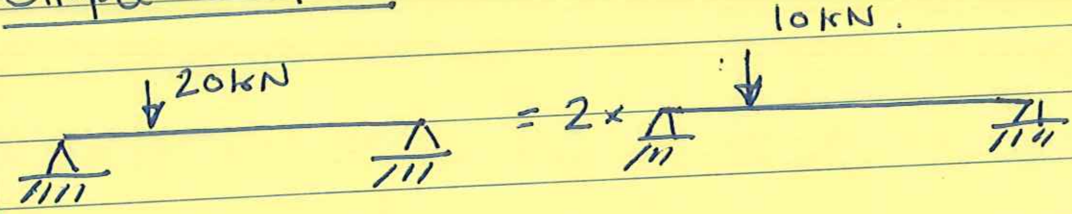
(A)

Superposition Examples

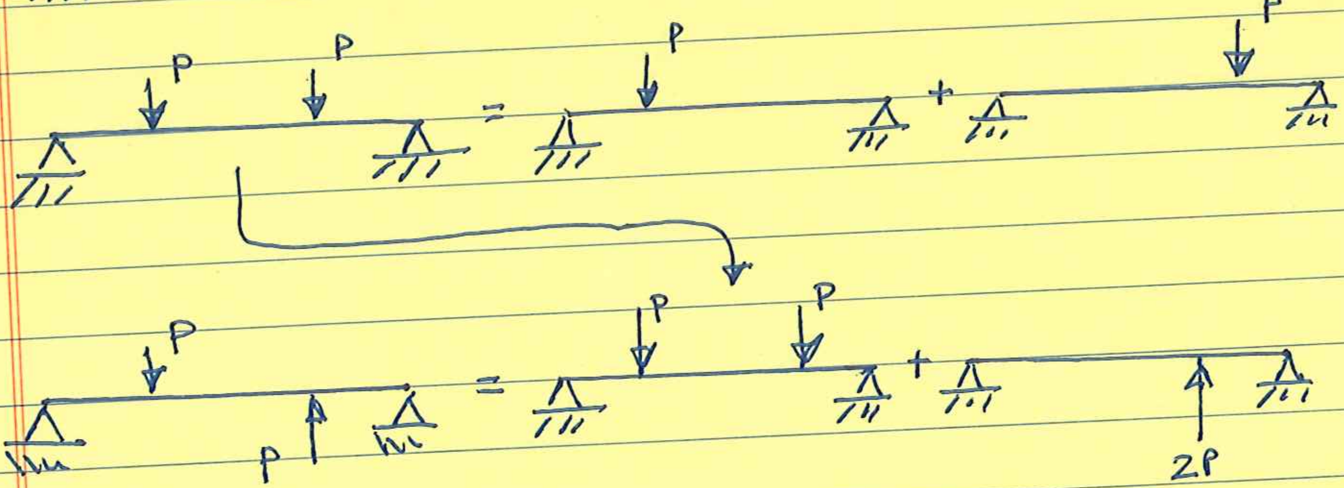
Simple Examples.

1.



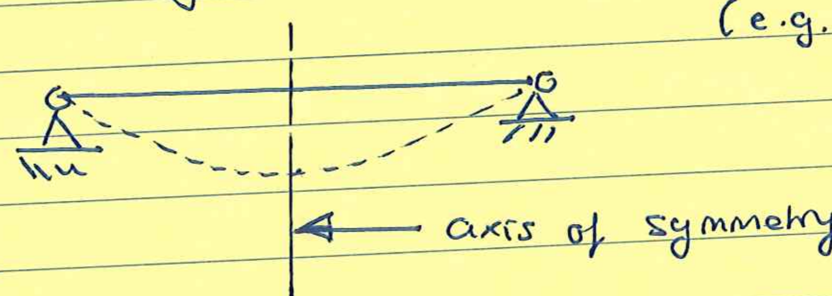
2.

