Introduction to Structural Analysis

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Overview

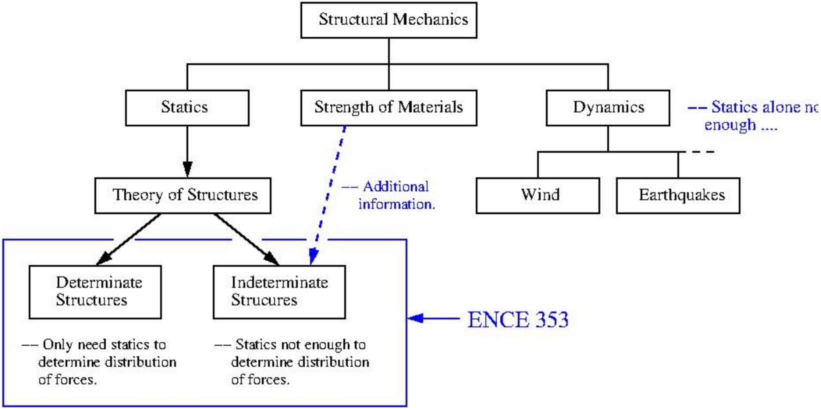
1. Introduction
   - Course Introduction
2. Connecting Mechanics to Analysis
3. Connecting Analysis to Structural Design
   - Connecting Analysis to Structural Design
4. Theory of Structures
   - Statically Determinate and Indeterminate Structures
5. Simplifying Assumptions
   - Small Displacements, Linear Systems Behavior
6. Symmetries

Part 3

Introduction

Structural Mechanics and Analysis

Structural Mechanics → Static / Dynamic Analysis of Structures:



Structural Mechanics and Analysis

Scope of this class:

- We will be concerned with structural systems that are attached to the ground.

Pathway forward:

- Connect mechanics to analysis ...
- Connect analysis to design ...
- Theory of structural analysis ...

  Statically determinate structures ...

  Statically indeterminate structures ...

- Simplifying assumptions ...

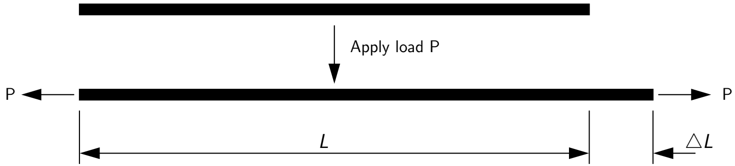
Simplifying Assumptions for ENCE 353

Small Displacements
Linear Systems Behavior

Assumption 1: Small Displacements

Definition. We assume that application of loads will cause a displacement (i.e., elements are not rigid).

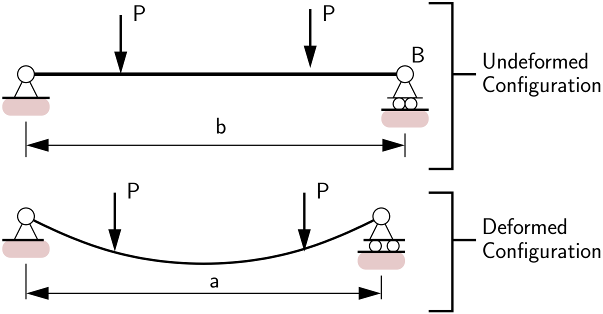
Example 1. Axial extension of a Rod



Small displacements means $\triangle L \ll L$, i.e., $\left[\frac{\triangle L}{L}\right] \ll 1$.

Assumption 1: Small Displacements

Example 2. Flexure of Beam Elements.



For steel/concrete structures: $a \approx b$ (i.e., $a > 0.99b$) $\rightarrow$ compute equilibrium with respect to the undeformed configuration.

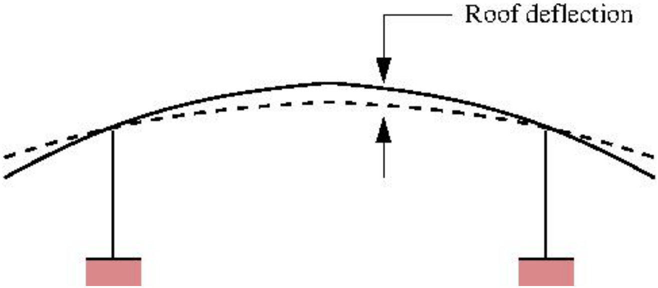
Assumption 1: Counter Examples

Arch Structures. Vintage Safeway Supermarkets.

Style: Use mid-century modern arch shape to create large open spaces.

Assumption 1: Counter Examples

Nice Trick: When heavy snow loads cause large roof deflections, arch mechanism gives illusion of safety!

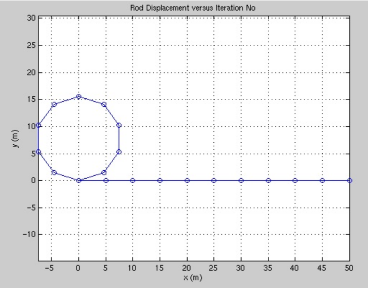


Google: safeway roof collapse.

