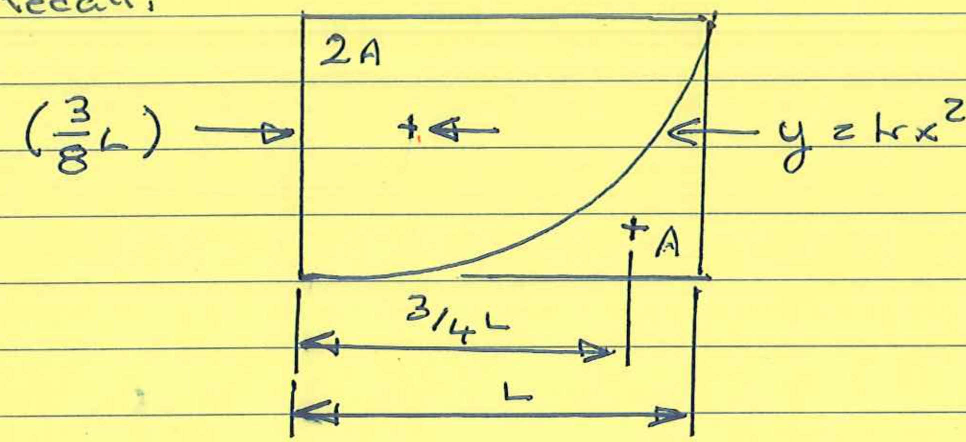
(1A)

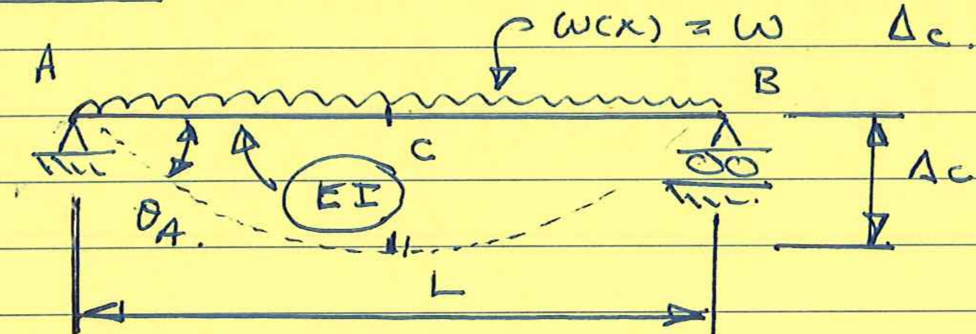
Moment-Area Examples

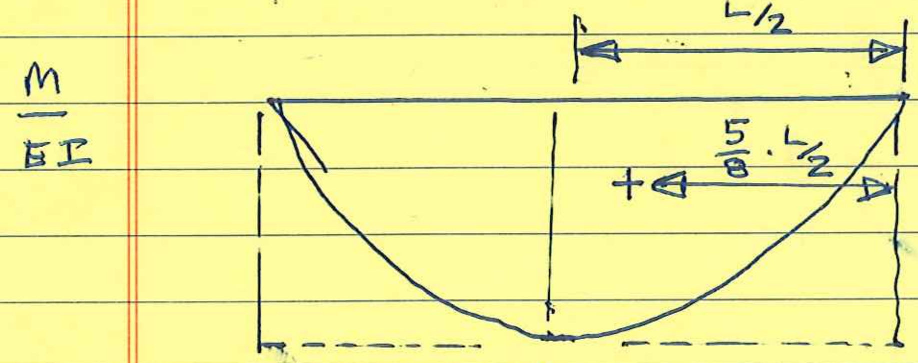
Recall:



Example 1.

What is θ_A & Δ_c.



