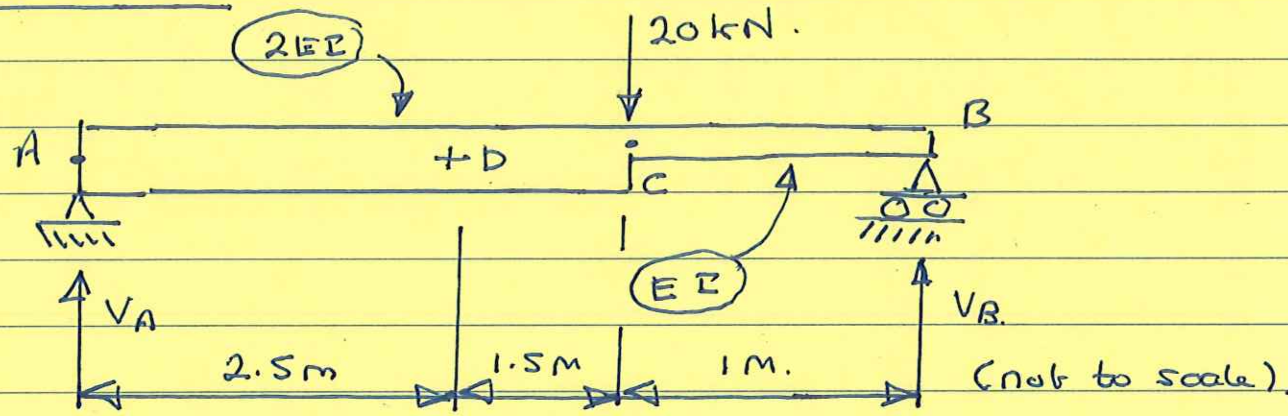
(F)

Method of Virtual Displacements

Goal: Use method of virtual displacements to find forces (e.g., reactions, member forces & moments).

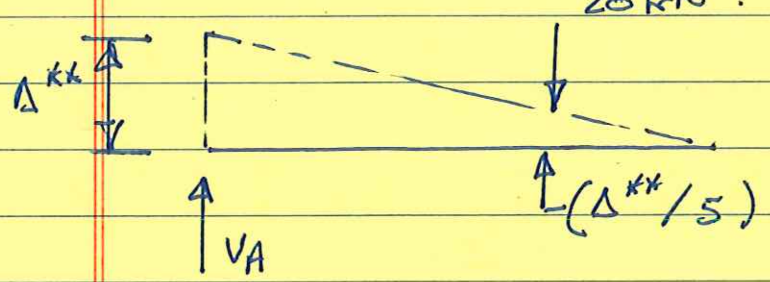
Test Problem.



(not to scale).

① Compute vertical reactions at A & B.

Apply a virtual deflection in the direction of V_A & V_B.



Energy Balance.

- $IWD = EWD$.

Ignore internal displacements

- $\epsilon^{**} = \phi^{**} = 0$.

$IWD = 0$

$EWD = 0 \Rightarrow V_A\,\Delta^{**} + (20kN)\,\frac{\Delta^{**}}{5} = 0$

$\Rightarrow V_A = 4kN$.

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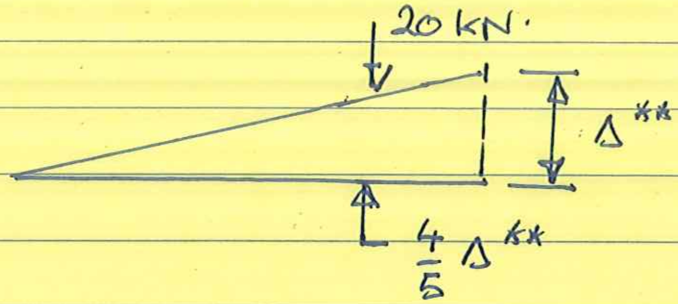
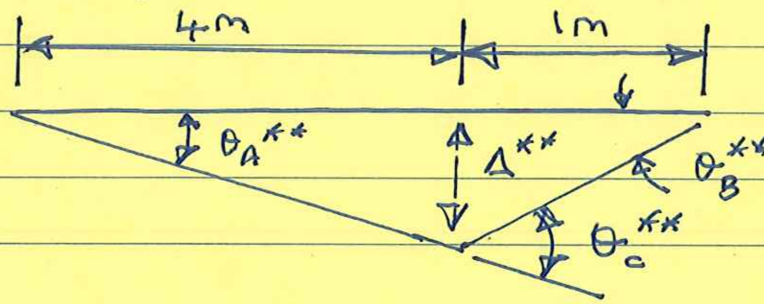


$V_B . \Delta^{**} + -20\left(\frac{4}{5}\Delta^{**}\right) = 0$

$\Rightarrow V_B = 16\text{ kN}.$

(2) Compute bending moment at C.