3

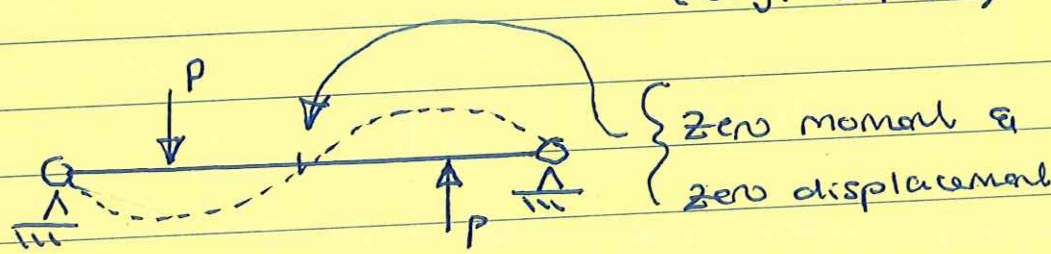


Also note:

1. Problem 2 is symmetric: $f(x) = f(-x)$
(e.g. $\cos(x)$).



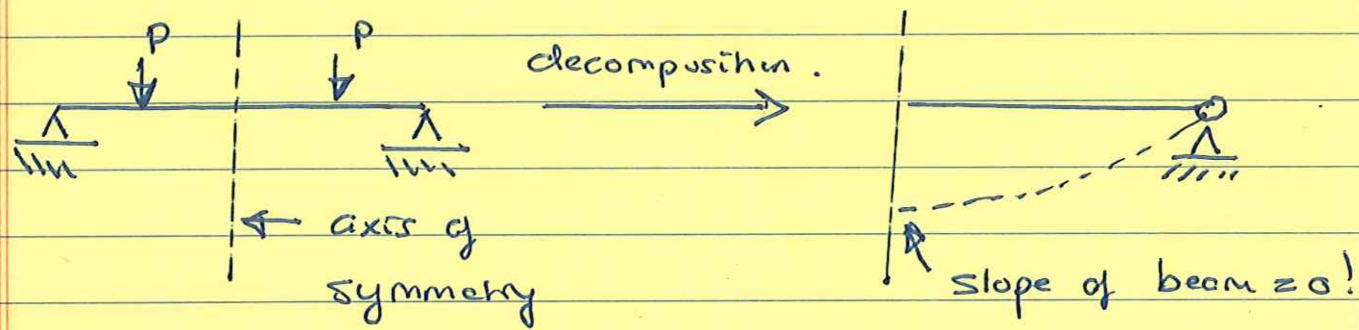
2. Problem 3 is skew-symmetric $f(x) = -f(-x)$.
(e.g. $\sin(x)$)



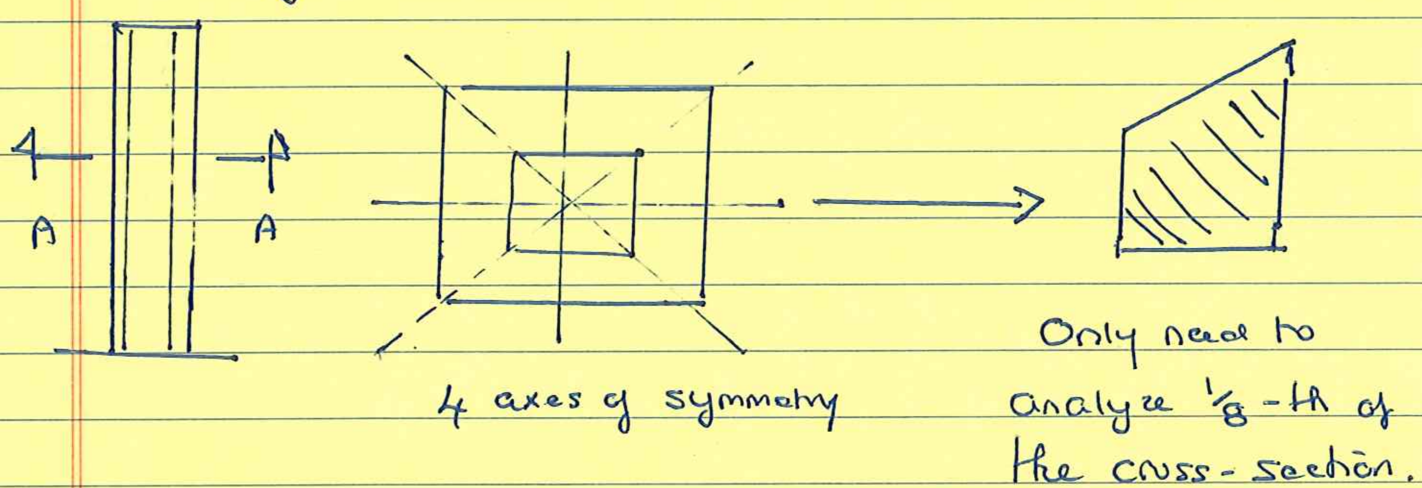
(B)

3. Symmetries nearly always allow for the simplification of a problem.

- In problem 2, only need to analyze 1/2 of the structure, ie.