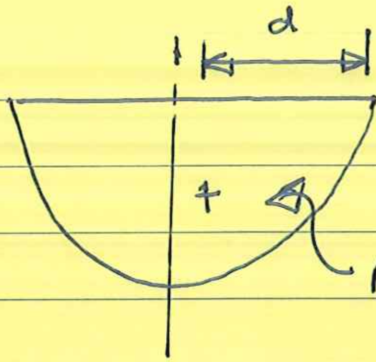


$M_c = \frac{wL^2}{8}$

$$\left(\frac{M}{EI}\right)_{Max} = \left(\frac{wL^2}{8EI}\right)$$

Δ_c = distance of B from the tangent at C

(B)



$$d = \frac{5}{8} \cdot \frac{L}{2}.$$

$$\text{Area}_1 = \frac{2}{3}\left(\frac{wL^2}{8EI}\right)\cdot\left(\frac{L}{2}\right) = \frac{wL^3}{24EI}$$

$$\Delta_c = \text{Area}_1 \times d = \left[\frac{wL^3}{24EI}\right]\cdot\left(\frac{5}{8}\frac{L}{2}\right)$$

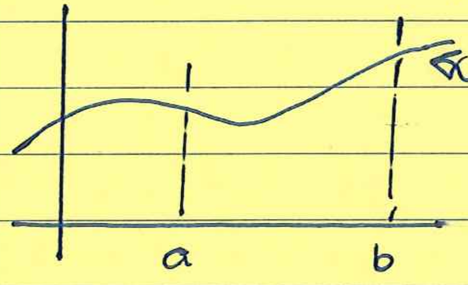
$$= \frac{5wL^4}{384EI}$$

Also note, problem is symmetric $\Rightarrow \theta_c = 0$.

$\theta_A - \theta_c$ = area under quadratic curve.

$$= \frac{wL^3}{24EI}.$$

<u>Aside:</u> Def'n of first moment of area



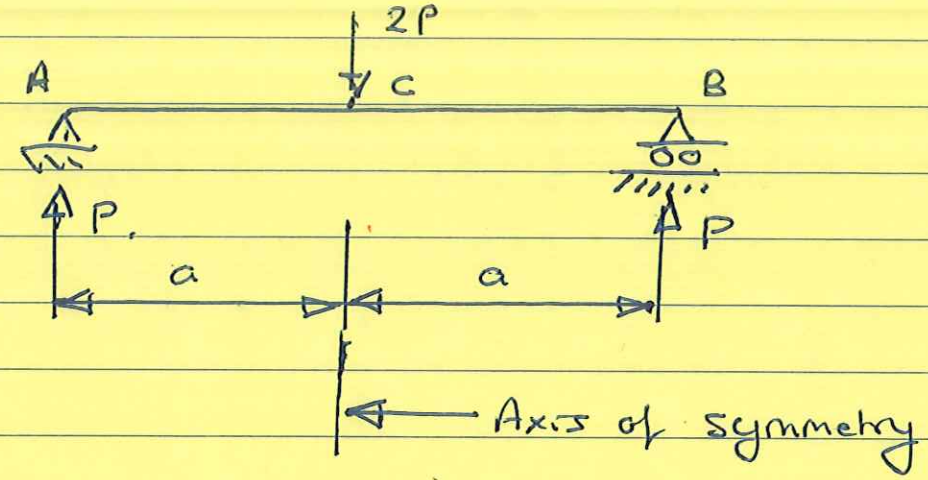
$y = f(x)$ Area $= \int_a^b f(x)\,dx.$

First moment of Area $= \int_a^b x f(x)\,dx = \text{Area} \times \bar{x}$

distance to centroid

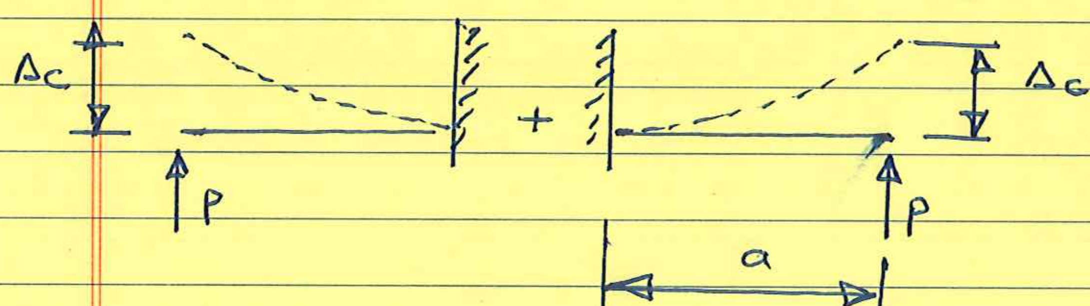
C

Example 2.



