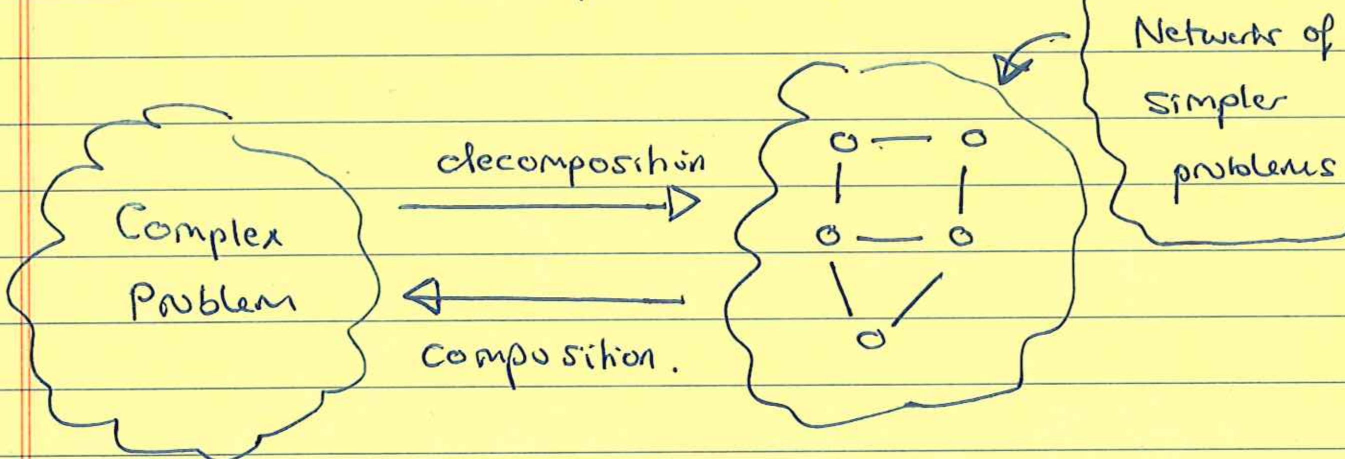
(A) **Principle of Superposition.**

Objective: Find a way to solve "complex problems" by decomposing them into simpler problems, and then composing the simple solutions into a solution for the original (hard!) problem.

We can visualize this process as follows:



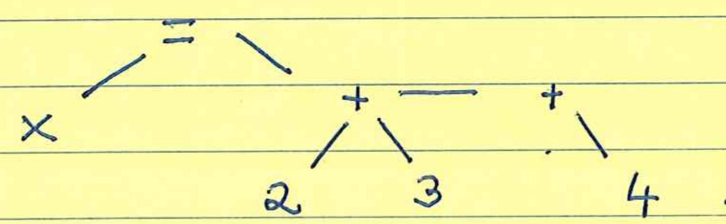
Simple Example: Evaluation of Arithmetic Expressions.

assignment. arithmetic operator

$$x = 2 + 3 + 4;$$

Variable number number

Convert original problem into a tree.



Traverse tree & evaluate result $x = 9$.

