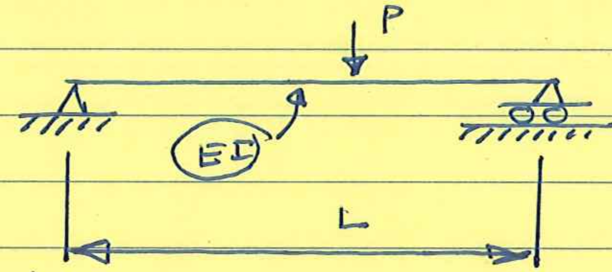
(A)

Method of Virtual Forces

Goal: Use method of virtual forces to compute displacements & rotations.

From Theorem II.



$$\underbrace{\Delta^{*} P^{**}}_{\text{EWD (External Work Done)}} = \underbrace{\int_0^L \phi^{*} M^{**} dx.}_{\text{IWD (Internal Work Done)}}$$

Δ^{*}: real displacement; P^{**}: virtual force; M^{**}: virtual moment; ϕ^{*}: real curvature. $\phi^{*} = \left(\frac{M^{*}}{EI}\right)$

EWD (External Work Done) IWD (Internal Work Done).

If $P^{**} = 1$,

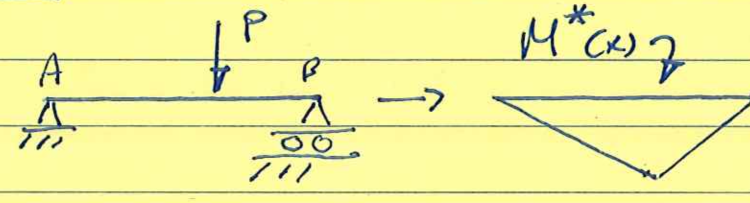
$$\Delta^{*} = \int_0^L \left(\frac{M^{*}}{EI}\right) M^{**} dx \qquad \text{——— (1)}$$

Here: M^{*} = real bending moment.

M^{**} = bending moment due to application of a virtual force.

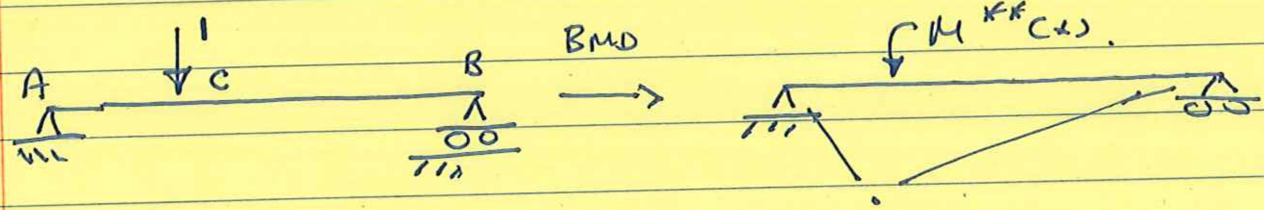
Computational Procedure.

1 Compute BMD due to real loads



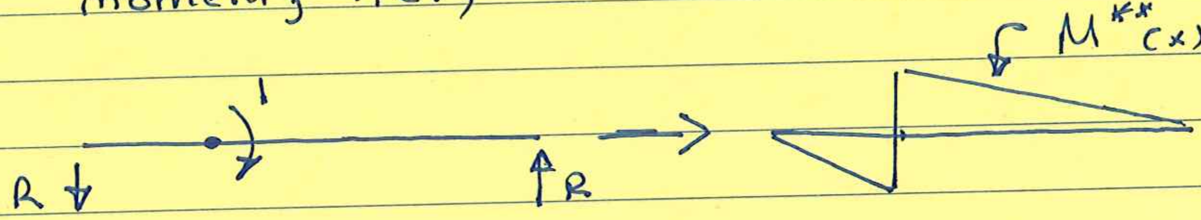
(B)

2. Select a point where we want to know the displacement. Apply a unit force at that point. e.g.,



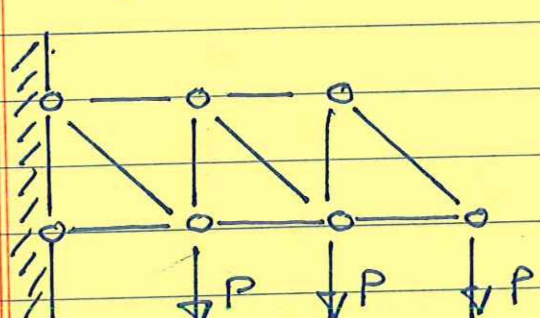
3. Compute
$$\Delta_c^{*} = \int_0^L \left(\frac{M^{*}(x)}{EI}\right) M^{**}(x)\,dx.$$

