Introduction to Structural Analysis

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Overview

Introduction

Definition of Structural Mechanics

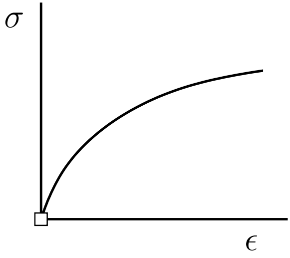
Mechanics. Branch of science that deals with response of matter to forces.

Civil Engineering:

- Structural mechanics (σ - ϵ): material displacement.
- Geomechanics (σ - ϵ): pressure, temperature, displacements.
- Fluid mechanics (σ - ϵ): pressure, velocities.

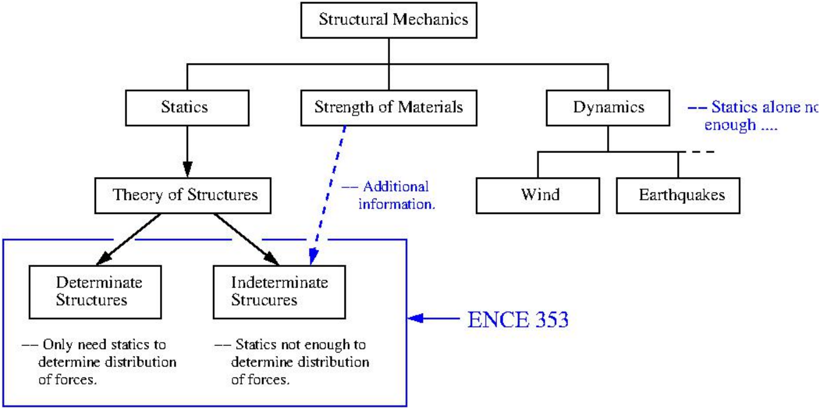
Other domains:

- Biomechanics (σ - ϵ): eye, heart, biological systems that grow!



Structural Mechanics and Analysis

Structural Mechanics → Static / Dynamic Analysis of Structures:



Structural Mechanics and Analysis

Scope of this class:

- We will be concerned with structural systems that are attached to the ground.

Pathway forward:

- Connect mechanics to analysis ...
- Connect analysis to design ...
- Theory of structural analysis ...

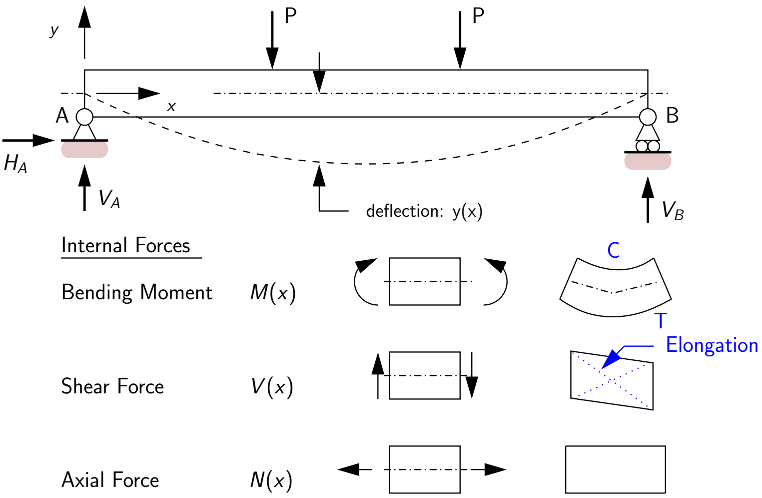
  Statically determinate structures ...

  Statically indeterminate structures ...

- Simplifying assumptions ...