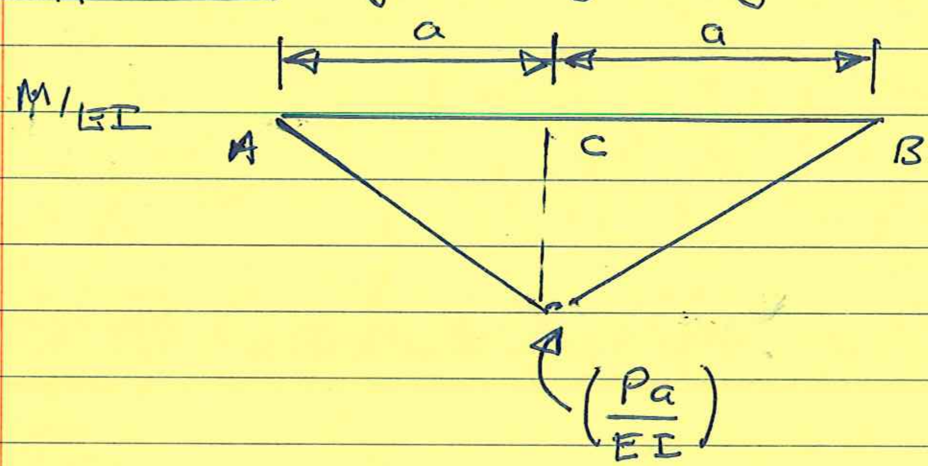
Approach 1: Take advantage of symmetry.

This problem is equivalent to two cantilever beams back-to-back. Rotation at C = 0.



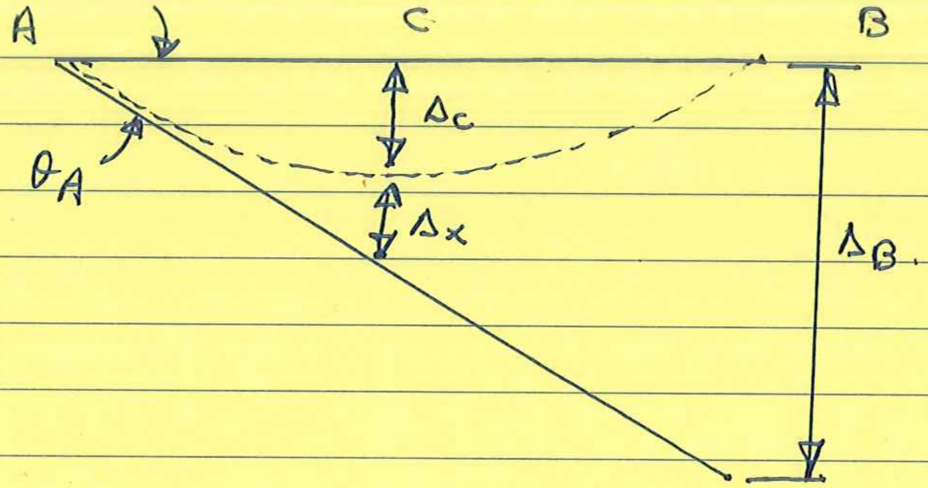
$$\Delta_C = \frac{Pa^3}{3EI}. \longleftarrow \text{Easy !!}$$

Approach 2: Ignore symmetry.

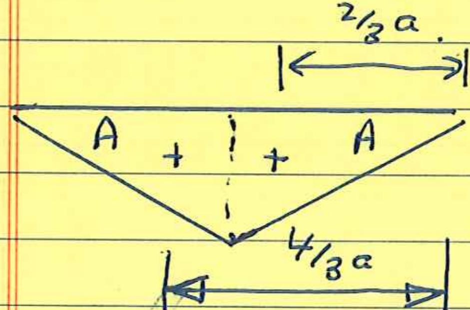


(D)

Deflected Shape.



