Qualitative Sketching of Displacements

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Overview

1. Qualitative Sketching of Displacements
   - Deflection of very simple structures
   - Rules and Simplifications
   - Types of Boundary Support
   - Rigid Corner Points

2. Case Study Examples

Qualitative Sketching of Displacements

Qualitative Sketching of Displacements

Qualitative Deflected Shape

A rough (usually exaggerated) sketch of the neutral surface (axis) of the structure in the deformed position under the action of a prescribed loading condition.

These sketches provide insight into the behavior of structures, e.g.,

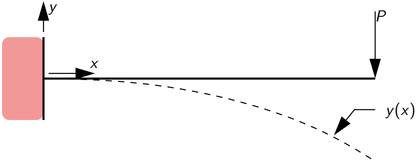
- Is there tension in the top or bottom of the beam?
- Does the beam move upwards or downwards?
- Does the beam rotate clockwise or anticlockwise?

Good answers to these questions can serve as valuable checks for correctness (catch mistakes) in more complicated approaches to analysis.

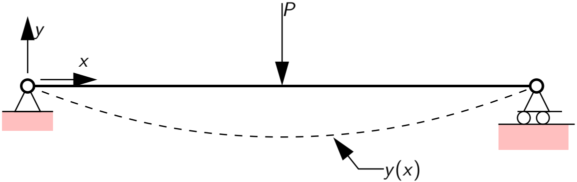
Qualitative Sketching of Displacements

Procedure: Very simple structures → can rely on intuition.

Simple Example 1. Cantilever Beam



Simple Example 2. Simply Supported Beam



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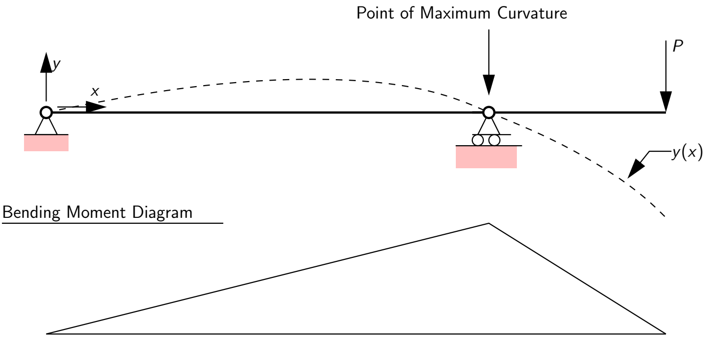
Simple Example 3. Cantilevered Simply-Supported Beam



For more complex cases, intuition starts to fail ...

Revised Procedure. Rely on math and mechanics.

Qualitative Sketching of Displacements

Here's what Mechanics tells us:

- Beam curvature is proportional to bending moment.

$$\phi(x) = \left[\frac{M(x)}{EI} \right]. \tag{1}$$

- Positive bending moment $\rightarrow$ deflection profile is concave upward (convex).
- Negative bending moment $\rightarrow$ deflection profile is concave downward.
- Zero bending moment $\rightarrow$ curvature of beam or frame element remains unchanged.

Qualitative Sketching of Displacements

The Rules:

- The curvature must be consistent with the profile of bending moment.
- The deflected shape must satisfy the boundary constraints.
- The original angle at a rigid joint (e.g., cantilever support) must be preserved.
- Points of inflection occur when $M(x) = 0$ and $V(x) \neq 0$.

Simplifications:

- The length of the deformed member is the same as the original length of the unloaded member (i.e., small displacements).
- Neglect axial deformations (i.e., very close to zero).
- Neglect self-weight of the structure.

Qualitative Sketching of Displacements

Types of Boundary Support:

Symbol	Name	Movements
	Roller (Horizontal)	δ_X $\cancel{\delta_Y}$ θ
	Vertical Roller	$\cancel{\delta_X}$ δ_Y θ
	Pin	$\cancel{\delta_X}$ $\cancel{\delta_Y}$ θ
	Fixed	$\cancel{\delta_X}$ $\cancel{\delta_Y}$ $\cancel{\theta}$
	Vertical Support (beam continuous over the support and can rotate)	δ_X $\cancel{\delta_Y}$ θ

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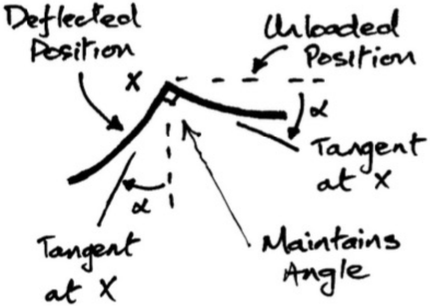
Rigid Corner Points: Must keep the same angle as they rotate.



