Statically Determinate Structures

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Overview

Part 2

Introduction

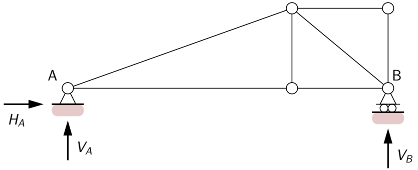
Need for Mathematical Test

Three cases to consider:

Test Structure A: Determinate.

Can compute:

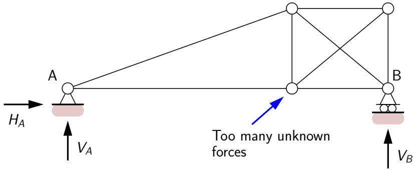
- Support reactions. ✓
- Member forces. ✓



Test Structure B: Indeterminate.

Can compute:

- Support reactions. ✓
- Member forces. ✗

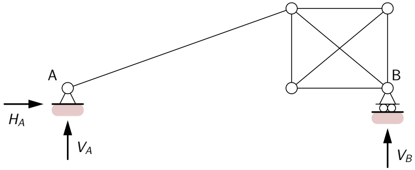


Need for Mathematical Test

Test Structure C: Unstable.

Can compute:

- Support reactions. ✗
- Member forces. ✗



Key Points:

- Intuition on notions of determinacy will not scale. We need a mathematical test to classify structures.
- Initial inclination is to design for A and avoid B – it's complicated and probably won't work. Unless, there are benefits to B?

Indeterminacy of Beams

Computing Degree of Indeterminacy

Definition. The degree of indeterminacy is equal to the number of additional equations needed to solve a problem uniquely.

Additional info:

- Compatibility of deformations – this is the force method.
- Equilibrium of forces – this is the displacement method.

Beams: $\hat{i}$ = f -3 - r, where:

- f = total no of external forces,
- r = total no of releases (hinges),
- 3 = no of equations from statics.

Indeterminacy of Beams

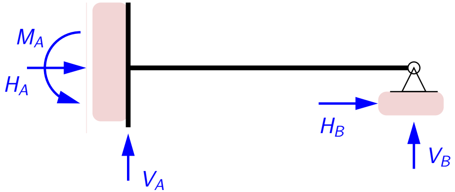
Example 1. Supported Cantilever Beam.

We have:

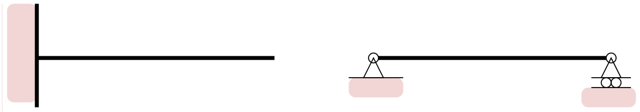
$r = 0$,

$f = \{V_A, H_A, \cdots, V_B\} =$ 5.

$\hat{i} =$ f - 3 - r $= 2$.

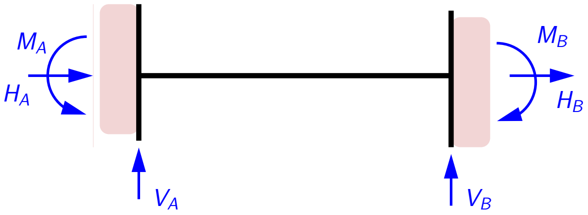


Need to release two restraints to create determinate structures, e.g.,



Indeterminacy of Beams

Example 2. Fixed-Fixed Beam.



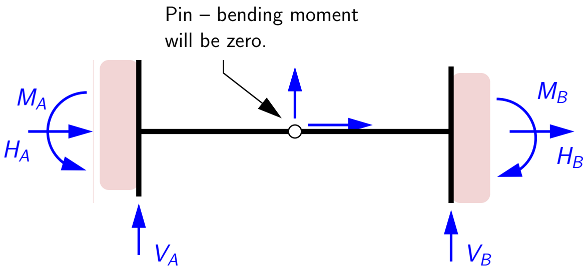
We have: $r = 0$,

$f = \{V_A, H_A, M_A, V_B, H_B, M_B\} = 6$.

$\hat{i} = f - 3 - r = 3$.

Indeterminacy of Beams

Example 3. Fixed-Fixed Beam + Hinge.



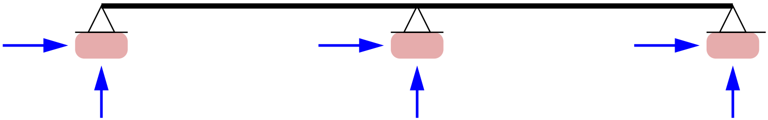
We have: $r = 1$,

$f = \{V_A, H_A, M_A, V_B, H_B, M_B\} = 6.$

$\hat{i} = f - 3 - r = 2.$

Indeterminacy of Beams

Example 4. Two-Span Beam.