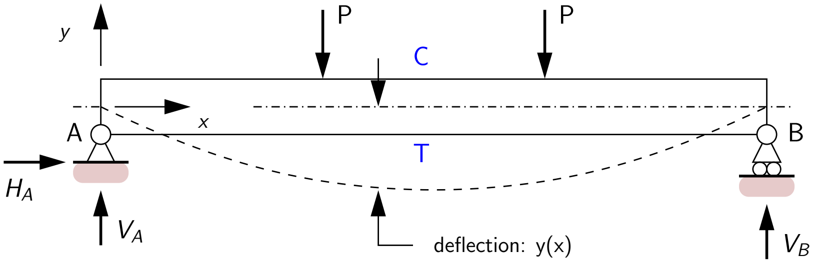
Connecting Mechanics to Analysis

Structural Mechanics and Analysis



Concrete Beam: Load-to-Failure Experiment

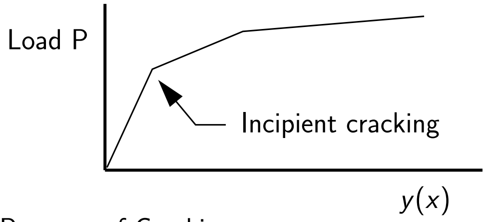
Experimental Setup



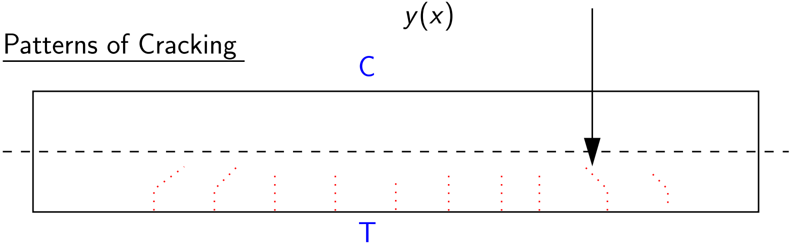
Bending Moment Diagram (BMD)

Concrete Beam: Load-to-Failure Experiment

Applied Load P versus Midspan Deflection



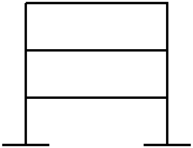
Patterns of Cracking



Pathway from Mechanics to System-Level Behavior

From material-level mechanics to building-system response:

Material Response —integrate→ Section Response —integrate→ Beam Response —assemble→ Building System Response

Material Response	Section Response	Beam Response	Building System Response
Stress $\sigma(x,y)$ Strain $\epsilon(x,y)$	Curvature $\phi(x) = \left[\frac{M(x)}{EI}\right]$	Deflection $y(x)$ Slope dy/dx	

How will the integration work?

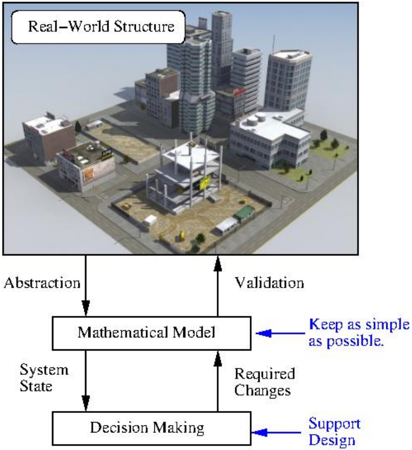
- Analytical Procedures: The math needs to be "nice" ...
- Numerical Procedures: Compute approximate solutions → linear algebra, numerical algorithms, structural analysis and finite elements.

Connecting Analysis to Design

Framework for Analysis and Design

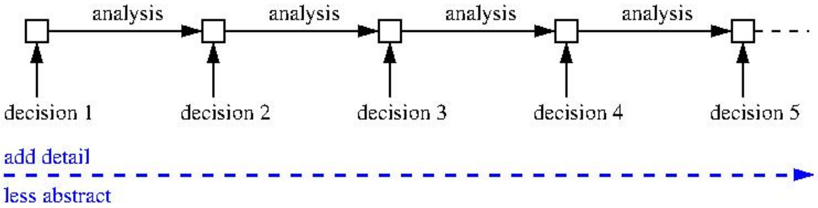
Creating an Analysis Model

- Abstract from consideration details not needed for decision making.
- Validate that model captures essential aspects of real-world behavior.
- Decision making needed for design.
- Perfect is the enemy of good. Mathematical model and decision making does not need to be perfect in order to be useful.



Connecting Analysis to Design

Structural Design. Sequence of analyses punctuated by decision making.

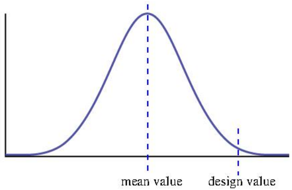


- Determine types and magnitudes of loads and forces acting on the structure.
- Determine context of project: geometric constraints, architectural constraints, geological conditions, urban regulations, cost, schedule, etc.