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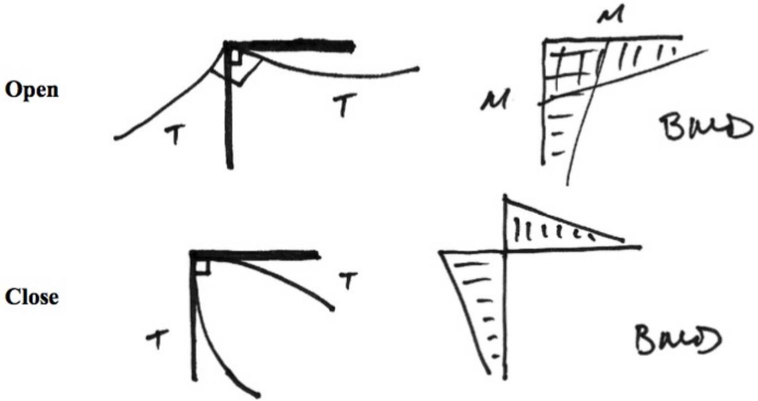
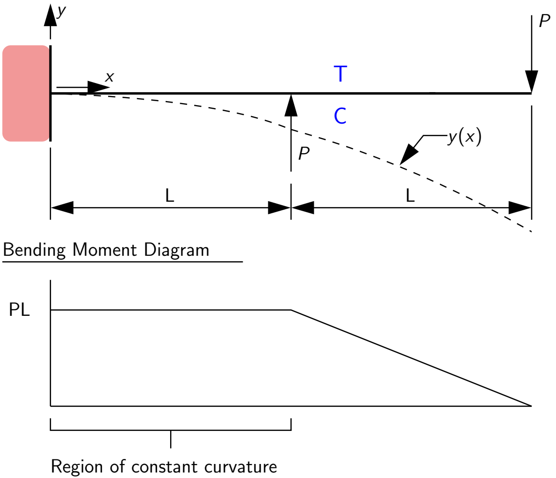
Rigid Corner Points: Can only open or close.



Qualitative Sketching of Displacements

Case Study 1. Simple application of mechanics.



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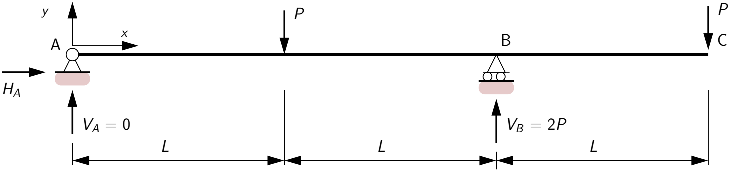
Case Study 2. Here's why you need math and mechanics:

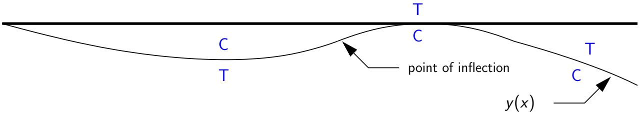


First Cut at Sketching Deflection

- Looks great!
- Completely WRONG !!!

Boundary Conditions: $y(0) = y(2L) = 0$.

Bending moment at A and C = 0



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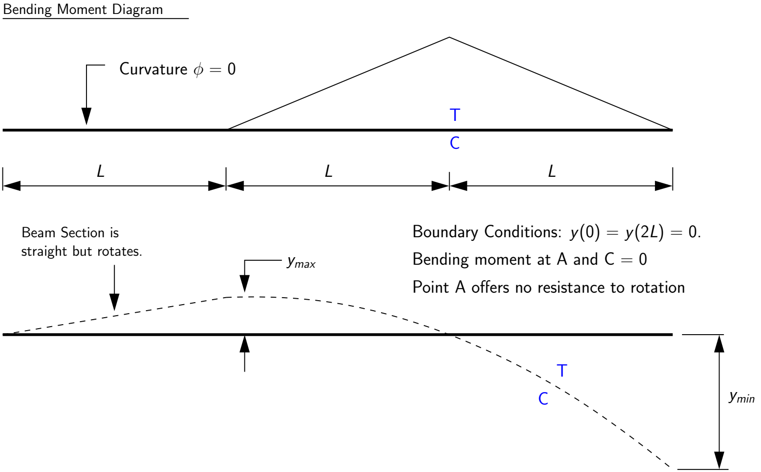
Here's what the mechanics says:



Boundary Conditions: $y(0) = y(2L) = 0$.

