(A)

Method of Virtual Forces (Trusses).

Analysis Procedure

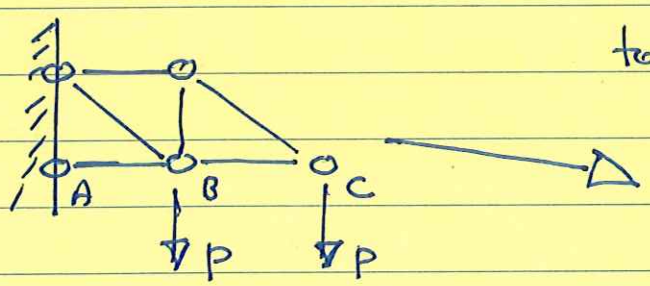
$$\Delta^* = \sum_{i=1}^{N} \left[\frac{F_i \, f_i \, L_i}{A_i \, E_i} \right]$$

such that:

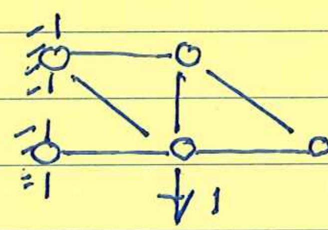
F_i = force in i^{th} element due to loads P.

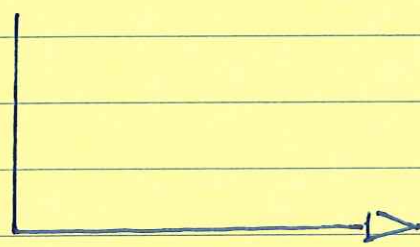
f_i = force in i^{th} element due to unit load applied at point where we want to know deflection.

ie

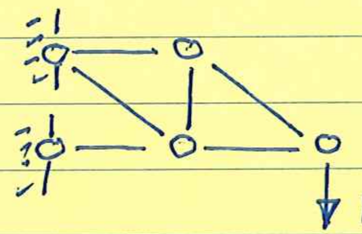


to compute deflection at B





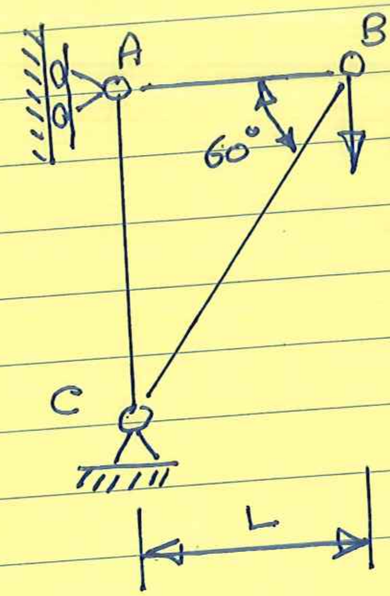
to compute deflection at C



Note: Can compute deflections at multiple locations with spreadsheet-like computation.

(B)

Example 1: Compute vertical & horizontal displacements at B.



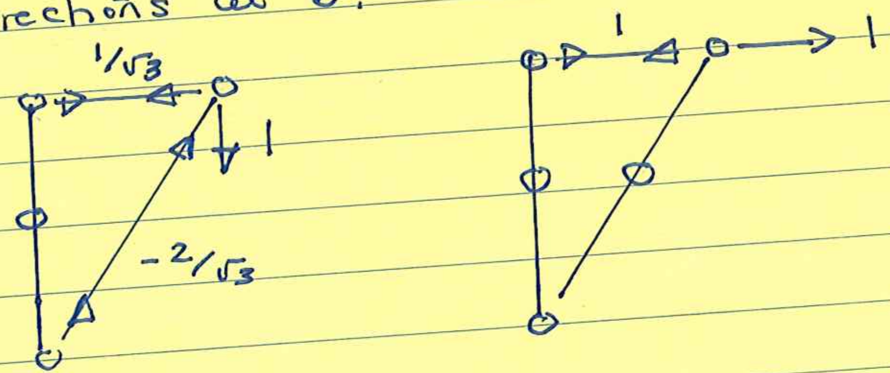
AE = constant in all members.