Δ_B = first moment of area between A & B evaluated about B.



$$\text{Area } A = \frac{1}{2}\,\frac{Pa}{EI}\cdot\frac{a}{2} = \frac{Pa^2}{2EI}.$$

$$\Delta_B = \left(\frac{2}{3}a\right)A + \left(\frac{4}{3}a\right)A$$

$$= 2aA = \frac{Pa^3}{EI}.$$

Also,

$$\Delta_x = A\cdot\left(\frac{a}{3}\right) = \frac{Pa^3}{6EI}.$$

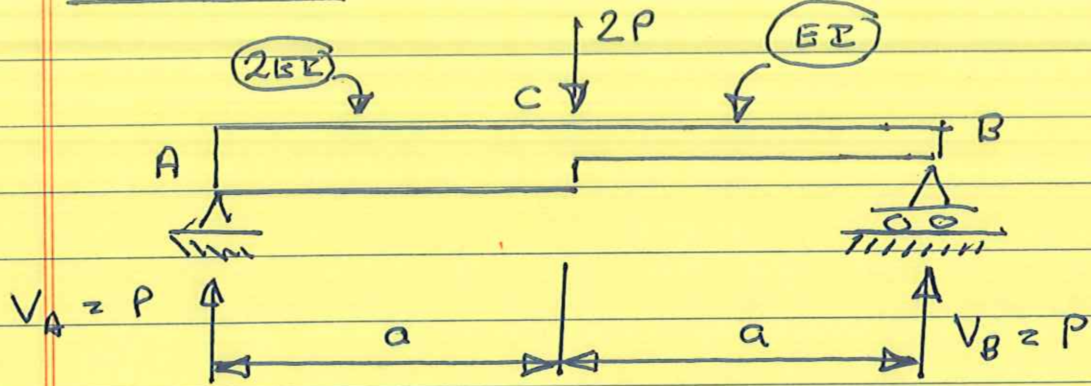
From geometry

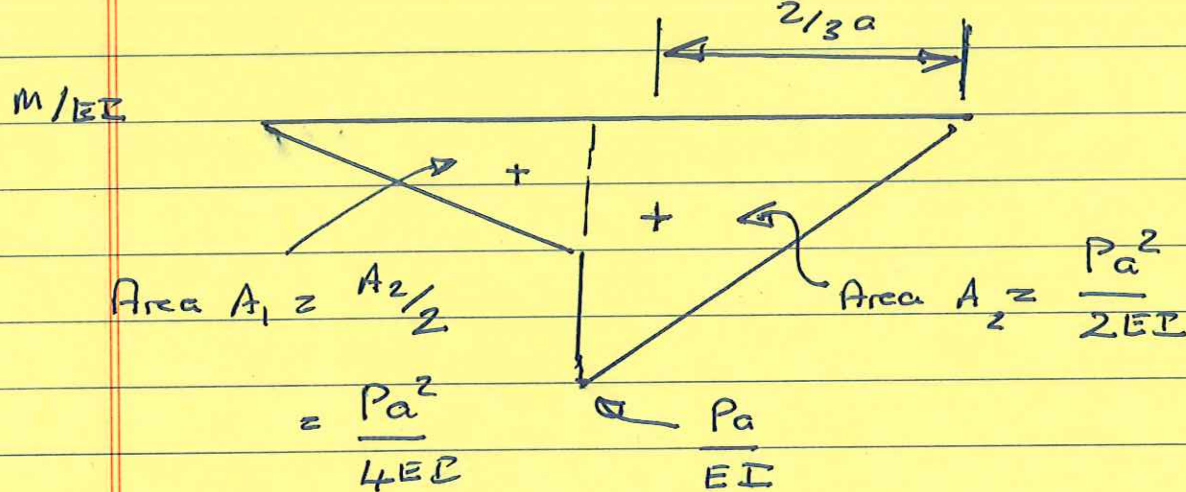
$$\Delta_x + \Delta_C = \frac{1}{2}\Delta_B.$$

$$\Rightarrow \Delta_C = \frac{\Delta_B}{2} - \Delta_x = \frac{Pa^3}{EI}\left[\frac{1}{2} - \frac{1}{6}\right] = \frac{Pa^3}{3EI}$$ ✓