Apply unit rotation at C.



Energy balance : EWD = IWD.

$$20\,\Delta^{**} = M_c^{*}\,\theta_c^{**} \quad \text{—— (1)}$$

Use geometry to connect Δ^{**} to rotations at A, B & C.

$$\Delta^{**} = 4\,\theta_A^{**} = \theta_B^{**} \quad \text{—— (2)}$$

But $\theta_c^{**} = \theta_A^{**} + \theta_B^{**}$ —— (3)

Plug (2) into (3) $\Rightarrow \theta_c^{**} = \frac{5}{4}\,\Delta^{**}$ —— (4)

Combine (1) & (4) $\Rightarrow 20\left(\frac{4}{5}\,\theta_c^{**}\right) = M_c^{*}\,\theta_c^{**}$

$\Rightarrow M_c^{*} = 16\text{ kNm}.$ (sagging).

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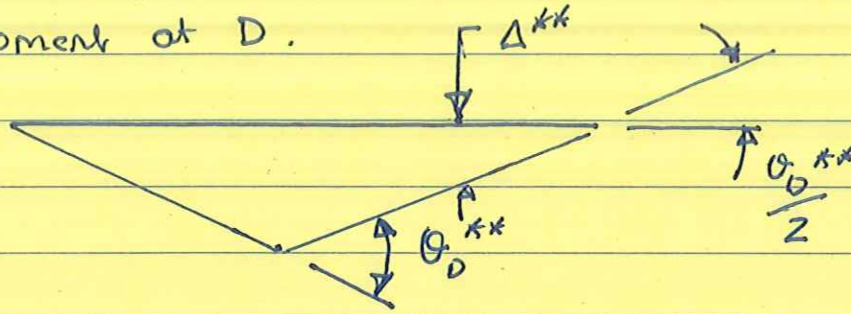
(3) Moment at D.



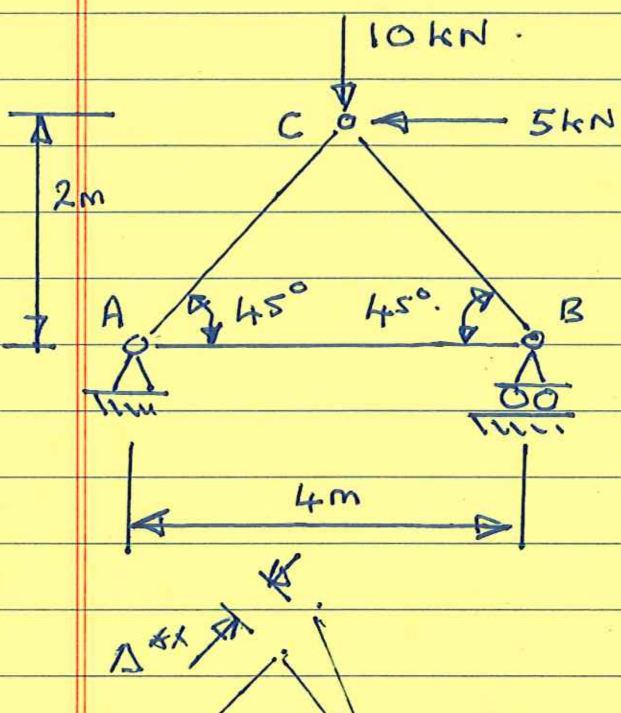
Energy Balance: IWD = EWD.

$$M_D^* \theta_D^{**} = 20 \Delta^{**} \quad \text{——— (5)}$$

From geometry at B: $\Delta^{**} = \left(\frac{\theta_D^{**}}{2}\right)$ ——— (6)

Combine equations (5) & (6): $M_D^* = 10$ kN.m.