- Thermal analysis of a chimney cross-section.

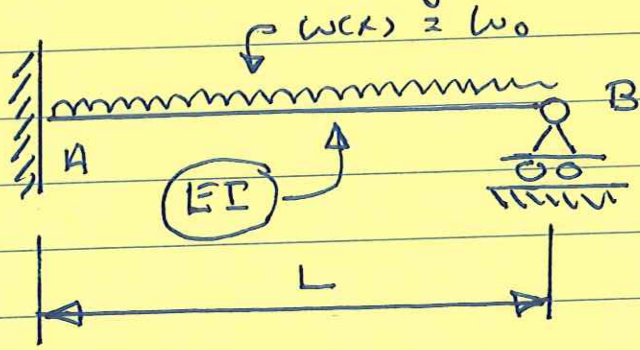


(c)

Solving Indeterminate Structures.

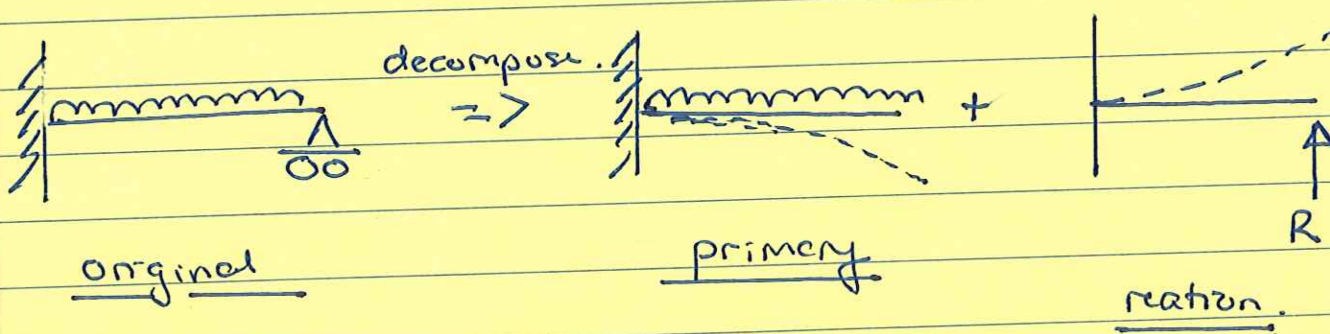
The principle of superposition can be used with "compatibility of displacements" to find reactions & bending moments in indeterminate structures.

Simple Example : Propped Cantilever carrying a uniformly distributed load.

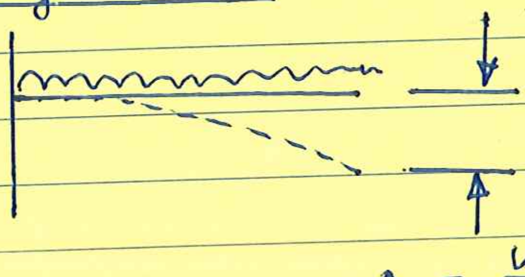


Statically indeterminate: $i = 1$.

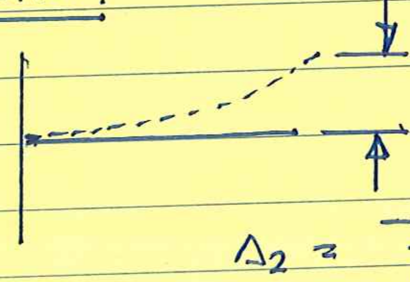
Let's decompose original problem into sum of two simple problems + displacement constraint.



Primary structure.



Reaction.



$$\Delta_1 = \frac{WL^4}{8EI}$$

$$\Delta_2 = \frac{-RL^3}{3EI}.$$

(D)

<u>Compatibility of Displacements.</u>

In the original problem, $\Delta_B = 0$

$$\Rightarrow \underbrace{\Delta_1 + \Delta_2}_{\text{compatibility of displacements.}} = \Delta_B = 0$$