STEP 1: Compute element forces due to applied loads

$\overline{AB} = P/\sqrt{3}\ (T)$, $\overline{BC} = \frac{-2}{\sqrt{3}} P\ (C)$

$\overline{AC} = 0$.

STEP 2: Apply unit loads in horizontal / vertical directions at B.



STEP 3 Compute Δ.

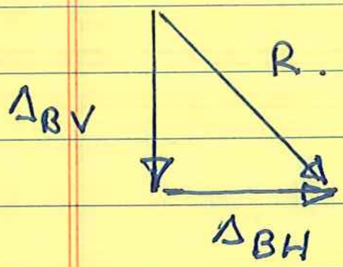
Member	L/AE	F_i	f_1	f_2	$\frac{F_i f_1 L_i}{AE}$	$\frac{F_i f_2 L_i}{AE}$
A – B	L/AE	$P/\sqrt{3}$	$1/\sqrt{3}$	1	$PL/3AE$	$PL/\sqrt{3}$
B – C	$2L/AE$	$-2P/\sqrt{3}$	$-2/\sqrt{3}$	0	$8PL/3AE$	0
A – C	$\sqrt{3}L/AE$	0	0	0	0	0
					$\frac{3PL}{AE}$	$\frac{PL}{\sqrt{3}A}$

Vertical deflection at B = $\frac{3PL}{AE}$

Horizontal deflection at B = $\frac{PL}{\sqrt{3}AE}$.

(c)

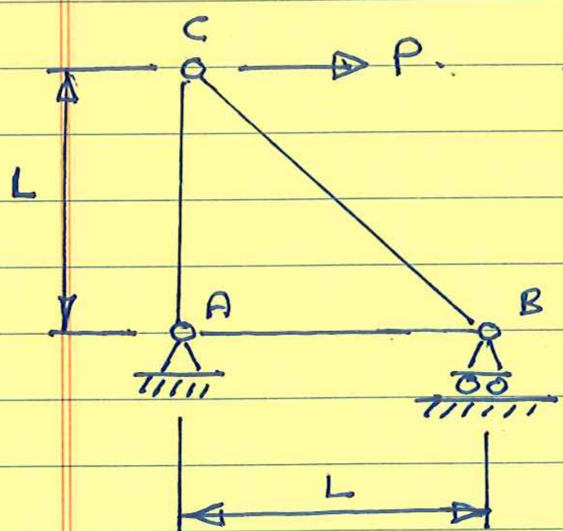
Resultant deflection at B



$$\text{Resultant deflection} = \left[\Delta_{BV}^2 + \Delta_{BH}^2\right]^{1/2}$$

$$= \frac{3.06\,PL}{AE}.$$

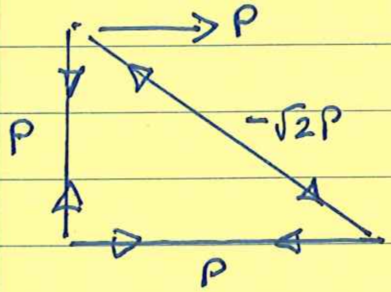
Example 2 (Final Exam 2017).



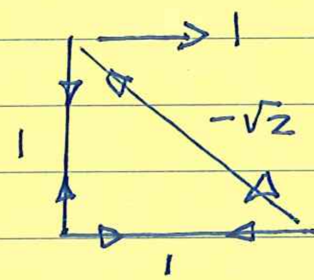
Question: Show that the horizontal deflection at C is