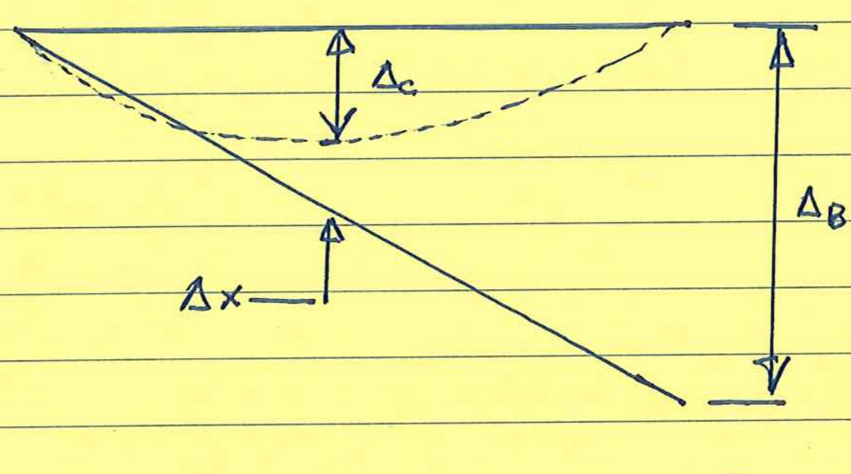
(E)

Example 3.





Deflection.



$$\Delta_B = \bar{x}_1 A_1 + \bar{x}_2 A_2 = \left(\frac{4}{3}a\right)\frac{Pa^2}{4EI} + \left(\frac{2}{3}a\right)\frac{Pa^2}{2EI}$$

$$= \frac{2}{3}\frac{Pa^2}{EI}.$$

$$\Delta_x = \left(\frac{a}{3}\right)\cdot\frac{Pa^2}{4EI} = \frac{Pa^3}{12EI}$$

(F)

Geometry (similar triangles).

$$\Delta_C + \Delta_X = \left(\frac{\Delta_B}{2}\right)$$

$$\Rightarrow \Delta_C = \left(\frac{\Delta_B}{2}\right) - \Delta X$$

$$= \frac{Pa^3}{3EI} - \frac{Pa^3}{12EI}.$$

$$\Delta_C = \frac{Pa^3}{4EI} \quad \text{———} \quad (A)$$

<u>Observation:</u> In example 2, we computed a midspan deflection:

$$\Delta_C = \frac{Pa^3}{3EI} \quad \text{———} \quad (B)$$

If we replace EI by 2EI, the midspan deflection is

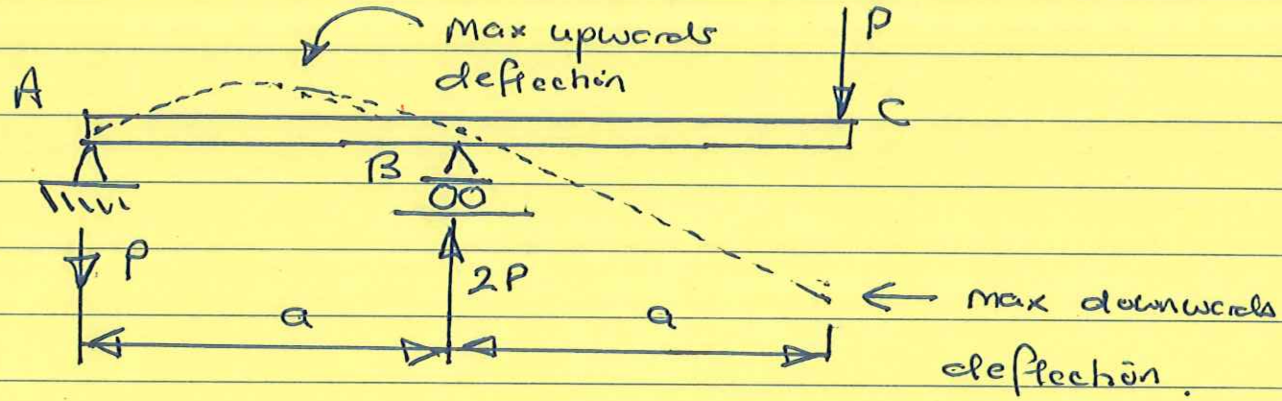
$$\Delta_C = \frac{Pa^3}{6EI}. \quad \text{———} \quad (C)$$