Connecting Analysis to Design

- Generate structural system alternatives.
- Analyze one or more of the alternatives.
- Select and perform detailed design.
- Implement/build.

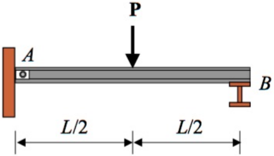
Analysis and decision making procedures complicated by uncertainties in loading, material properties, etc. State-of-the-art methods compensate for uncertainties with safety factors.



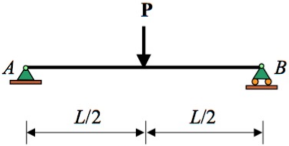
New structural systems may also require an experimental testing phase to verify behavior and achievable system performance.

Connecting Analysis to Design

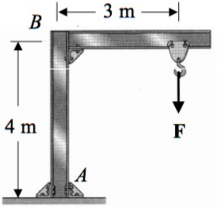
Real-World and Idealized Abstractions



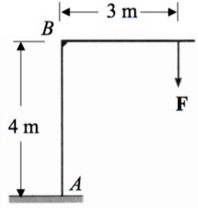
actual beam



idealized beam



actual structure



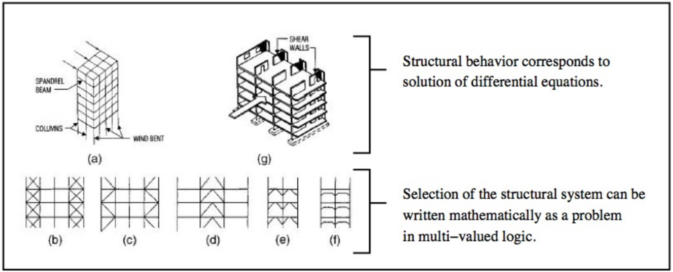
idealized structure

Connecting Analysis to Design

Formal Approaches to Behavior Modeling and Decision Making

Appropriate formalisms depend on the design domain of interest.

- Physical aspects of behavior are often characterized by differential equations.
- Logical aspects of system design can be captured by binary and multi-valued logic variables and boolean equations.



Connecting Analysis to Design

Structural Behavior

Time-dependent behavior corresponds to solutions of:

$$[M]\,\frac{d^2x}{d^2t} + [C]\,\frac{dx}{dt} + [K]\,x = P(t). \tag{1}$$

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Here,

- M, C, and K are (n × n) matrices,
- x is a (n × 1) vector of displacements,
- P(t) is a vector of external loads applied to the structural degrees of freedom.

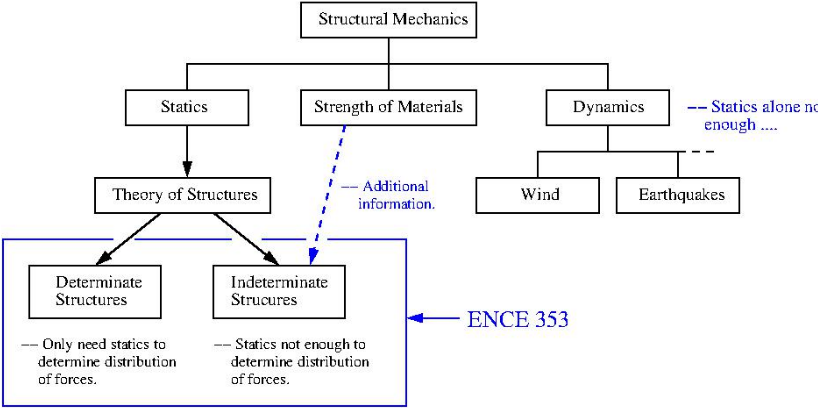
Design Parameters

- Selection of the best structural system (e.g., braced system) from a list of options.
- Size of the beams, columns, and bracing (if required).

Theory of Structures

Theory of Structures

Structural Mechanics → Static / Dynamic Analysis of Structures:



Statically Determinate Structures

Definition. Can use statics to determine reactions and distribution of element-level forces. Determinacy is not affected by details of loading.