$$\Delta = \frac{2PL}{AE}(1 + \sqrt{2}).$$

Member forces due to P



Apply unit load in horizontal direction



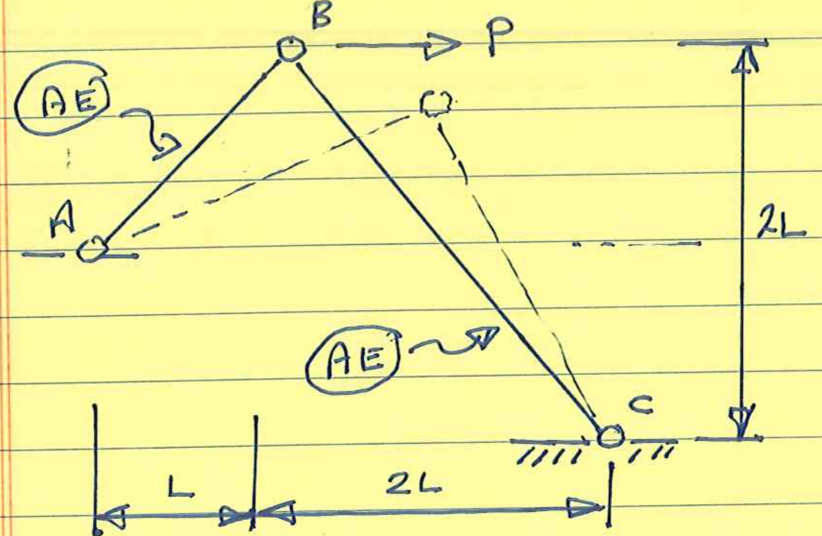
Compute deflection

$$\Delta = \sum_{i=1}^{3}\left(\frac{f_i F_i L}{AE}\right) = \frac{1}{AE}\Big[\underbrace{\sqrt{2}P\sqrt{2}\sqrt{2}L}_{BC} + \underbrace{P.1.L}_{AC} + \underbrace{P.1.L}_{AB}\Big]$$

$$= \frac{2PL}{AE}\left[1 + \sqrt{2}\right].$$

D

Example 3: (Midterm II, 2015).

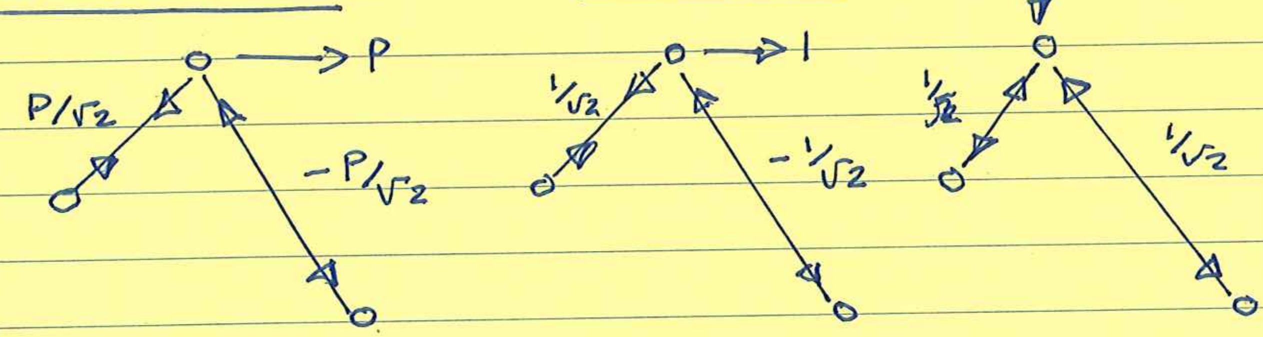


Question: Show that the horizontal displacement is:

$$\Delta_x = \frac{3}{\sqrt{2}} \frac{PL}{AE}.$$

Member Forces

Unit loads



Simple Table

Member	F	f	L/AE	$\frac{FfL}{AE}$
A − B	$P/\sqrt{2}$	$1/\sqrt{2}$	$\frac{\sqrt{2}L}{AE}$	$PL/\sqrt{2}AE$
B − C	$-P/\sqrt{2}$	$-1/\sqrt{2}$	$\frac{2\sqrt{2}L}{AE}$	$\sqrt{2}PL/AE$

