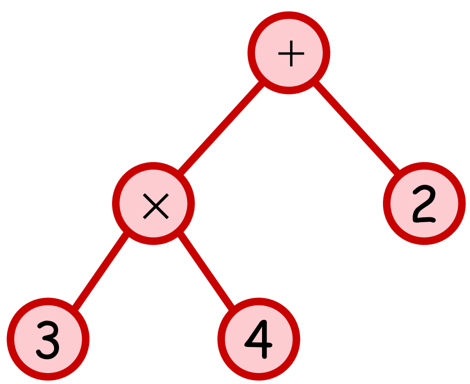
Two versions of Inorder:
EBCF vs EBCBF



Expression Tree

Expression Tree

- We can use rooted binary trees to represent mathematical expressions that involve only binary operators
- Each internal node stores an operator
- Each leaf stores an operand
- Ex :

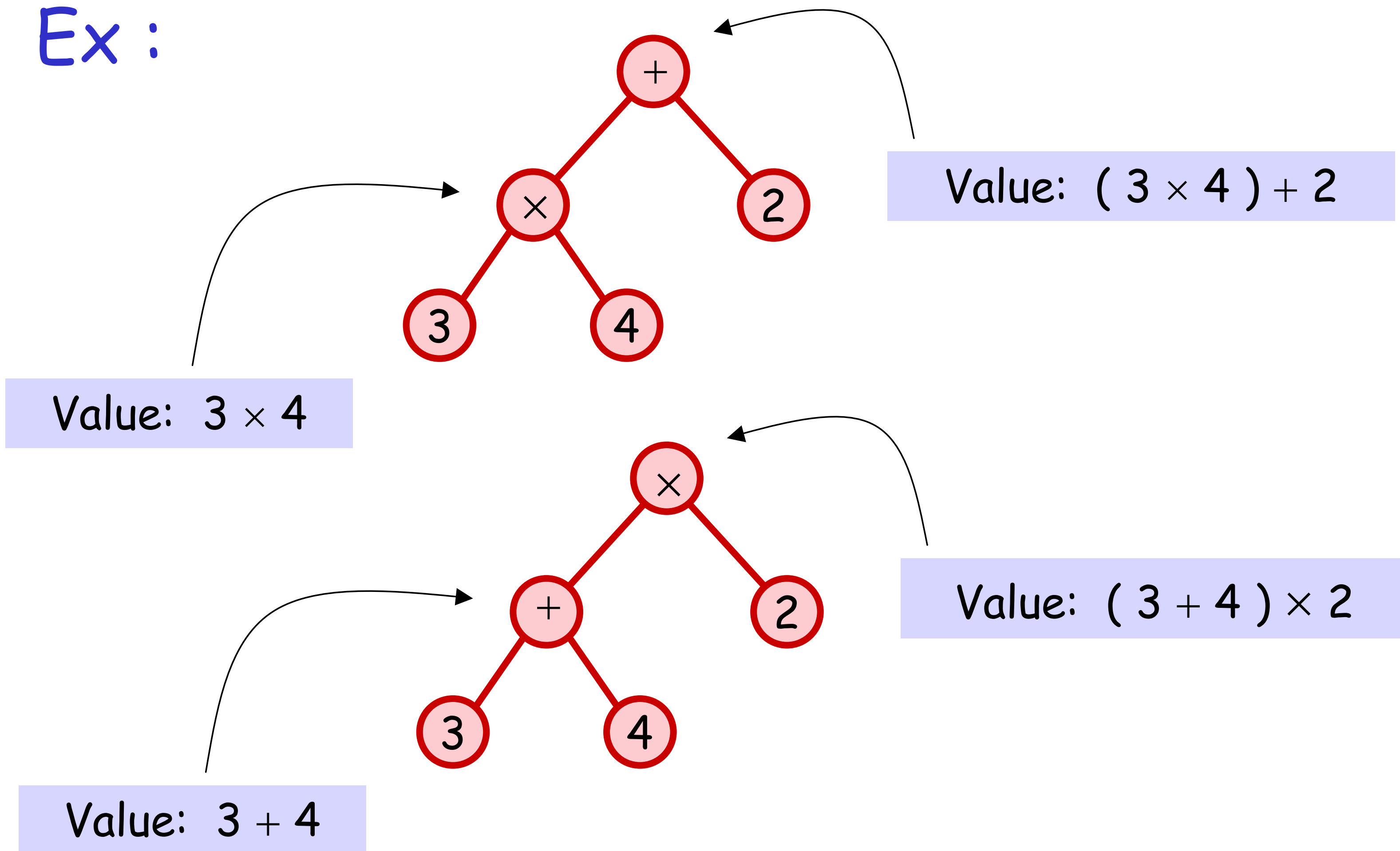


Expression Tree

- Each internal node u corresponds to a value computed recursively as follows:
  1. Compute the value x corresponding to left child of u
  2. Compute the value y corresponding to right child of u