Since the geometry of this problem lies halfway between that for (B) & (C), we expect (A) to lie in the middle; let's check:

$$\frac{1}{6} < \frac{1}{4} < \frac{1}{3} \quad \checkmark$$

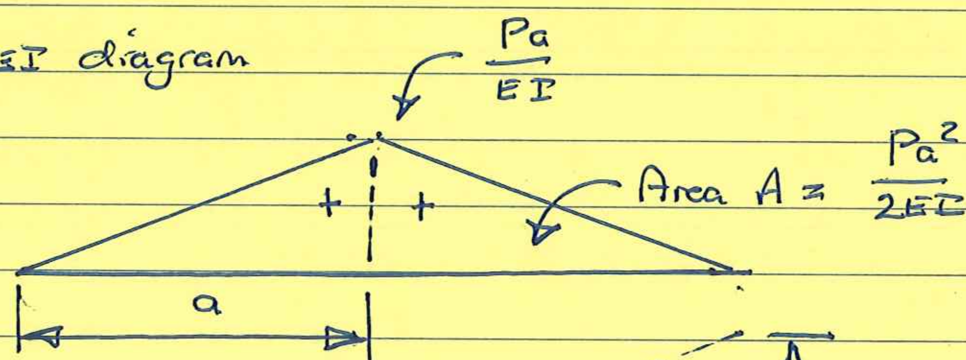
It works!!

Example 4: Cantilevered beam structure.

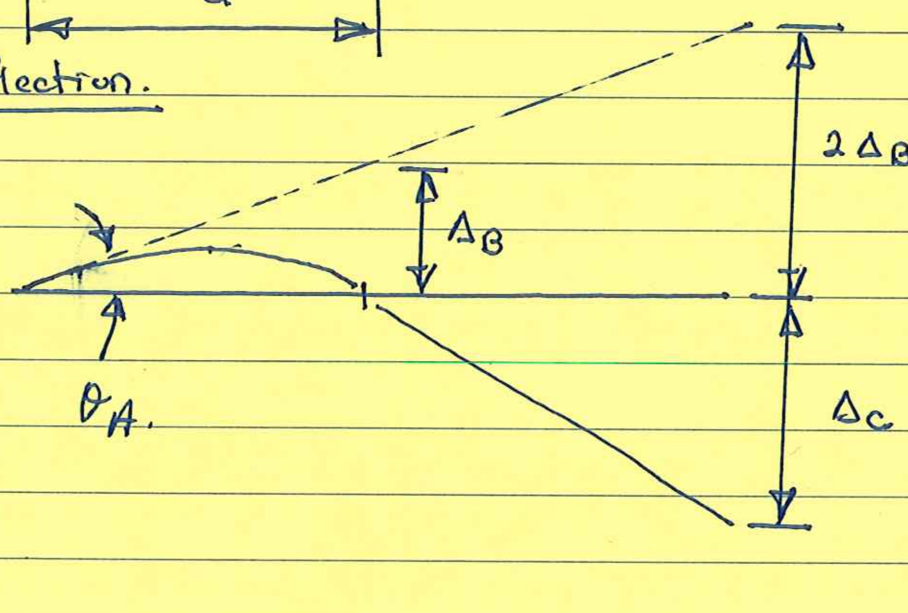


Question: What are the values of the max upwards / downwards deflections? And where do they occur?