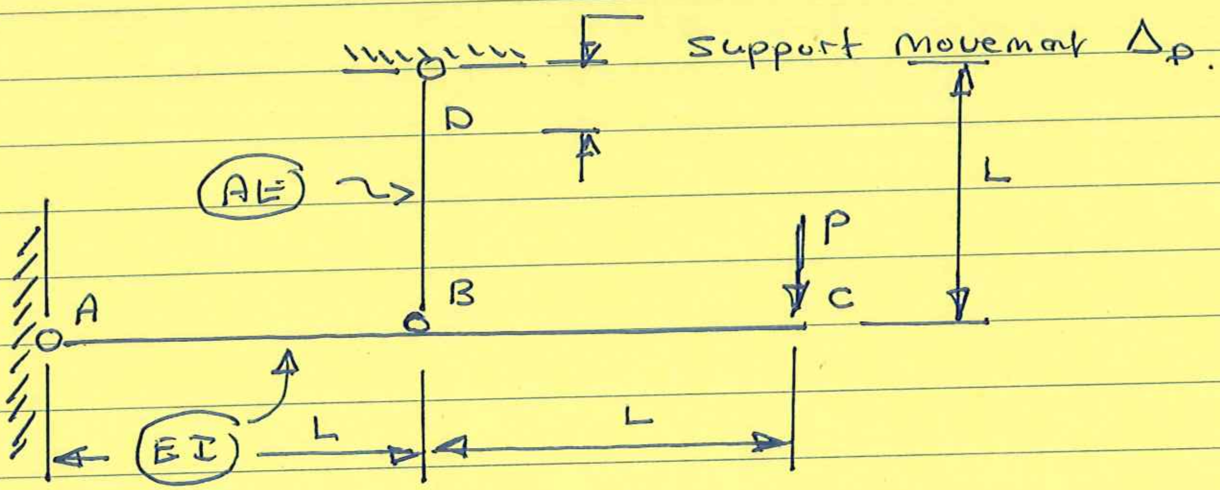
$$\Delta_x = \frac{PL}{\sqrt{2}AE} + \sqrt{2}\frac{PL}{AE} = \frac{3}{\sqrt{2}}\frac{PL}{AE} \checkmark$$

Can also show $\Delta_y = \frac{-1}{\sqrt{2}}\frac{PL}{AE}$. Total deflection $= (\Delta_x^2 + \Delta_y^2$

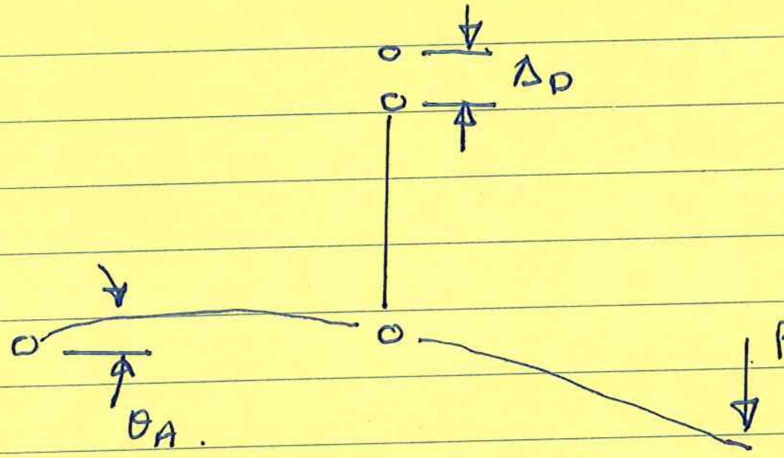
$= \sqrt{5}PL/AE$.

E

Example 4: In this problem, the applied loads cause both flexure (bending) & axial extension of the element.



