

**ENCE 353 Midterm 1, Open Notes and Open Book**

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**Exam Format and Grading.** This take home midterm exam is open notes and open book. You need to comply with the university regulations for academic integrity.

There are three questions. Partial credit will be given for partially correct answers, so please show all your working.

Please see the class web page for instructions on how to submit your exam paper.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

**Question 1 (15 points): Support Reactions and Bending Moments in a connected Beam Structure.**

Consider the multi-span beam structure shown in Figure 1.

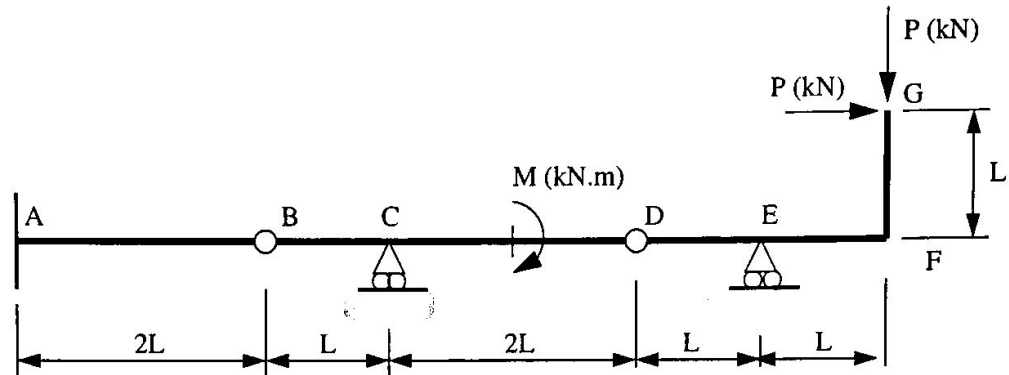


Figure 1. Front elevation view of multi-span beam structure.

The cantilever is fully-fixed to the wall at Point A. Points B and D are hinges. Horizontal and vertical point loads  $P$  (kN) are applied at point G. A clockwise moment  $M$  (kN.m) is applied at the midpoint of beam segment C-D. Assume that  $P$  and  $M$  are both positive values.

[1a] (3 pts). Compute the degree of indeterminacy for the beam structure.

$$\hat{i} = f - 3 - r; \quad f = 5, \quad 2 \text{ releases.}$$

$$= 5 - 3 - 2 = 0 \leftarrow \text{Statically determinate.}$$

[1b] (3 pts). Show that the total force at hinge D is  $\sqrt{5}P$ .

$$\sum \mathcal{M}_d = 0$$

$$\Rightarrow V_e \cdot L - PL - P(2L) = 0$$

$$\Rightarrow V_e = 3P.$$

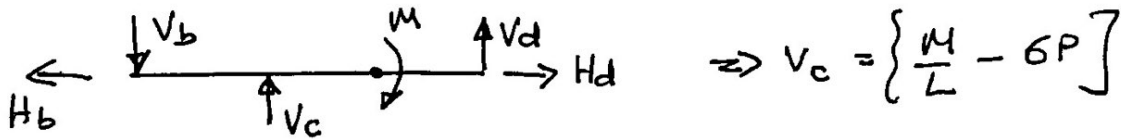
$$\sum F_y = 0 \Rightarrow V_d = 2P.$$

$$\sum F_x = 0 \Rightarrow H_d = P.$$

$$\text{Total force at } d = [V_d^2 + H_d^2]^{1/2} = \sqrt{5}P.$$

[1c] (3 pts). Derive an expression for the vertical reaction force at C as a function of P, M and L.

$$\sum M_b = 0 \Rightarrow V_c \cdot L + V_d(3L) - M = 0$$



[1d] (6 pts). Now suppose that  $M = k PL$ , where  $k$  is a constant that can take values 0, 3, 6 or 9. Draw and label a diagram that shows how the bending diagram for beam element B-C-D varies as function of  $k$ .

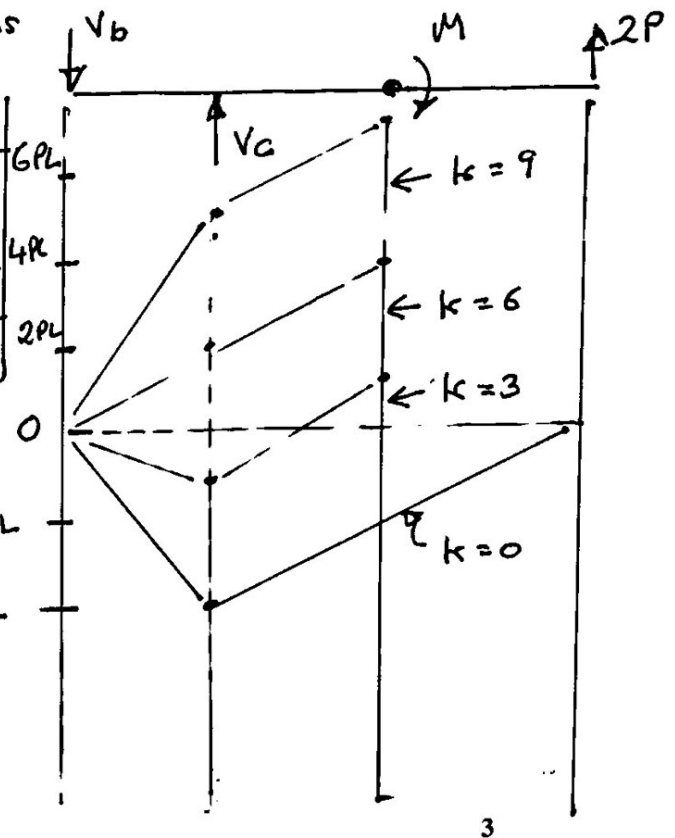
I suggest that you create one diagram with four bending moment diagram profiles. Clearly indicate on the bending moment diagram regions where the fibre will be in tension and compression, and any conditions on P and M that also apply.