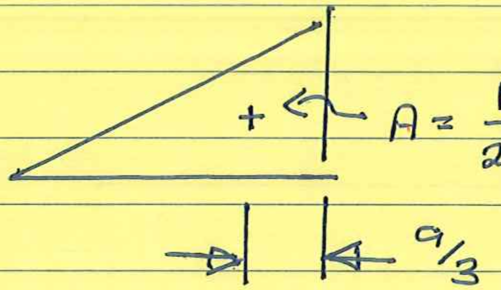
M / EI diagram



Deflection.



(4) <u>Step 1</u>: Compute first moment of area for A-B, evaluated about B.



$$A = \frac{Pa^2}{2EI}$$

$$\Delta_B = \left(\frac{a}{3}\right)\cdot\left(\frac{Pa^2}{2EI}\right)$$

centroid distance   area A.

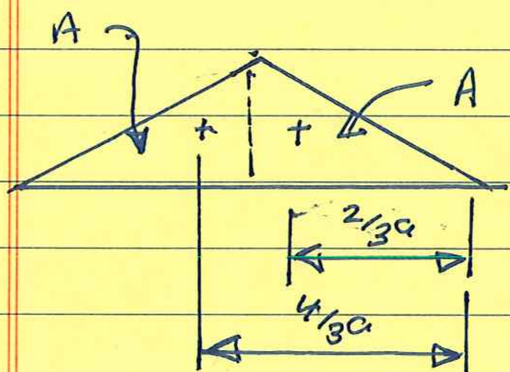
$$\Delta_B = \frac{Pa^3}{6EI}. \quad \text{——— (1)}$$

From geometry (small angles):

$$\theta_A \cdot a - \Delta_B = 0.$$

$$\Rightarrow \theta_A = \left[\frac{Pa^2}{6EI}\right]$$

<u>Step 2</u>: Compute ⊥r distance $2\Delta_B + \Delta_C$ $\Longleftrightarrow$ First moment of area between A-C, evaluated about C.



$$2\Delta_B + \Delta_C = \left(\tfrac{2}{3}a\right)A + \left(\tfrac{4}{3}a\right)A$$

$$= 2aA$$

$$= \frac{Pa^3}{EI} \quad \text{——— (2)}$$

<u>Step 3</u>: Compute Δ_C. Plug (1) into (2),

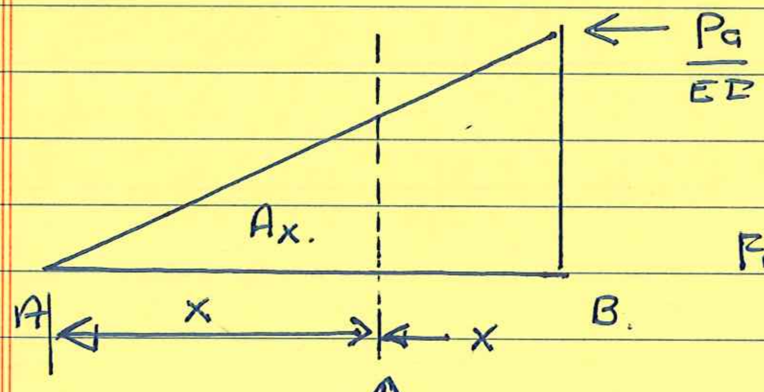
(I)

$$\Delta_C = \frac{Pa^3}{EI} - 2\,\frac{Pa^3}{6EI}$$

$$= \frac{Pa^3}{EI}\left[1 - \tfrac{2}{6}\right] = \frac{2}{3}\frac{Pa^3}{EI} \quad \text{——— (3)}$$

<u>Step 4:</u> Location of max upwards deflection?

We have $\theta_A = \dfrac{Pa^2}{6EI}$. At location of max upwards deflection, slope of beam $= 0$!!



Need to find x such that $\theta_A = A_x$.

