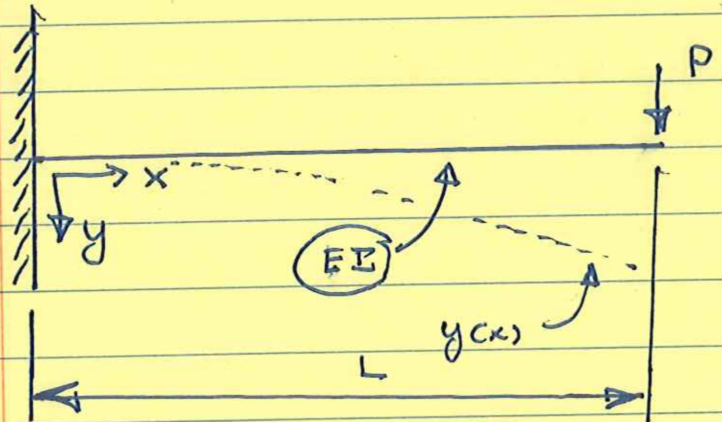
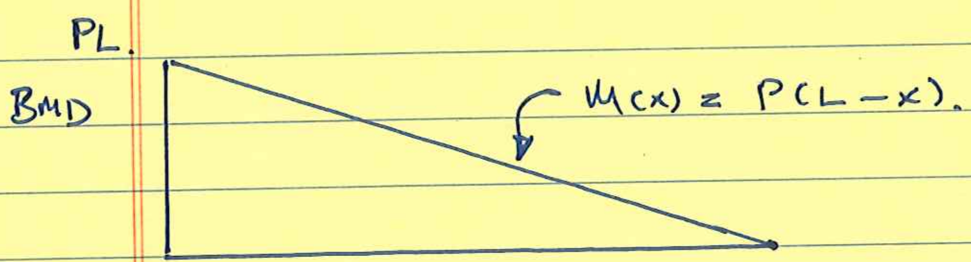
(A)

Integration of Beam Equations

Example 1: Midterm II, 2017





— (A)

Question: Starting from $\dfrac{d^2y}{dx^2} = \dfrac{M(x)}{EI}$ — (B)

and appropriate boundary conditions, show that

$$y(x) = \left(\frac{P}{6EI}\right) x^2 (3L - x).$$

Plug (A) into (B), rearrange:

$$EI \frac{d^2y}{dx^2} = P(L - x)$$

Integrating:

$$EI\, dy/dx = PLx - \frac{Px^2}{2} + A.$$

$$\rightarrow \quad EI\, y(x) = \frac{PLx^2}{2} - \frac{Px^3}{6} + Ax + B.$$

(B)

Boundary Conditions:

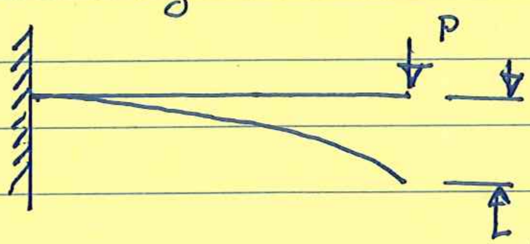
$$y(0) = 0 \longrightarrow B = 0$$

$$\left.\frac{dy}{dx}\right|_{x=0} = 0 \longrightarrow A = 0$$

$$\Rightarrow EI\, y(x) = \frac{PLx^2}{2} - \frac{Px^3}{6}$$

$$\Rightarrow y(x) = \frac{Px^2}{6EI}(3L - x) \quad ✓$$

Previously we saw:



$\Delta = \frac{PL^3}{3EI}$

Note: $y(L) = \frac{PL^3}{3EI}$. ✓

<u>Question</u>: Derive a formula for the slope of the beam as a function of x. Show that

$$\theta_B = \frac{PL^2}{2EI}.$$

<u>Sol'n</u>: From the derivation, we have:

$$EI \frac{dy}{dx} = PLx - \frac{Px^2}{2}.$$

Also: $\tan(\theta(x)) = dy/dx$. But $\tan(\theta) = \theta + \frac{\theta^3}{3} + \ldots$

For small θ, $\tan(\theta) \doteq \theta = \frac{dy}{dx}$.