Bending moment at A and C = 0

Point A offers no resistance to rotation

Points to note:

- The bending moment diagram, M(x), is related to beam curvature, which, in turn, is related to curvature in the deflected shape.
- In order for the first deflected shape to be correct, the point-of-inflection in the beam deflection would need to correspond to a change in sign in the bending moment diagram. No such point exists.
- The bending moment in the left-most segment is zero – thus, any displacements will be due to rigid body rotations of the beam segment.
- In the center and right segments of the beam the top fibre is in tension (T) and the bottom fiber is in compression (C).
- The second draft of the deflected shape is consistent with these notions.

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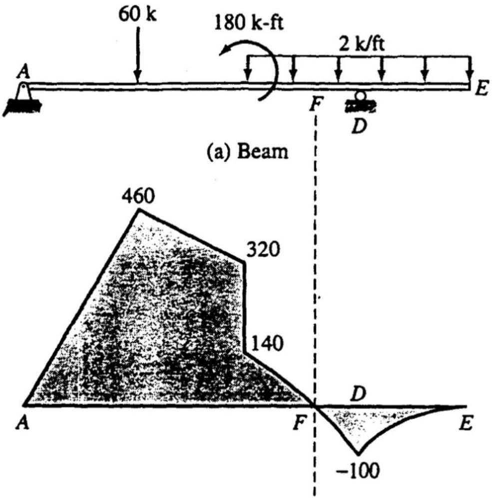
Case Study 3. Cantilevered Simply-Supported Beam



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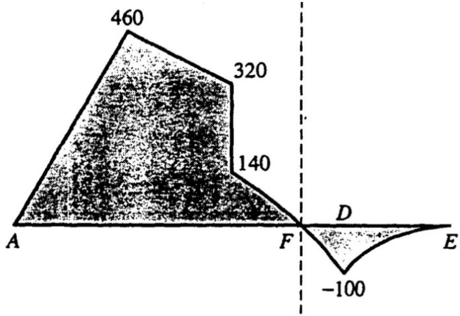
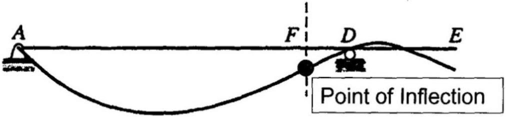
Case Study 3. Cantilevered Simply-Supported Beam



(b) Bending Moment Diagram (k-ft)



(c) Qualitative Deflected Shape

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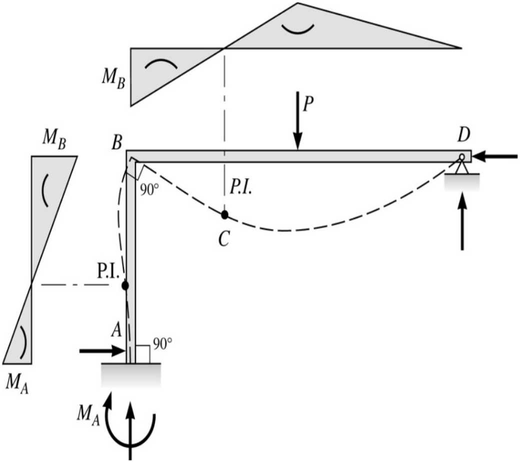
Case Study 4. Points of inflection (P.I.) in deflected shape occur at locations where bending moment equals zero.



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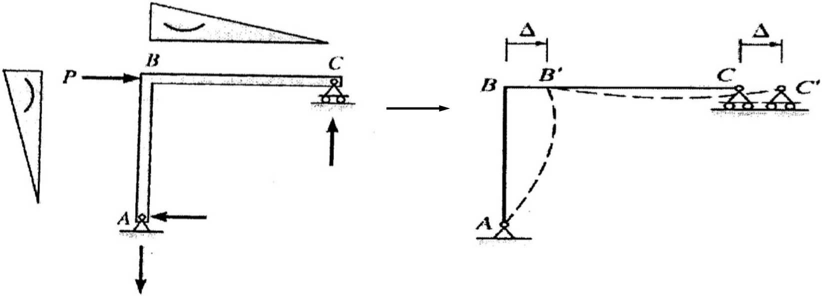
Case Study 5. Sideways displacement of inverted L-frame.



Note. We assume that the length of elements A-B and B-C will not change.

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Case Study 6. Discontinous slope of beam elements at pin C.