From geometry $A_x = \dfrac{Px^2}{2EI}$.

$$\theta_x = \theta_A - A_x = 0.$$

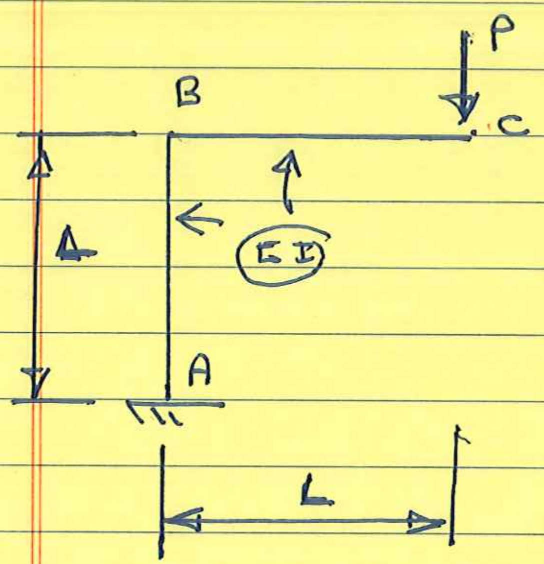
$$\Rightarrow \frac{Pa^2}{6EI} = \frac{Px^2}{2EI} \Rightarrow x^2 = \frac{a^2}{3}$$

$$\Rightarrow x = a/\sqrt{3}.$$

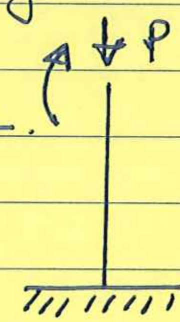
Notice that location of max upwards deflection <u>is not</u>. in the middle of A–B.

J

Example 5: What is Δ_C? Ignore compression in A-B.

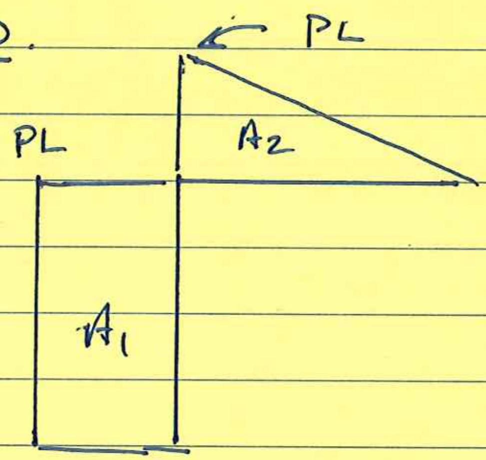


For segment A-B we have.



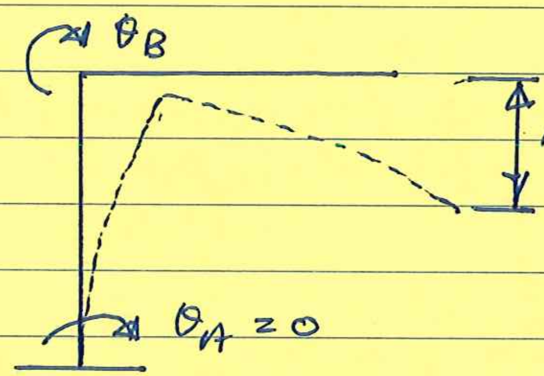
$M = PL.$

BMD.



$$A_1 = \frac{PL^2}{EI}, \quad A_2 = \frac{PL^2}{2EI}$$

Deflected Shape.



Δ_C = first moment area between B-C, evaluated about C

$+ \underbrace{\theta_B \cdot L}$.

deflection due to rotation at B.

(k).

Apply moment area :

$$\theta_B - \theta_A = \text{Area } A_1 = \frac{PL^2}{EI}.$$

Deflection in B-C due to flexural bending :

$$= A_2 \times \left(\tfrac{2}{3}L\right)$$

$$= \frac{PL^3}{3EI}.$$

$$\Rightarrow \Delta_C = \theta_B \cdot L + \frac{PL^3}{3EI}$$

$$\Delta_C = \frac{PL^3}{EI} + \frac{PL^3}{3EI} = \frac{4}{3}\frac{PL^3}{EI}.$$