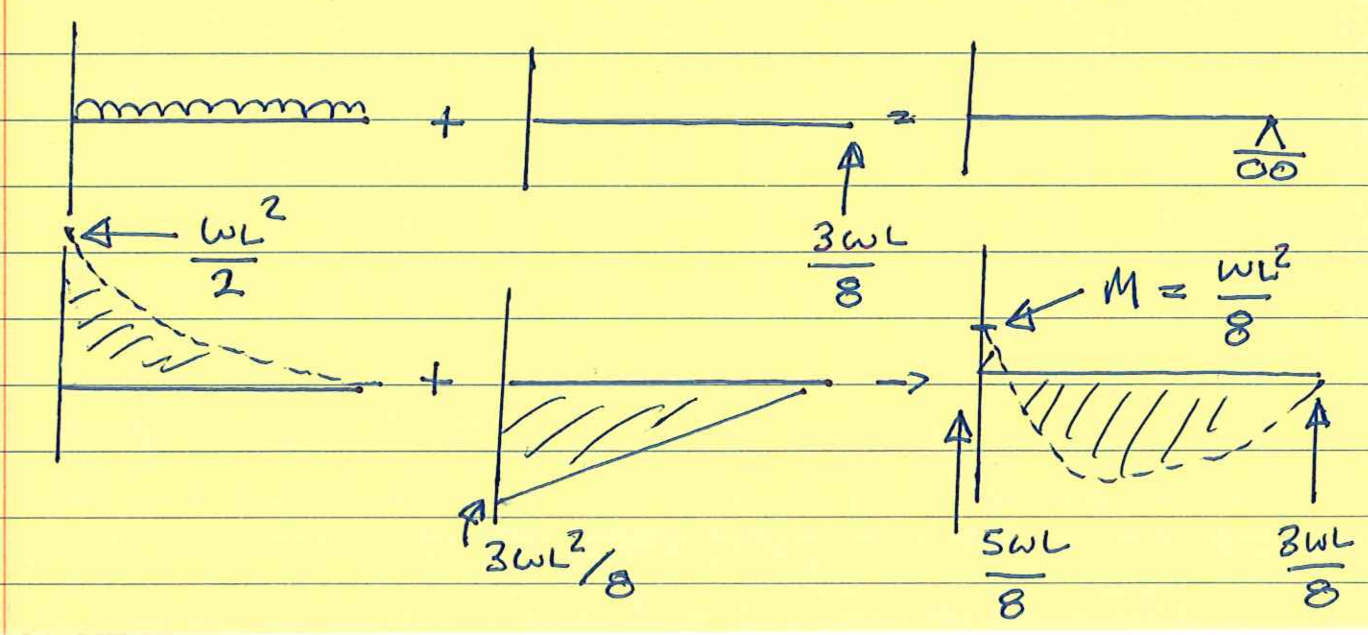
$$\Rightarrow \frac{wL^4}{8EI} + -\frac{RL^3}{3EI} = 0$$

$$\Rightarrow R = \frac{3wL}{8}.$$

Thus, in the original problem $V_B = \dfrac{3wL}{8}.$

$$\Rightarrow V_A = \frac{5wL}{8}.$$

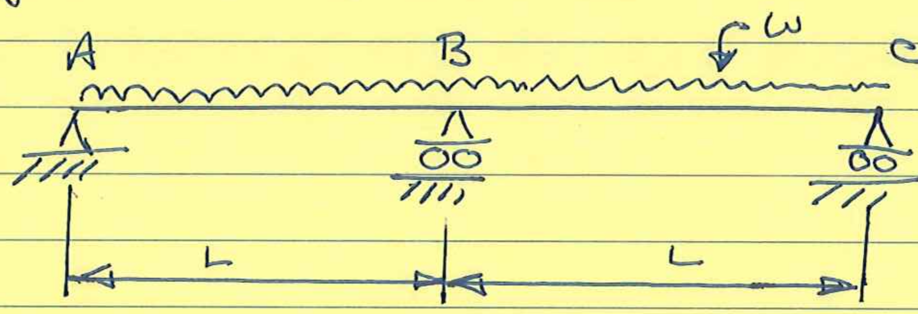
<u>Bending Moment Diagram</u> – linearity means we can <u>add</u> bending moment components!