Two-Dimensional Problems

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum M_z = 0. \tag{1}$$

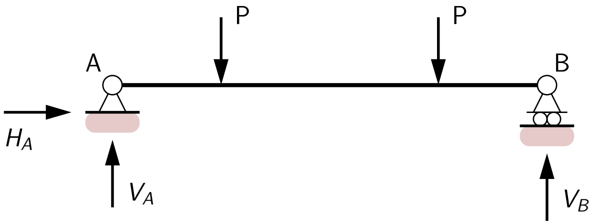
Three-Dimensional Problems

$$\sum F_x = 0, \ \sum F_y = 0, \ \sum F_z = 0. \tag{2}$$

$$\sum M_x = 0, \ \sum M_y = 0, \ \sum M_z = 0. \tag{3}$$

Statically Determinate Structures

Example 1. Simply supported beam:

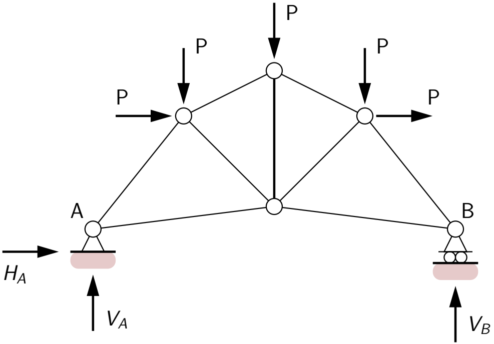


Three equations of equilibrium: $\sum F_x = 0, \ \sum F_y = 0, \ \sum M_z = 0.$

Three unknowns: V_A, H_A and V_B $\rightarrow$ Can use statics to solve.

Statically Determinate Structures

Example 2. Small truss structure:

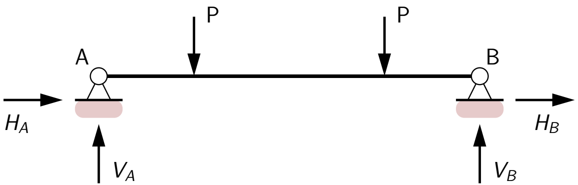


- Use statics to find support reactions V_A, H_A and V_B.
- Compute member forces by considering equilibrium of individual joints.

Statically Indeterminate Structures

Definition. Statics alone are not enough to find reactions. Need to find additional information (e.g., material behavior).

Example 1. Simply supported beam:

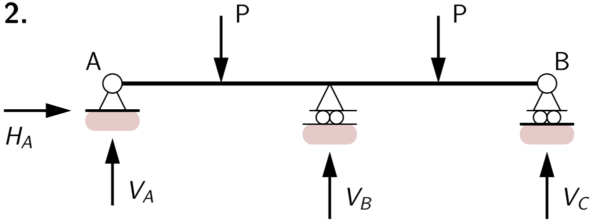


Three equations of equilibrium: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M_z = 0$.

Four unknowns: V_A, H_A, V_B and H_B $\rightarrow 4 > 3 \rightarrow$ statically indeterminate to degree 1.

Statically Indeterminate Structures

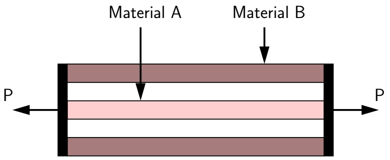
Example 2.



Three equations of equilibrium. Four unknowns: V_A, H_A, V_B and V_C $\rightarrow 4 > 3 \rightarrow$ statically indeterminate to degree 1.

Example 3. Multi-material Truss Element.

Material behavior defined by $\sigma - \epsilon$ characteristics. Need to maintain geometric compatibility.



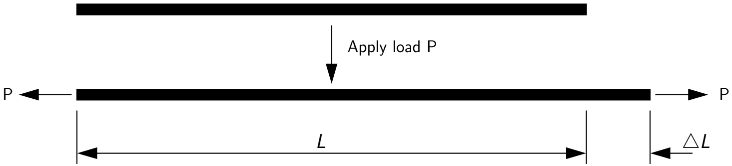
Simplifying Assumptions for ENCE 353

Small Displacements
Linear Systems Behavior

Assumption 1: Small Displacements

Definition. We assume that application of loads will cause a displacement (i.e., elements are not rigid).