(c)

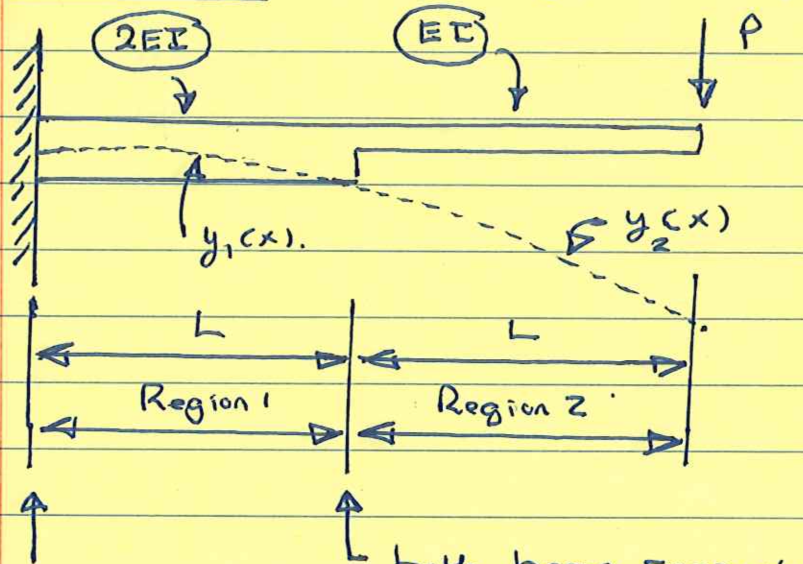
Hence,

$$\theta(x) = \frac{Px}{EI}\left(L - \frac{x}{2}\right)$$

When $x = L/2$,

$$\theta(L) = \frac{PL}{EI}\left(L - \frac{L}{2}\right) = \frac{PL^2}{2EI} \quad \checkmark.$$

Example 2: Compute $y_1(x)$ & $y_2(x)$.

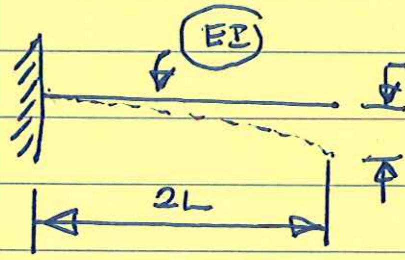


both beam segments must have same displacement & slope.

$y(0) = 0$

$\left.\frac{dy}{dx}\right|_{x=0} = 0$

Observation:



$$\Delta_1 = \frac{P(2L)^3}{3EI} = \frac{8}{3}\frac{PL^3}{EI}.$$

We expect that:

If EI $\rightarrow$ 2EI $\quad \Delta_2 = \frac{8}{6}\frac{PL^3}{EI}$

$$\Delta_2 \le y_2(2L) \le \Delta_1$$

(D)

Region 1.

$$\frac{d^2y}{dx^2} = \frac{P}{2EI}(2L - x)$$

$$\Rightarrow 2EI \frac{d^2y}{dx^2} = 2PL - Px$$

$$\Rightarrow 2EI\left(\frac{dy}{dx}\right) = 2PLx - \frac{Px^2}{2} + A$$

$$\Rightarrow 2EI\, y_1(x) = PLx^2 - \frac{Px^3}{6} + Ax + B.$$

Region 2.

$$\left(\frac{EI}{P}\right)\frac{d^2y}{dx^2} = (2L - x)$$

$$\Rightarrow \left(\frac{EI}{P}\right)\left(\frac{dy}{dx}\right) = 2Lx - \frac{x^2}{2} + C$$

$$\left(\frac{EI}{P}\right) y_2(x) = \frac{2Lx^2}{2} - \frac{x^3}{6} + Cx + D.$$

Boundary Conditions

$y_1(0) = 0$ At $x = L$ $y_1(L) = y_2(L)$

$\left.\frac{dy_1}{dx}\right|_{x=0} = 0$ $\left.\frac{dy_1}{dx}\right|_{x=L} = \left.\frac{dy_2}{dx}\right|_{x=L}.$

We have 4 boundary conditions, 4 equations.

$\Rightarrow$ can solve for $y_1(x)$ & $y_2(x)$

This is a Mess!