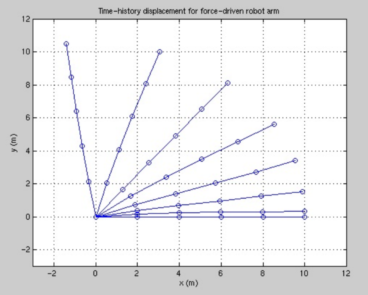
Assumption 1: Counter Examples

Large Geometric Displacements. Wind turbine blades, flexible robot arms, etc ...

Roll cantilever into circle.



Flexible robot arm maneuver.

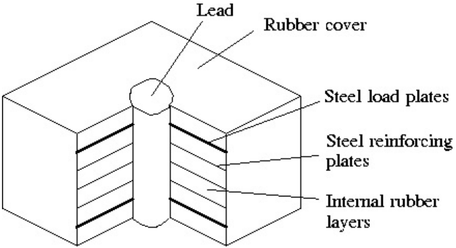


Source: Simo, Vu-Quoc, 1986.

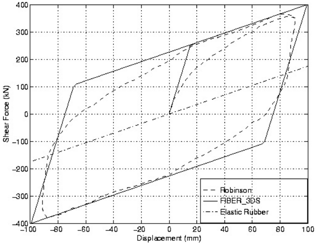
Assumption 1: Counter Examples

Large Material Displacements. Behavior of Lead-Rubber Isolators under Large Cyclic Earthquake Loads.

Lead-Rubber Laminated Bearing



Hysteresis Loops



Source: Lin W-J., 1997.

Assumption 2: Linear Systems Behavior

Mathematical Definition. Let k be a non-zero constant. A function y = f(x) is said to be linear if it satisfies two properties:

- $y = f(kx_1)$ is equal to $y = kf(x_1)$.
- $f(x_1 + x_2) = f(x_1) + f(x_2)$.

For constants k and m these equations can be combined:

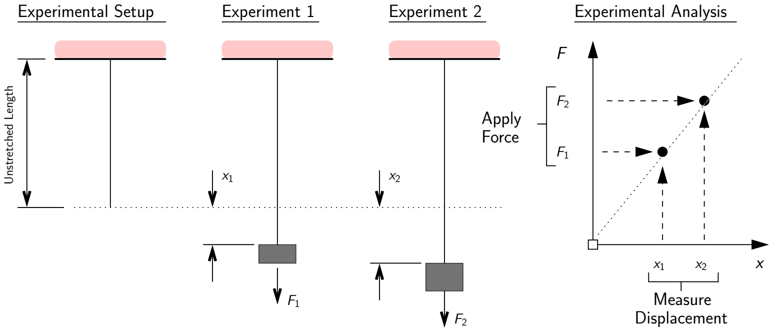
$$kf(x_1) + mf(x_2) \rightarrow f(kx_1 + mx_2). \tag{4}$$

Economic Benefit. Often evaluation of $y = f(x)$ has a cost.

Linearity allows us to compute $y_1 = f(x_1)$ and $y_2 = f(x_2)$ and then predict the system response for $kx_1 + mx_2$ via linear combination of solutions. This is free!

Assumption 2: Linear Systems Behavior

Example 1. Consider an experiment to determine the extension of an elastic chord as a function of applied force.



Linearity allows us to predict solutions:

$$Kx_1 = F_1, Kx_2 = F_2, \rightarrow K(mx_1 + nx_2) = mF_1 + nF_2. \qquad (5)$$

Assumption 2: Linear Systems Behavior

Example 2. Analysis of Linear Structural Systems:

Suppose that matrix equations $AX = B$ represent behavior of a structural system:

- Matrix A will capture the geometry, material properties, etc.
- Matrix B represents externally applied loads (e.g., dead/live gravity loads).
- Column vector X represents nodal displacements.

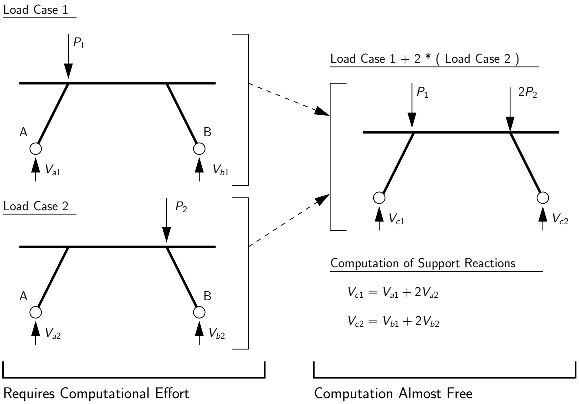
Solving $AX = B$ has computational cost $O(n^3)$.

However, if matrix system is linear, then:

$$AX_1 = B_1, AX_2 = B_2 \rightarrow A(mX_1 + kX_2) = mB_1 + kB_2. \quad (6)$$

Assumption 2: Linear Systems Behavior

We can simply add the results of multiple load cases:



Works for support reactions, bending moments, displacements, etc.