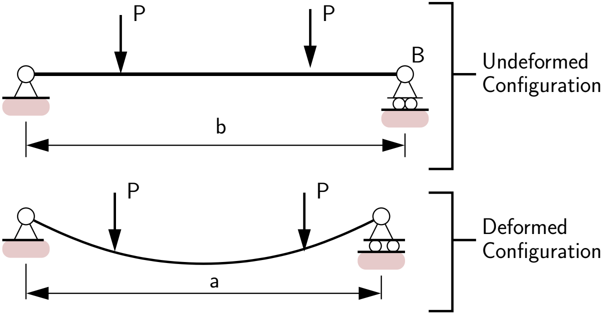
Example 1. Axial extension of a Rod



Small displacements means $\triangle L \ll L$, i.e., $\left[\frac{\triangle L}{L}\right] \ll 1$.

Assumption 1: Small Displacements

Example 2. Flexure of Beam Elements.



For steel/concrete structures: $a \approx b$ (i.e., $a > 0.99b$) $\rightarrow$ compute equilibrium with respect to the undeformed configuration.

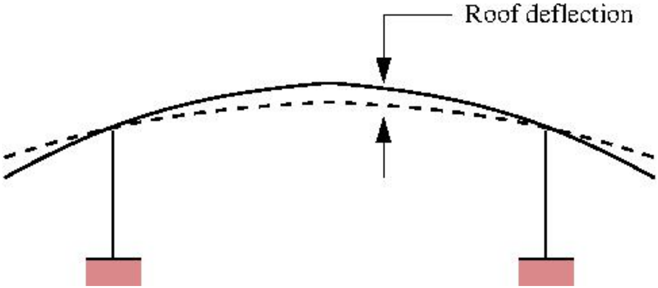
Assumption 1: Counter Examples

Arch Structures. Vintage Safeway Supermarkets.

Style: Use mid-century modern arch shape to create large open spaces.

Assumption 1: Counter Examples

Nice Trick: When heavy snow loads cause large roof deflections, arch mechanism gives illusion of safety!

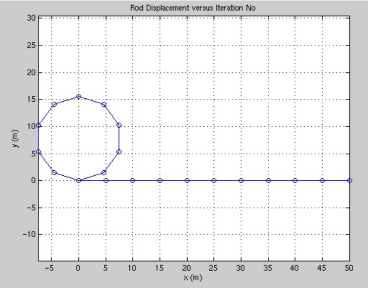


Google: safeway roof collapse.

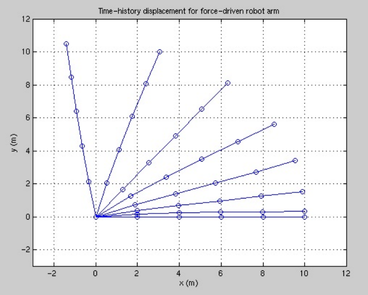
Assumption 1: Counter Examples

Large Geometric Displacements. Wind turbine blades, flexible robot arms, etc ...

Roll cantilever into circle.



Flexible robot arm maneuver.

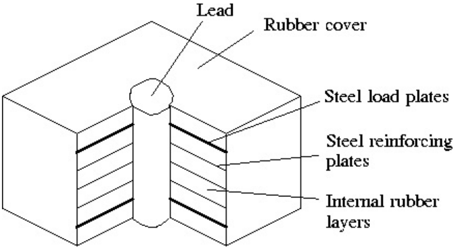


Source: Simo, Vu-Quoc, 1986.

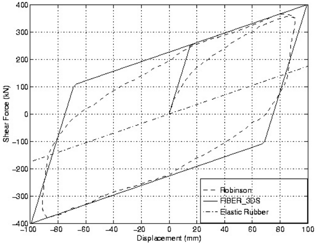
Assumption 1: Counter Examples

Large Material Displacements. Behavior of Lead-Rubber Isolators under Large Cyclic Earthquake Loads.

Lead-Rubber Laminated Bearing



Hysteresis Loops



Source: Lin W-J., 1997.

Assumption 2: Linear Systems Behavior

Mathematical Definition. Let k be a non-zero constant. A function y = f(x) is said to be linear if it satisfies two properties:

- $y = f(kx_1)$ is equal to $y = kf(x_1)$.
- $f(x_1 + x_2) = f(x_1) + f(x_2)$.