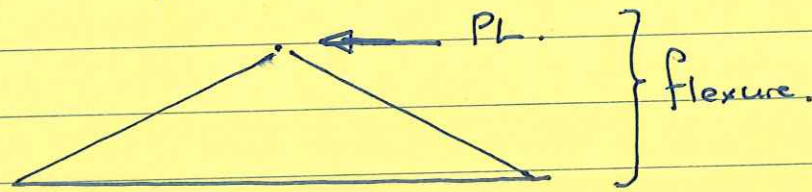
Qualitative estimate of deflection.



Question: What is θ_A?

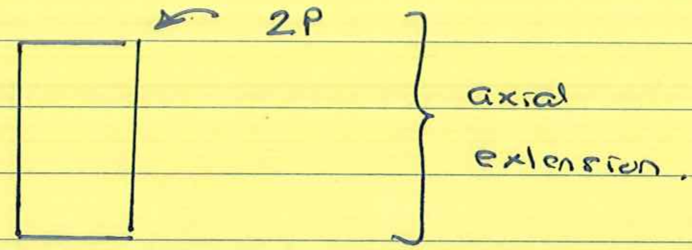
Compute internal forces / bending moments.

Member A - B - C.

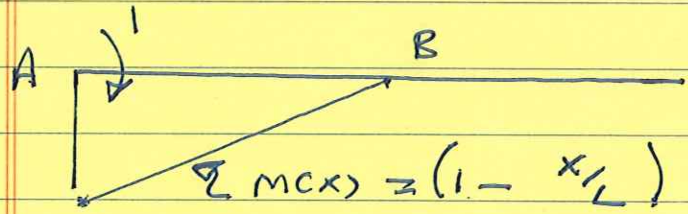


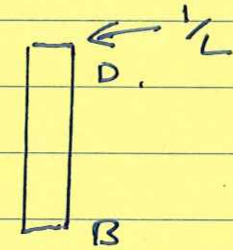
(F)

Member B - D



To compute θ_A, apply a unit moment at A.





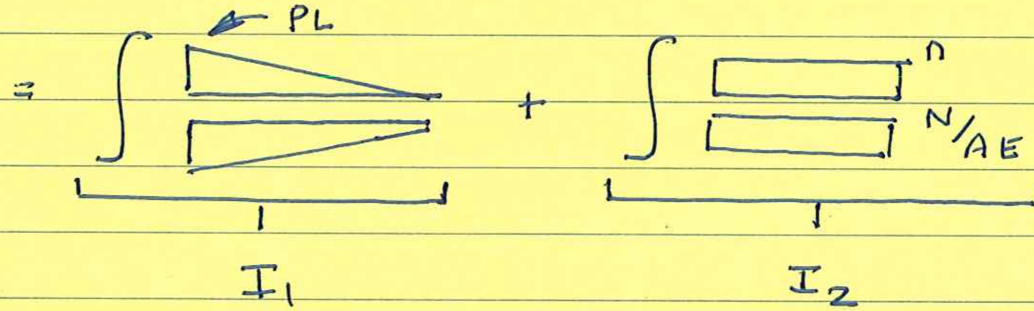
<u>Energy Balance.</u>

$$W_{Ext} = \left[\theta_A - \frac{\Delta_D}{L} \right]$$

rotation at A

rigid body rotation due to support movement.

$$W_{int} = \underbrace{\sum_{i=1}^{N} \int m \frac{M}{EI} dx}_{\text{flexure}} + \underbrace{\sum_{i=1}^{N} \int n \frac{N}{AE}}_{\text{axial extension}}.$$



(G)

$$I_1 = \frac{-1}{EI}\int_0^L \left(1 - x/L\right) Px \, dx = \frac{-PL^2}{6EI}$$

$$I_2 = \frac{1}{AE}\int_0^L \left(\frac{1}{L}\right)(2P)\, dx = \frac{2P}{AE}.$$

$$W_{ext} = W_{int} \Rightarrow \theta_A - \frac{\Delta_D}{L} = \frac{2P}{AE} - \frac{PL^2}{6EI}$$

$$\Rightarrow \theta_A = \frac{\Delta_D}{L} + \frac{2P}{AE} - \frac{PL^2}{6EI}.$$