  3. The value of $u = x \Delta y$ where Δ is the operator stored in u
- value of expression = value of the root

Expression Tree

- Ex :

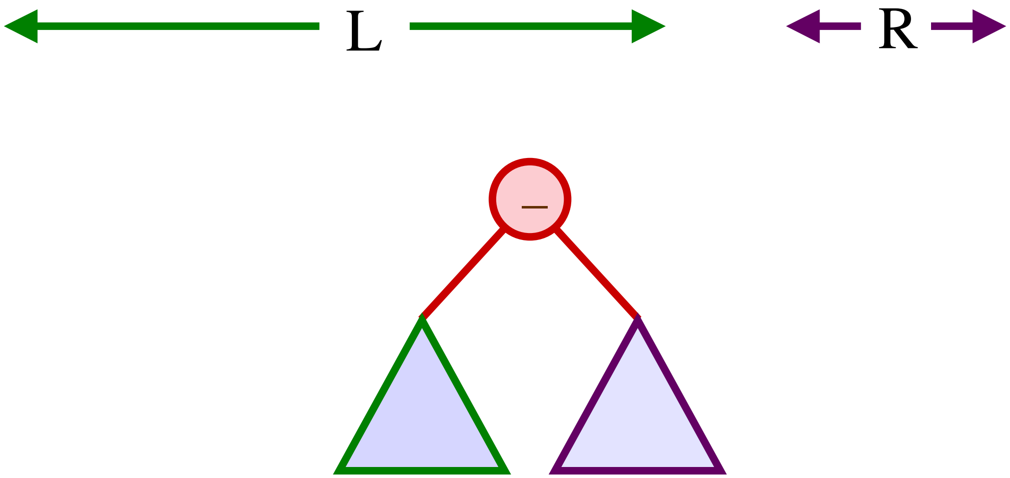


Expression Tree

- Each mathematical expression has a corresponding expression tree
- To find such a tree, we can :
  1. First determine which operator is last applied, then put it inside the root ;
  2. After that, recursively construct the left and right subtrees of the root based on the contents on the left and right sides of the operator