$$\text{Let } M = kPL \Rightarrow V_c = \frac{M}{L} - 6P = (k - 6)P$$

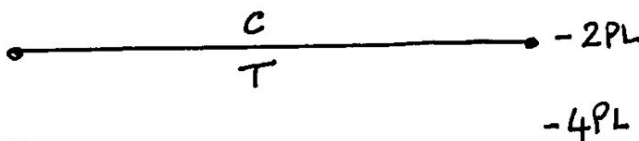
$$V_b = (k - 4)P$$

Table of Moments & Reactions

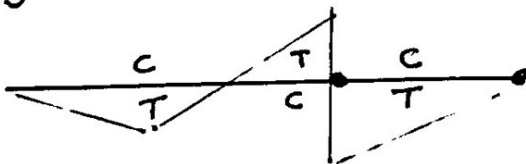
k	M	$V_b$	$V_c$	$V_d$
0	0	$-4P$	$-6P$	$2P$
3	$3PL$	$-P$	$-3P$	$2P$
6	$6PL$	$2P$	0	$2P$
9	$9PL$	$5P$	$3P$	$2P$



$k = 0$ .



$k = 3$



etc...

**Question 2 (15 points): Tension, Compression and Zero-Force Members in a Truss Structure.**

Consider the truss structure shown in Figure 2.

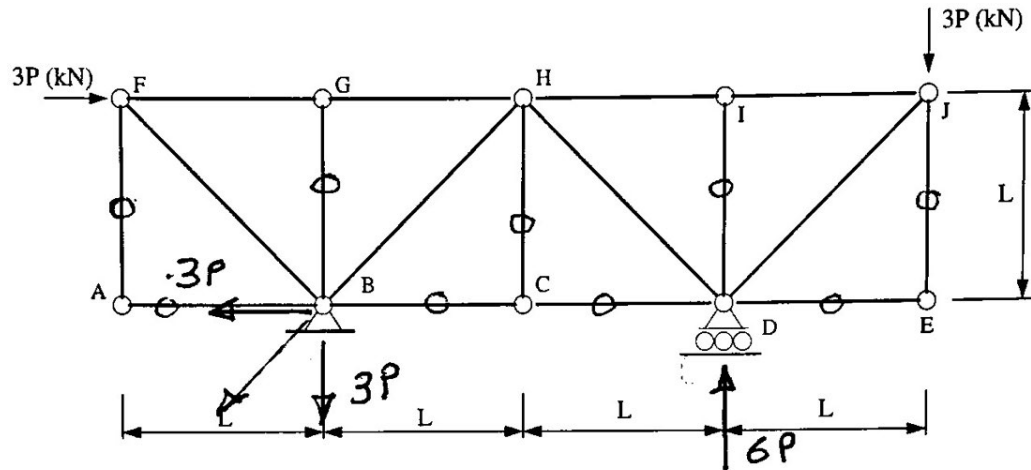


Figure 2. Elevation view of 17 bar truss structure.

Horizontal and vertical loads of  $3P$  kN are applied at nodes F and J, respectively.

[2a] (3 pts). Compute the **magnitude** and **direction** of the total support reactions at points B and D.

$$\sum \mathcal{M}_B = 0 \Rightarrow (3P) \cdot L + 3P(3L) - V_D \cdot (2L) = 0$$