(4) Analysis of Truss Structure: Determine internal forces $\overline{CA}$ & $\overline{CB}$ & reaction V_B.



Member force $\overline{CA}$: Impose deflection Δ^{**} in direction of CA.

Energy balance: IWD = EWD.

$$\overline{CA}\, \Delta^{**} = -5\left(\frac{\Delta^{**}}{\sqrt{2}}\right) - 10\left(\frac{\Delta^{**}}{\sqrt{2}}\right)$$

force in member $\overline{CA}$.

$\Rightarrow \overline{CA} = -10.61$ kN (compression).

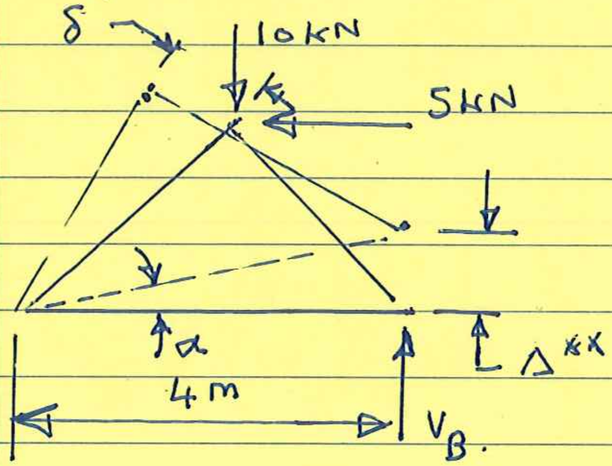
(I)

Similarly, for $\overline{CB}$: IWD = EWD

$$\overline{CB}\ \Delta^{**} = 5\cdot\left(\frac{\Delta^{**}}{\sqrt{2}}\right) - 10\left(\frac{\Delta^{**}}{\sqrt{2}}\right)$$

$$\Rightarrow \overline{CB} = -3.54\ kN \text{ (compression).}$$

To find vertical reaction V_B, impose ~~vertical~~ virtual displacement at B



From geometry. $\alpha = \dfrac{\Delta^{**}}{4}$

At point C, the movement is:

$$\delta = 2\sqrt{2}\,\alpha$$

$$= \frac{\sqrt{2}}{2}\Delta^{**}$$

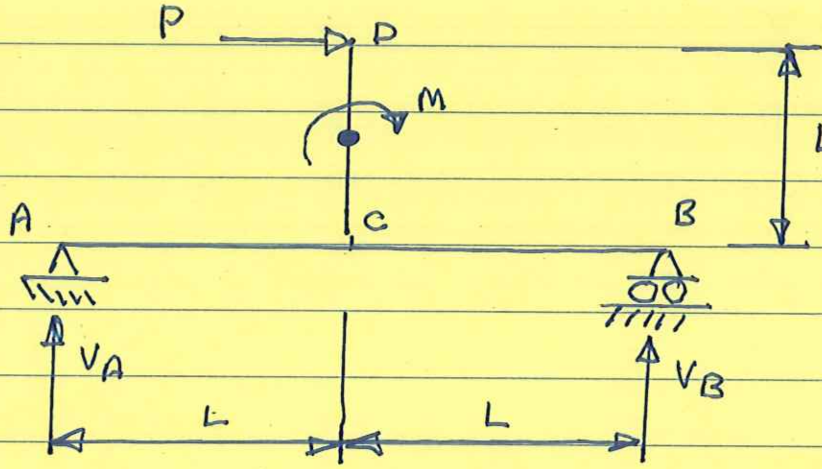
Energy balance : EWD = IWD , IWD = 0

$$\Rightarrow V_b\,\Delta^{**} - \frac{10\,\delta^{**}}{\sqrt{2}} + 5\left(\frac{\delta^{**}}{\sqrt{2}}\right) = 0$$