We have: $r = 0$,

$f = \{V_A, H_A, V_B, H_B, V_C, H_C\} = 6.$

$\hat{i}$ = f - 3 - r = 3.

Indeterminacy of Frames

Tree Method

Approach: Systematically release redundant forces until trees are formed.

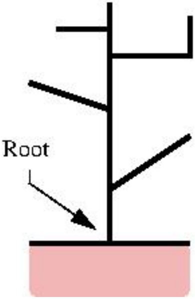
Formula: $\hat{i}$ = f - 3t, where:

- f = no of external forces,
- t = no of trees.

Constraints: Frame cannot have internal releases (no loops in trees).

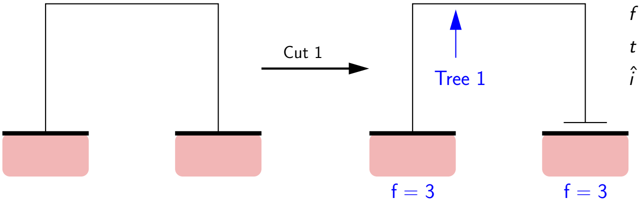
Trees:

- A tree has one root.
- A tree cannot have a closed loop branch.



Tree Method

Example 1a.

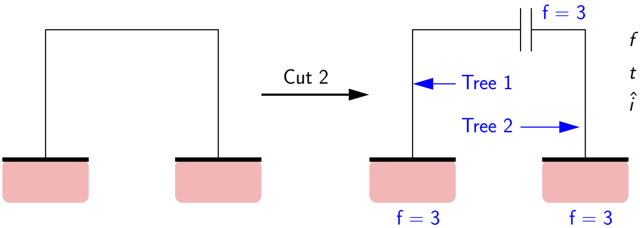


$f = 6$

$t = 1$

$\hat{i} = 6 - 3 = 3$

Example 1b.



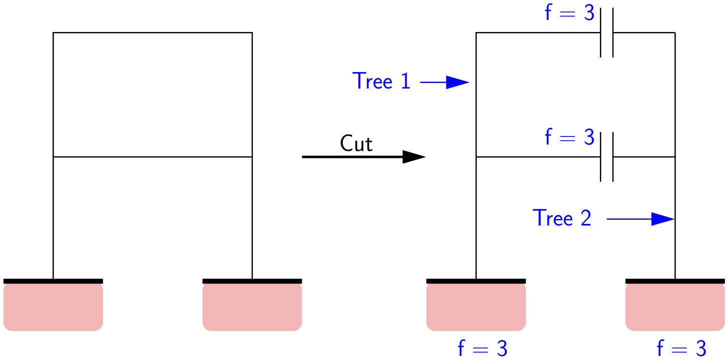
$f = 9.$

$t = 2.$

$\hat{i} = 9 - 3 * 2 = 3$

Tree Method

Example 2.



$f = 12.$

$t = 2.$

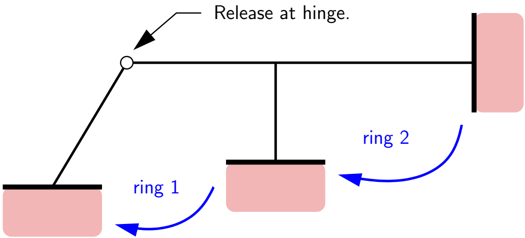
$\hat{i} = 12 - 3 * 2 = 6.$

Ring Method

Formula: $\hat{i}$ = 3n - r, where:

- n = no of rings.
- r = no of releases (each ring has 3 degrees of indeterminacy).

Example 1.



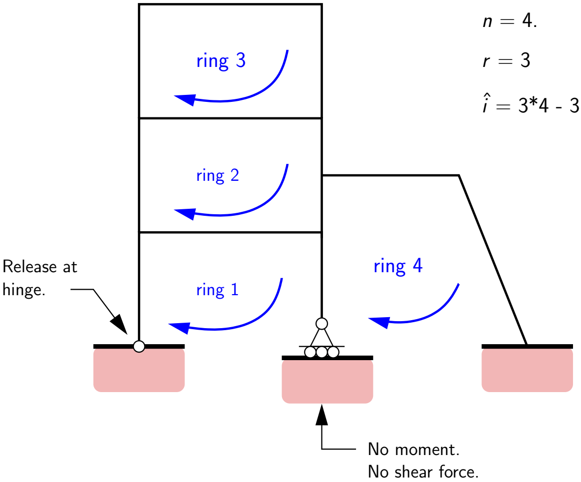
$n = 2.$

$r = 1.$

$\hat{i}$ = 6 - 1 = 5.

Ring Method

Example 2.



n = 4.

r = 3

$\hat{i}$ = 3*4 - 3 = 9.