$$\Rightarrow V_D = 6P.$$

$$\sum F_x = 0 \Rightarrow H_B = -3P.$$

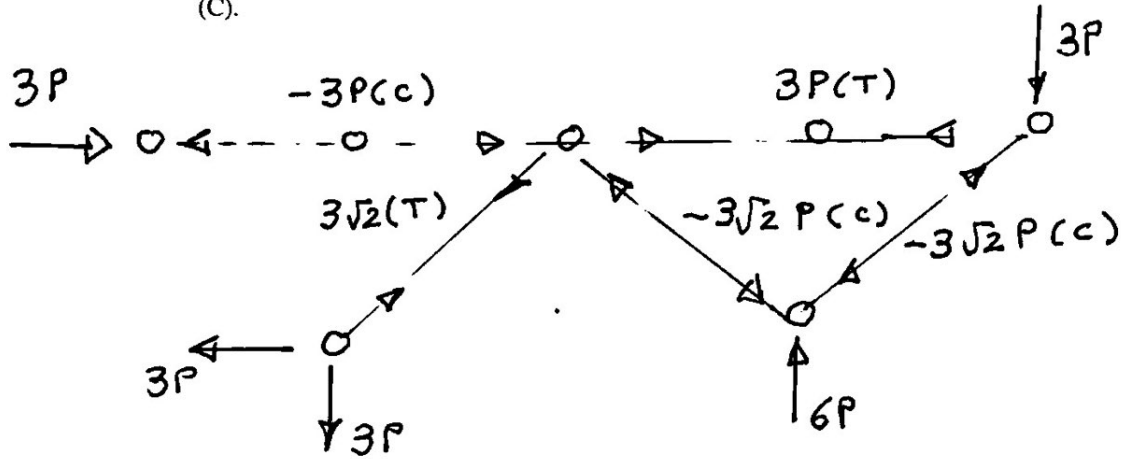
$$\sum F_y = 0 \Rightarrow V_B = -3P$$

[2b] (3 pts). Identify the zero-force members (If you wish, you can simply annotate Figure 2).

Notice that the total force at B aligns with element BH.

$$\Rightarrow BC = 0.$$

[2c] (7 pts). Using the method of joints (or otherwise) show that: (1) The maximum tensile force in the structure is  $3\sqrt{2}P$  kN (T), and (2) The maximum compressive force in the structure is  $-3\sqrt{2}P$  kN (C).



[2d] (2 pts). Draw a simplified version of Figure 2 with all of the zero force elements removed.

See part 2c.

Question 3 (10 points): Degree's of Indeterminacy.

[3a] (4 pts). Compute the degree of indeterminacy for the structure shown in Figure 3.

Ring Method

$$\hat{i} = 3n - r$$

$$\left. \begin{array}{l} n = 2 \\ r = 2 \end{array} \right\} \Rightarrow \hat{i} = 4.$$

Method 2

$$\hat{i} = f - 3 - r; \quad \begin{array}{l} f = 9 \\ r = 2 \end{array}$$

$$= 4.$$

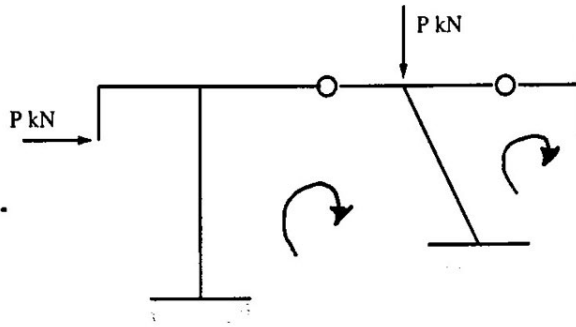


Figure 3. Simple portal frame.

[3b] (6 pts). Using the method of trees (or otherwise), compute the degree of indeterminacy for the moment-resistant frame shown in Figure 4.

Strictly speaking,  
tree method does  
not apply because  
there are releases.

But if you ignore  
that detail.

$$t = 4.$$

$$f = 37$$

$$\hat{i} = f - 3t = 37 - 12$$

$$= \underline{\underline{25}}$$

6

Ring Method

$$\hat{i} = 3n - r.$$

$$\left. \begin{array}{l} n = 10 \\ r = 5 \end{array} \right\} \hat{i} = 30 - 5$$

$$= \underline{\underline{25}}$$

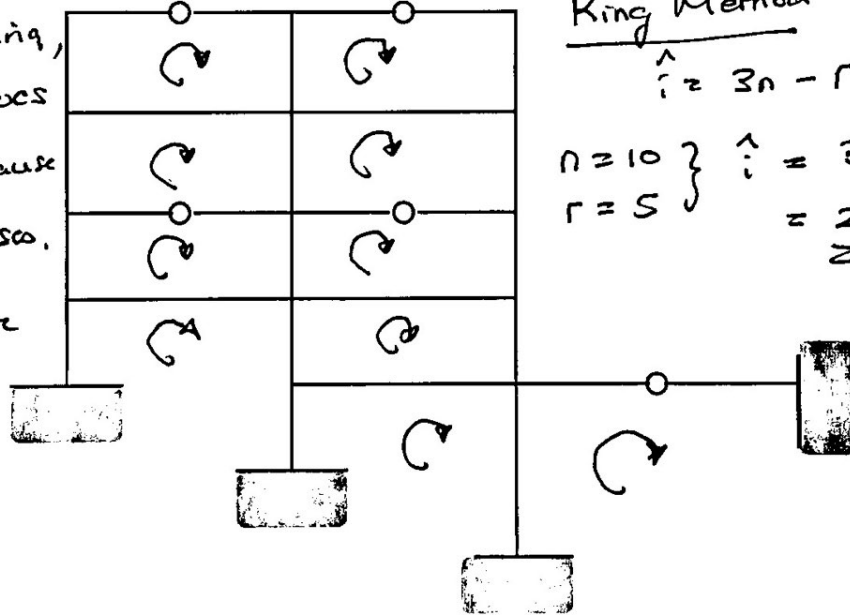


Figure 4. Elevation view of a moment-resistant frame.