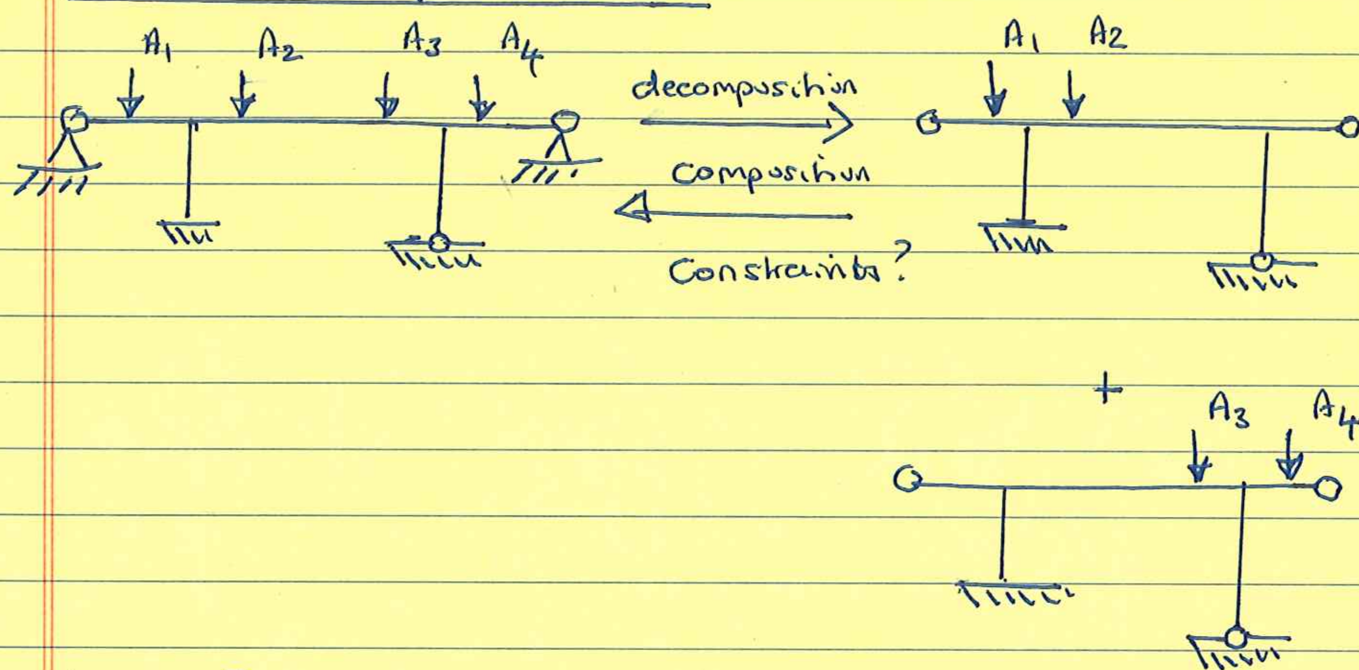
What about constraints on evaluation?

Consider: $y = 1 + \left[\dfrac{2}{x-1} \right]$ ← only works when $x \neq 1$.

Operators for composition of arithmetic expressions:

$$+, \; -, \; \times, \; /$$

Structural Analysis Example.



Basic Questions:

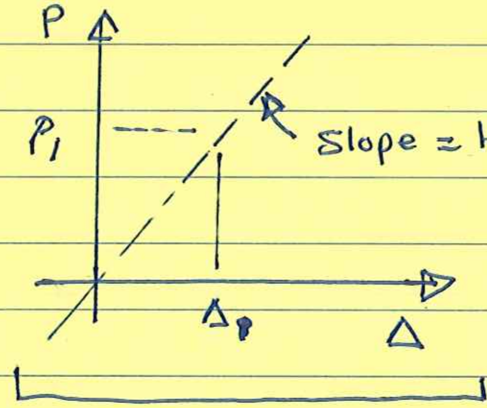
1. What are the shear forces, bending moments & displacements?
2. What conditions need to exist in order for the decomposition & composition to work?

(c)

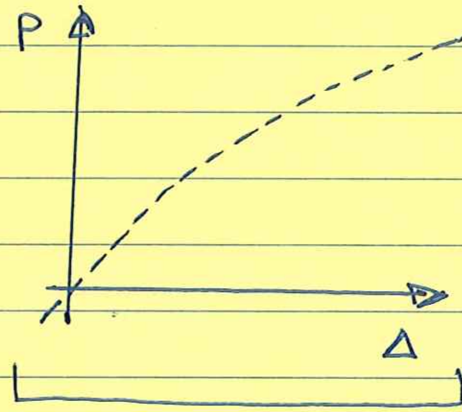
Principle of Superposition.