Note: To find the <u>rotation</u> <u>at C</u> apply a unit moment, i.e.,



<u>Analysis of Trusses</u>.

$$\Delta^{*} P^{**} = \int_0^L \epsilon^{*} N^{**}\,dx$$



$$\xrightarrow[\text{version}]{\text{discrete}} \sum_{n=1}^{N} \epsilon_n^{*} L_n f_n$$

Here: ϵ_N^{*} = strain in truss element = $\{F_n / A_N E_N\}$

L_n = length of element n.

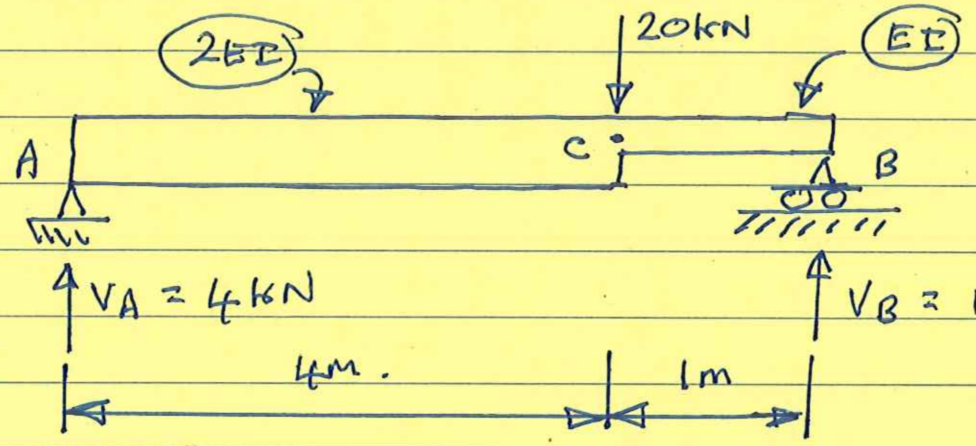
f_n = force in element due to unit load applied at point where we want to know displacement

(c)

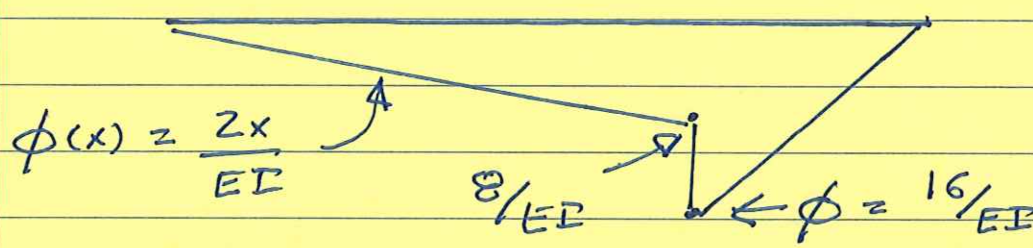
$$\Delta^* = \sum_{i=1}^{N} \left(\frac{F_i f_i L_i}{A_i E_i} \right)$$ ← use spreadsheet for evaluation.

Examples

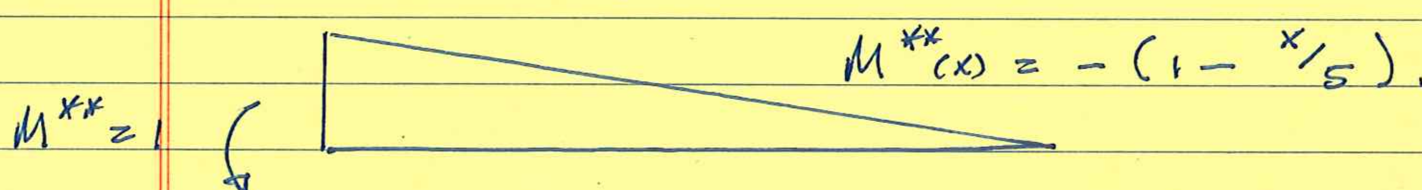
Let's go back to our test problem & compute displacements.



Curvature diagram $\phi^*(x) = \left[\frac{M^*(x)}{EI} \right]$



To get θ_A, apply unit virtual moment at A.