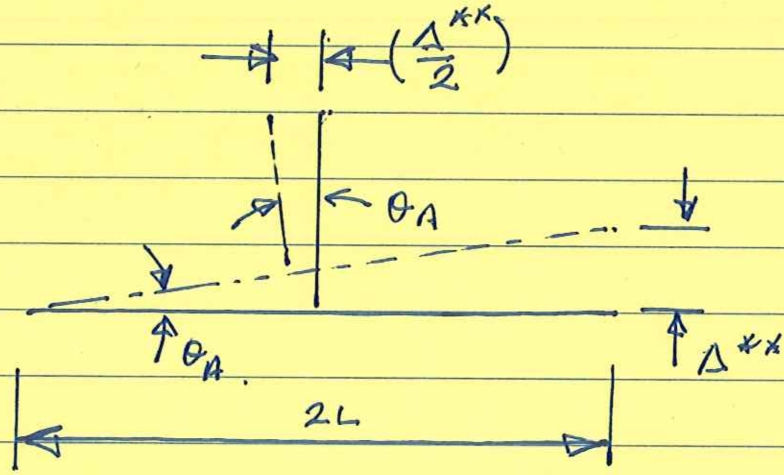
$$\Rightarrow V_b = 2.5\,kN.$$

(J)

Midterm II Exam Question, Use virtual displacements to find V_A & V_B.



Start with V_B



$\sum EWD = 0.$ $\quad V_B \Delta^{**} + P\left(-\frac{\Delta^{**}}{2}\right) + M(-\theta_A) = 0$

$$\Rightarrow V_B \Delta^{**} = \frac{P\Delta^{**}}{2} + M\theta_A. \quad \text{— (1)}$$

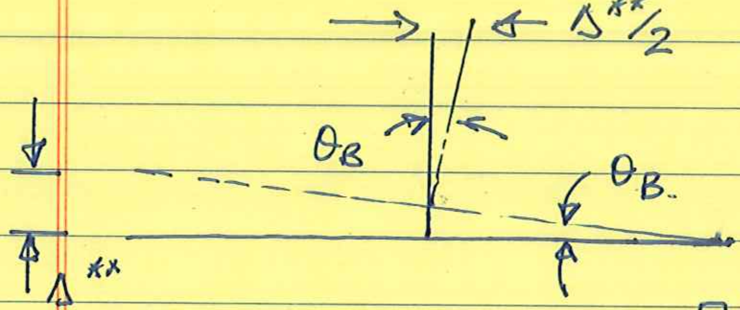
From geometry: $\theta_A = \left(\frac{\Delta^{**}}{2L}\right)$ — (2)

Plug (2) into (1). $V_B \Delta^{**} = P\frac{\Delta^{**}}{2} + M\left(\frac{\Delta^{**}}{2L}\right)$

$$\Rightarrow V_B = \frac{P}{2} + \frac{M}{2L}$$ ← units are dimensionally consistent

(k)

Repeat for V_A.



EWD = 0 $\Rightarrow$ $V_A \Delta^{**} + \frac{P \Delta^{**}}{2} + M\theta_B^{**} = 0$ — (3)

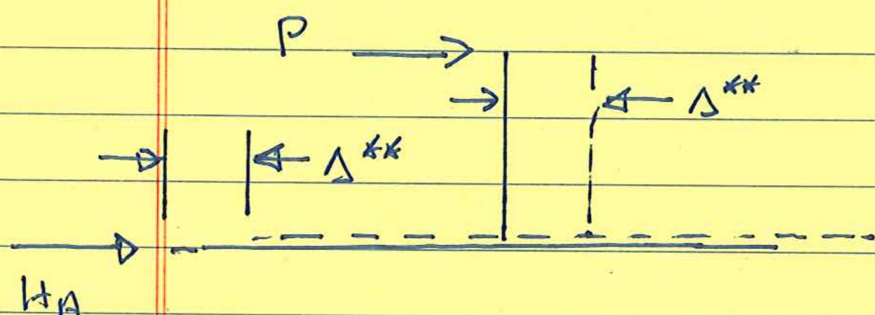
From geometry: $\theta_B^{**} = \left(\frac{\Delta^{**}}{2L}\right)$ — (4)

Plug (4) into (3).

$$V_A = -\left(\frac{P}{2} + \frac{M}{2L}\right) \quad \text{— (5)}$$

Notice: $V_A + V_B = 0$ ✓.

Horizontal Reaction at A? Impose virtual horizontal displacement to find H_A.



EWD = 0

$\Rightarrow P\Delta^{**} + H_A \Delta^{**} = 0$

$\Rightarrow H_A = -P$. — (6)