For constants k and m these equations can be combined:

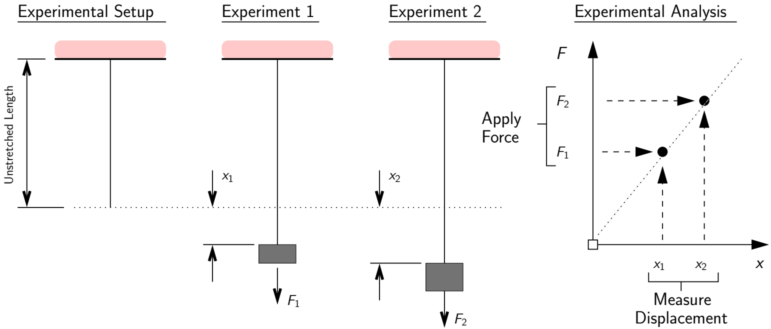
$$kf(x_1) + mf(x_2) \rightarrow f(kx_1 + mx_2). \tag{4}$$

Economic Benefit. Often evaluation of $y = f(x)$ has a cost.

Linearity allows us to compute $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and then predict the system response for $kx_1 + mx_2$ via linear combination of solutions. This is free!

Assumption 2: Linear Systems Behavior

Example 1. Consider an experiment to determine the extension of an elastic chord as a function of applied force.



Linearity allows us to predict solutions:

$$Kx_1 = F_1, Kx_2 = F_2, \to K(mx_1 + nx_2) = mF_1 + nF_2. \qquad (5)$$

Assumption 2: Linear Systems Behavior

Example 2. Analysis of Linear Structural Systems:

Suppose that matrix equations $AX = B$ represent behavior of a structural system:

- Matrix A will capture the geometry, material properties, etc.
- Matrix B represents externally applied loads (e.g., dead/live gravity loads).
- Column vector X represents nodal displacements.

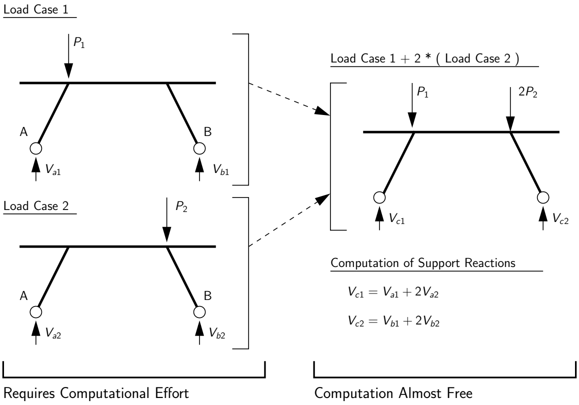
Solving $AX = B$ has computational cost $O(n^3)$.

However, if matrix system is linear, then:

$$AX_1 = B_1, AX_2 = B_2 \rightarrow A(mX_1 + kX_2) = mB_1 + kB_2. \quad (6)$$

Assumption 2: Linear Systems Behavior

We can simply add the results of multiple load cases:



Works for support reactions, bending moments, displacements, etc.

Symmetries

Taking Advantage of Symmetry

Observation. Symmetries provide engineers with an opportunity to reduce model size and computational effort.

Definitions. Here's what a mathematician would say:

- A function is even (symmetric) when y = f(x) = f(-x).
  Examples: y = x^2 and y = cos(x).
- A function is odd (skew-symmetric) when y = g(x) = -g(-x).
  Examples: y = x^3 and y = sin(x).

Home Exercise. Show that:

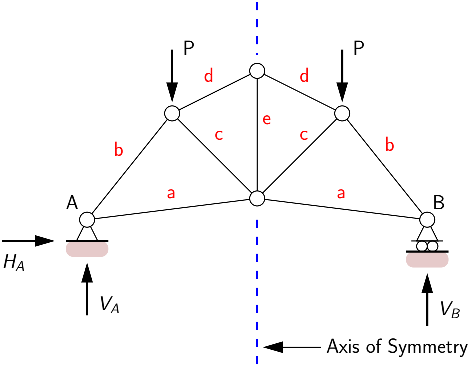
$$\int_{-a}^{a} f(x)g(x)dx = 0. \quad (7)$$

We will use this later in the course to simplify analysis.