Def'n: For a linearly elastic structure the effects caused by two or more loadings are the sum of the load effects ~~by~~ caused by each loading separately.

Note:



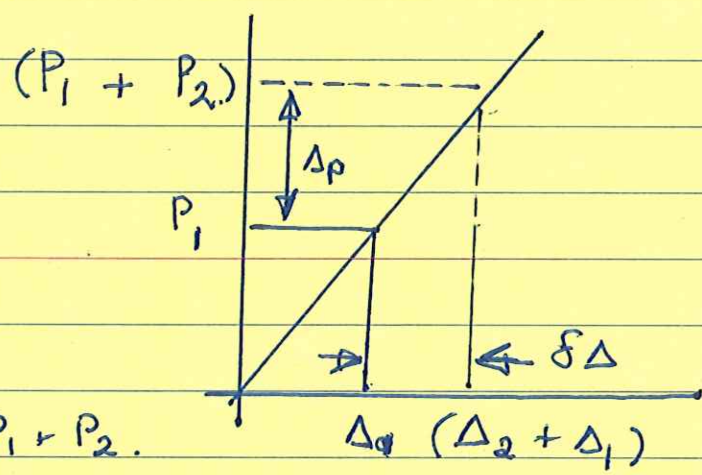
Linear Elastic.



Non-linear elastic.

For a linear elastic structure; structural loads (P) & displacements (Δ) are related through the stiffness (k)

$$P = k\Delta.$$



For load P_1 :

$$P_1 = k\Delta_1$$

For load P_2 :

$$P_2 = k\Delta_2.$$

For the combined load. $P_1 + P_2$.

$$P_1 + P_2 = k(\Delta_1 + \Delta_2) = (k\Delta_1) + (k\Delta_2)$$

(D)

In other words, the total displacement due to combined loads $P_1 + P_2$ is equal to Δ_1 (due to P_1) + Δ_2 (due to P_2).

This works because the relationship between applied loads & displacements is <u>linear</u>

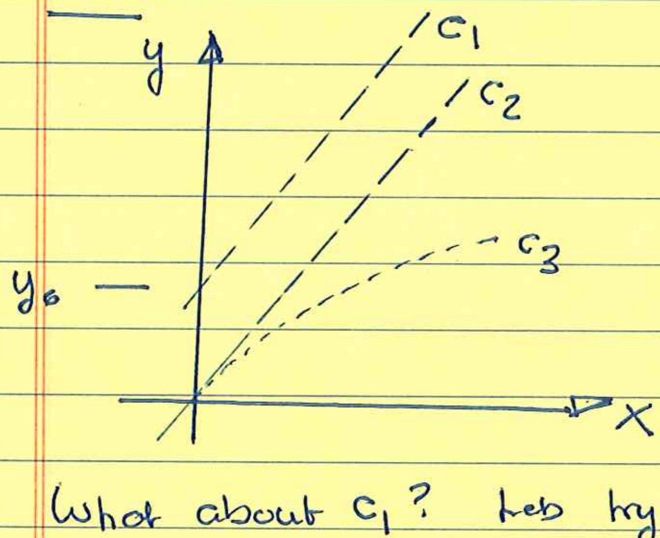
For structures, superposition applies:

1. small displacements (assumption)
2. material behavior in the linear range.

These assumption <u>do not apply</u> to non-linear elastic behavior (e.g. cable structures).

<u>Aside:</u> What does math say? A function is linear if:

- $f(kx) = k f(x)$
- $f(x+y) = f(x) + f(y)$



Among the three curves, only c_2 is linear.

What about c_1? Lets try $f(x_0 + x_1) = f(x_1) + f(x_0)$

$$f(x_0) = y_0 + k x_0 \quad \overset{?}{\longleftrightarrow} \quad f(x_1 + x_0) = y_0 + k(x_0 + x_1)$$
$$f(x_1) = y_0 + k x_1$$

Only works when $y_0 = 0$!