E

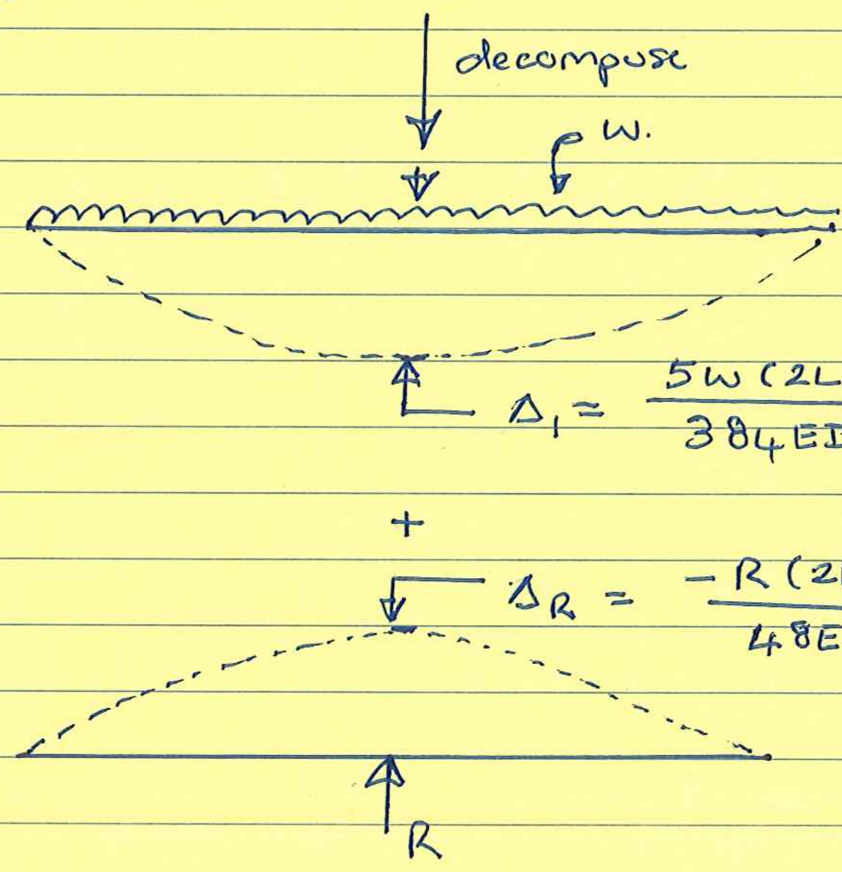
Example 2: Two-span Beam.

Consider a two-span beam with two equal spans carrying a uniformly distributed load.



This structure is statically indeterminate: $\hat{i} = 1$.

We can decompose the original problem into two simple (statically determinate!) problems, and then apply compatibility of displacements.



$$\Delta_1 = \frac{5w(2L)^4}{384EI}.$$

$$\Delta_R = \frac{-R(2L)^4}{48EI}$$

(F)

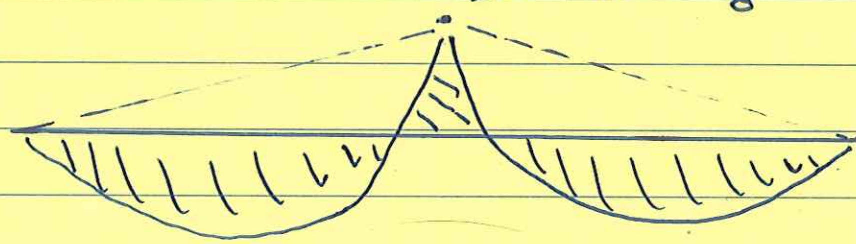
Apply compatibility of displacements:

$$\Delta_B = \Delta_1 + \Delta_R = 0$$

$$\Rightarrow R = \frac{10wL}{8}.$$

<u>Bending Moment Diagram.</u>

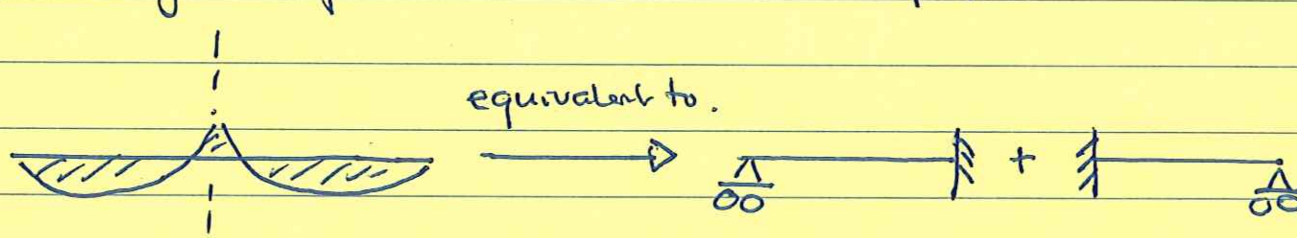
$M = \frac{WL^2}{8}$



<u>Note:</u> $R = \frac{10wL}{8}$. Total load $= 2wl \Rightarrow$

$V_A = V_C = 3WL/8$.

Also, the problem is symmetric about point B. Thus, the original problem can also be simplified:



← axis of symmetry.

beam rotation at B = 0