Taking Advantage of Symmetry

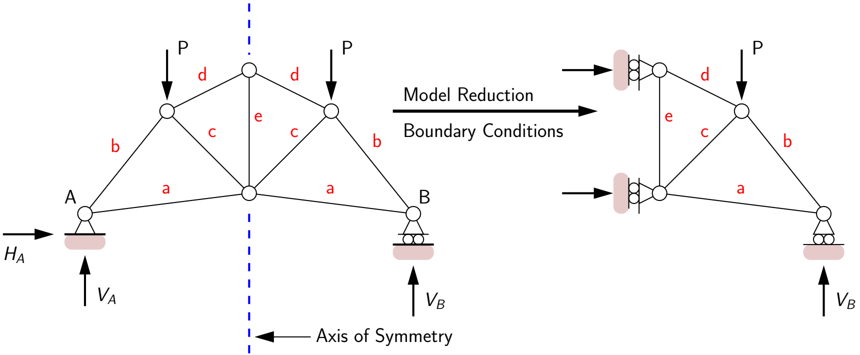
Example 1. Consider the Small Truss Structure:

- Axis of symmetry works for geometry, loading patterns and reactions.
- Only need to compute member forces $a - e$.
- Model reduction requires careful treatment of boundary conditions.



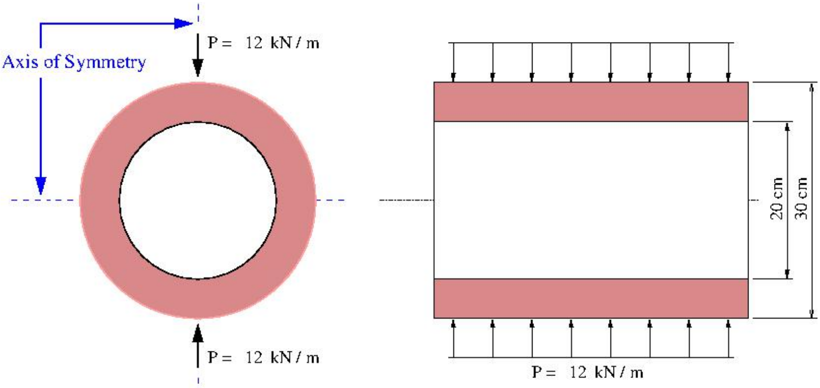
Taking Advantage of Symmetry

Model Reduction: Cut model size in half, then adjust boundary conditions along axis of symmetry.



Taking Advantage of Symmetry

Example 2. Stress Analysis in Cross Section of a Long Pipe



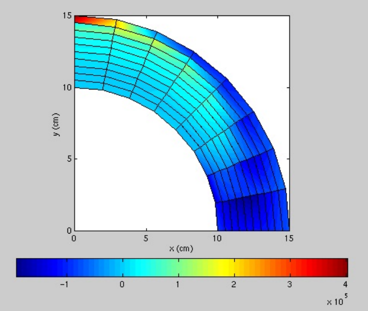
Pipe Cross Section

Elevation View of Pipe

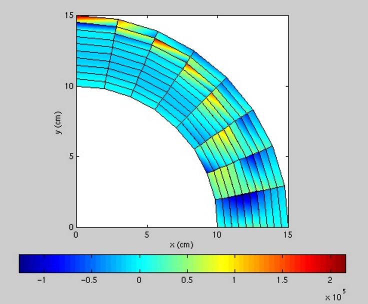
Taking Advantage of Symmetry

Two axes of symmetry for geometry and loading → Only need to analyze 1/4 of the cross section.

Stresses: $\sigma_{yy}(x, y)$



Stresses: $\sigma_{xx}(x, y)$



References

- Wane-Jang Lin, Modern Computational Environments for Seismic Analysis of Highway Bridge Structures, PhD Thesis, University of Maryland, College Park, MD, 1997.
- Simo J.C., Vu-Quoc L., On the Dynamics of Flexible Beams Under Large Overall Motions—The Plane Case: Part II, Journal of Applied Mechanics, (53) 4, 855-863, 1986.