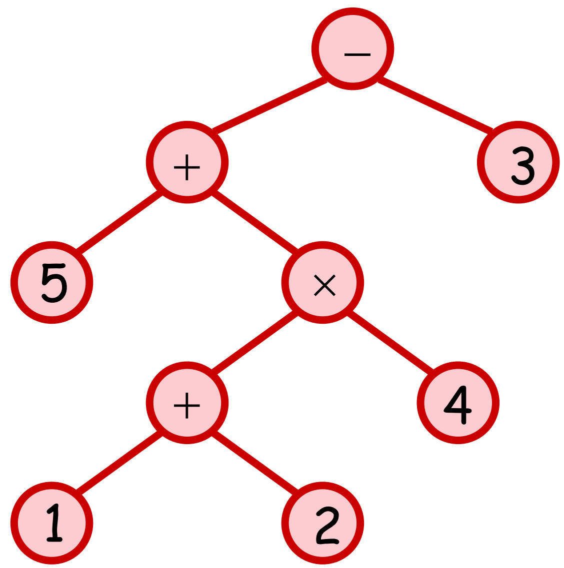
Expression Tree

- Ex : $5 + ((1 + 2) \times 4) - 3$



Expression Tree

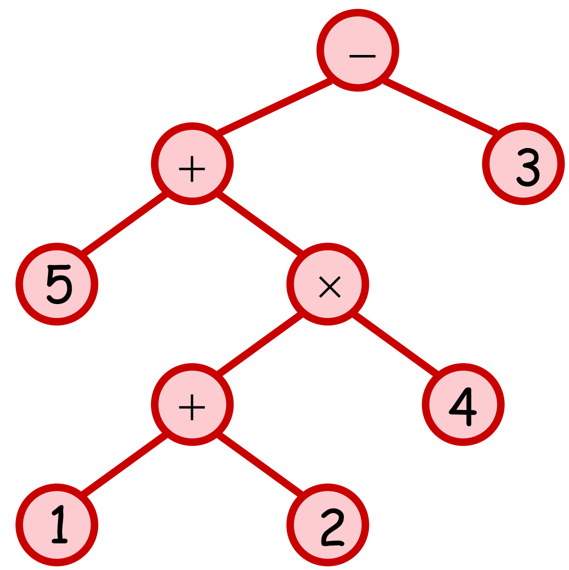
- Ex : $5 + ((1 + 2) \times 4) - 3$



Expression Tree

- If we now perform preorder traversal on the expression tree, we get the **prefix notation** of the expression

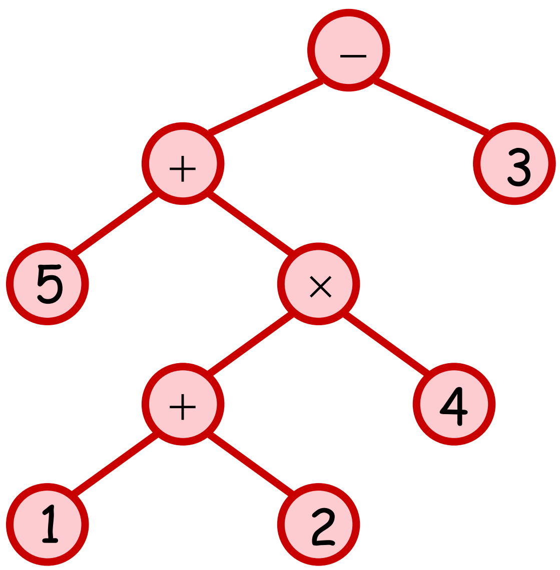
Prefix Notation :
$- + 5 \times + 1\ 2\ 4\ 3$



Expression Tree

- If we perform postorder traversal instead, we get the postfix notation of the expression

Postfix Notation :
5 1 2 + 4 × + 3 –



Evaluation

- In prefix or postfix notations, we do not need any parentheses
  - Both notations can allow us to compute the value of the original expression
  - Idea : Using a stack

- Remark : the original expression is stored in the infix notation

Evaluating Prefix Notation

- In prefix notation, when there are two consecutive "values", we can apply the operator before the two values
- So the evaluation can be done as follows:
  - Push operator or value on a stack, but ..
  - Whenever there are two values x and y on top of the stack, pop x and y, and also the next operator Δ. Then push a new value x Δ y back to stack

Evaluating Prefix Notation

- Ex : $- + 5 \times + 1\ 2\ 4\ 3$

  (Prefix notation of $5 + ((1 + 2) \times 4) - 3$)

contents of stack after key operations

$- + 5 \times + 1\ 2$
$- + 5 \times 3$
$- + 5 \times 3\ 4$
$- + 5\ 12$
$- 17$
$- 17\ 3$
14

Evaluating Postfix Notation

- In postfix notation, when we see an operator, we can apply the operator to the two values before the operator
- So the evaluation can be done as follows:
  - Push operator or value on a stack, but ..
  - Whenever we see an operator Δ, we pop Δ, and the next two values x and y on top of the stack. Then push a new value x Δ y back to stack

Evaluating Postfix Notation

- Ex : $5\ 1\ 2 + 4 \times + 3 -$

  (Postfix notation of $5 + ((1 + 2) \times 4) - 3$)

contents of stack after key operations

$5\ 1\ 2\ +$
$5\ 3$
$5\ 3\ 4\ \times$
$5\ 12$
$5\ 12\ +$
17
$17\ 3\ -$
14

Remarks

- Prefix or postfix notations are very useful because they can evaluate an expression easily (in one pass)

- In the next assignment, we will examine how to convert an expression from infix to postfix
  - This can also be done with a stack !!