Symmetries

Taking Advantage of Symmetry

Observation. Symmetries provide engineers with an opportunity to reduce model size and computational effort.

Definitions. Here's what a mathematician would say:

- A function is even (symmetric) when y = f(x) = f(-x). Examples: y = x^2 and y = cos(x).
- A function is odd (skew-symmetric) when y = g(x) = -g(-x). Examples: y = x^3 and y = sine(x).

Home Exercise. Show that:

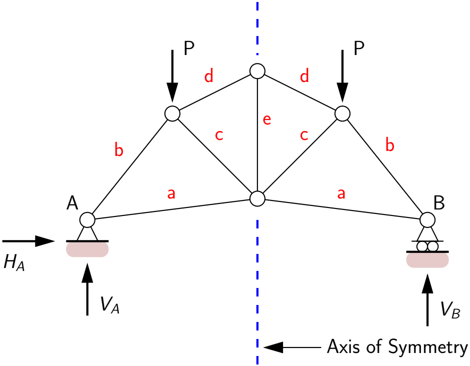
$$\int_{a}^{-a} f(x)g(x)dx = 0. \tag{7}$$

We will use this later in the course to simplify analysis.

Taking Advantage of Symmetry

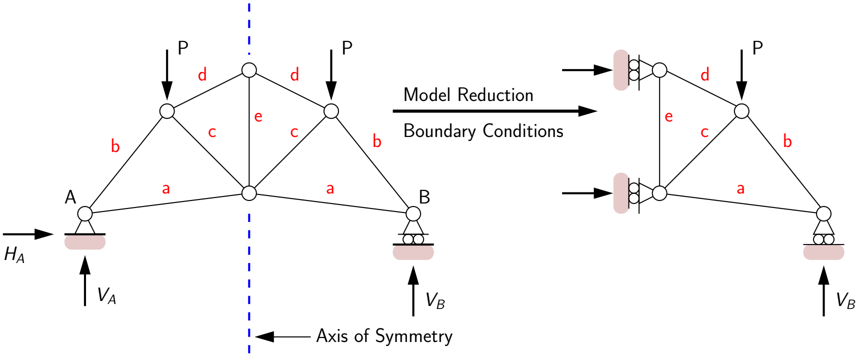
Example 1. Consider the Small Truss Structure:

- Axis of symmetry works for geometry, loading patterns and reactions.
- Only need to compute member forces $a - e$.
- Model reduction requires careful treatment of boundary conditions.



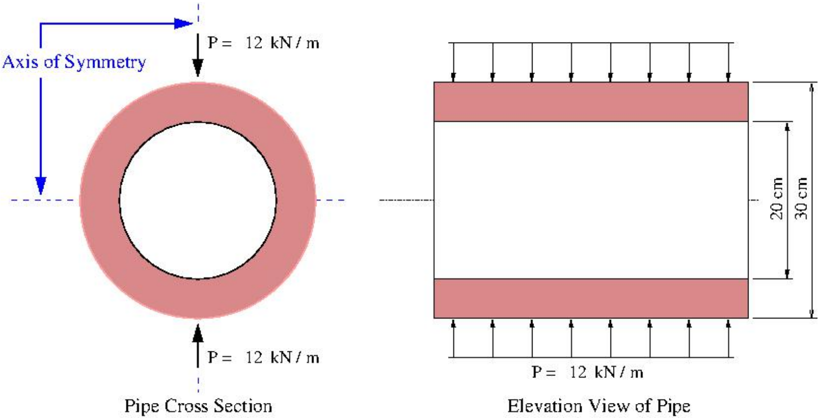
Taking Advantage of Symmetry

Model Reduction: Cut model size in half, then adjust boundary conditions along axis of symmetry.



Taking Advantage of Symmetry

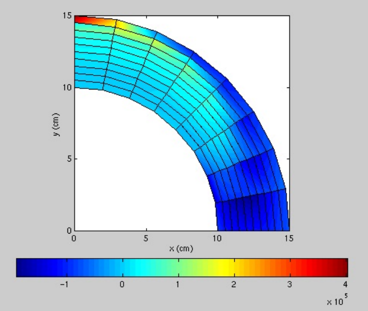
Example 2. Stress Analysis in Cross Section of a Long Pipe



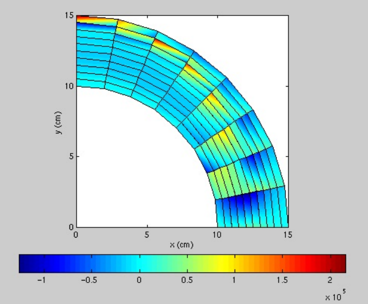
Taking Advantage of Symmetry

Two axes of symmetry for geometry and loading → Only need to analyze 1/4 of the cross section.

Stresses: $\sigma_{yy}(x, y)$



Stresses: $\sigma_{xx}(x, y)$



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- Simo J.C., Vu-Quoc L., On the Dynamics of Flexible Beams Under Large Overall Motions—The Plane Case: Part II, Journal of Applied Mechanics, (53) 4, 855-863, 1986.