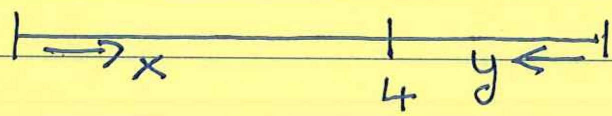
(D)

$$\theta_A = \int_0^5 \underbrace{\frac{M^*(x)}{EI(x)}}_{} M^{**}(x)\, dx.$$

Notice that EI is not constant along beam!.

$$= \int_0^5 \phi^*(x)\, M^{**}\, dx. \longrightarrow (3)$$

Can simplify integration by splitting (3) into two two parts.



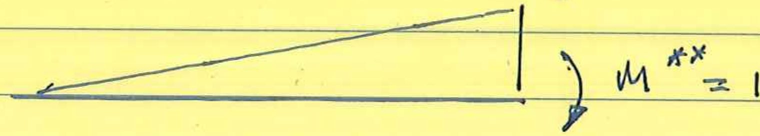
$$\theta_A = -\int_0^4 \underbrace{\left(\frac{2x}{EI}\right)}_{\phi^*(x)} \left(1 - \frac{x}{5}\right) dx + \int_0^1 \underbrace{\left(\frac{16y}{EI}\right)}_{\phi^*(y)} \underbrace{\left(\frac{-y}{5}\right)}_{M^{**}(y)} dy$$

$$= \frac{-1}{EI}\left[\int_0^4 2x\left(1 - x/5\right) dx + \int_0^1 (16y)(y/5)\, dy\right]$$

$$\theta_A = \left(\frac{-8.53}{EI}\right)$$

Note: We assumed anticlockwise rotation at A ($M^{**} = 1$), but answer is negative, indicating real rotation is clockwise. This makes sense!!

(E) To compute θ_B, apply unit moment at B



for $0 \le x \le 4 \quad M(x) = \left(\frac{-x}{5}\right)$

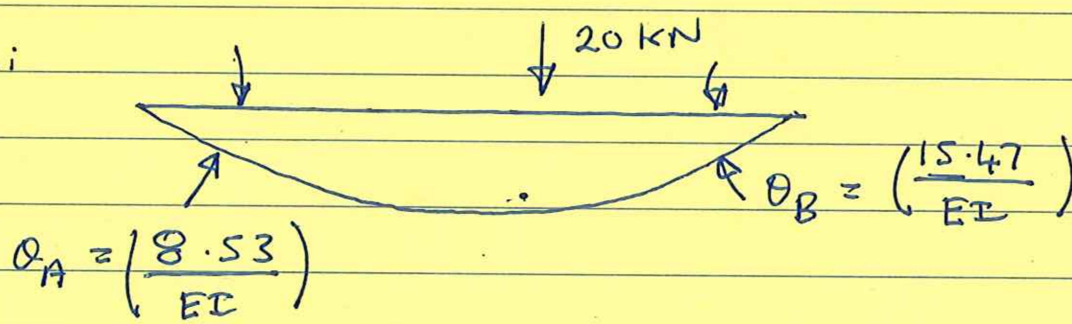
$4 \le x \le 5 \quad M(x) = -\left(1 - x/5\right).$

Hence,

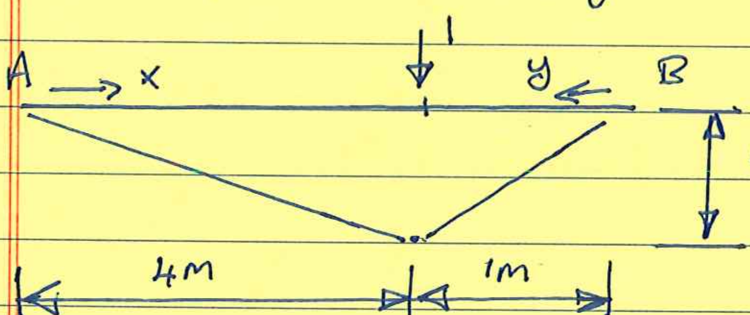
$$\theta_B = \frac{-1}{EI}\left[\int_0^4 2x\left(\frac{x}{5}\right)dx + \int_0^1 \left(\frac{16y}{1}\right)\left(1 - \frac{y}{5}\right)dy\right]$$

$$= -\left(15.47/EI\right)$$

Summary:



To compute Δc, apply unit load at C.



for $0 \le x \le 4$, $M(x) = \frac{x}{5}$

$0 \le y \le 1 \quad M(y) = \frac{4y}{5}.$

$$\Delta_c = \frac{1}{EI}\left[\int_0^4 (2x)\left(\frac{x}{5}\right)dx + \int_0^1 16y \cdot \frac{4y}{5}\,dy\right] = \left[\frac{12.30}{EI}\right]$$