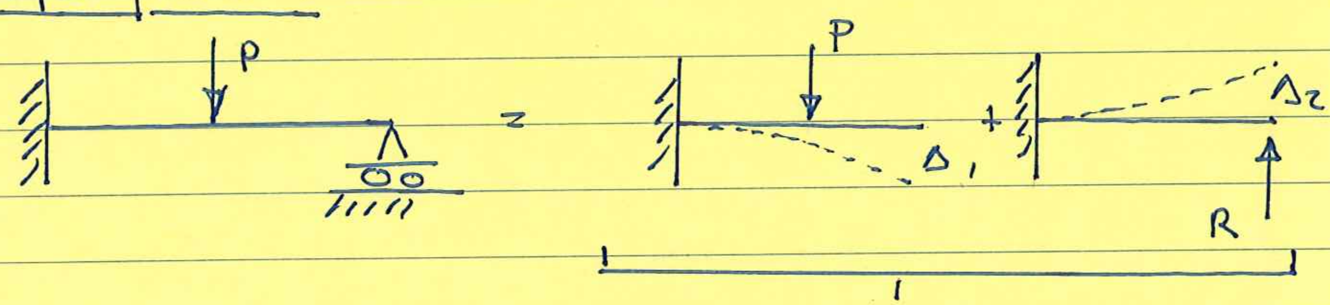
(A)

Force & Displacement Methods

Force Method – maintain equilibrium of forces
– use compatibility of displacements to solve problem.

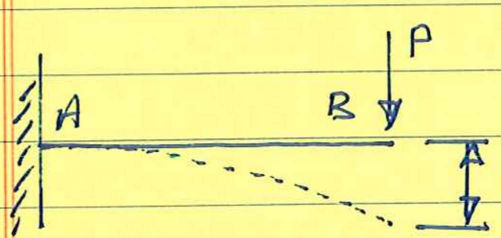
Displacement Method – maintain compatibility of displacements
– use equilibrium of forces to solve problem.

Simple problem.



Linear system → can use superposition.

Use compatibility of displacements to solve problem: $\Delta_1 + \Delta_2 = 0$



$$y_B = \frac{PL^3}{3EI}.$$

Using the flexibility approach: displacements ∝ external loads

(B) In matrix form:

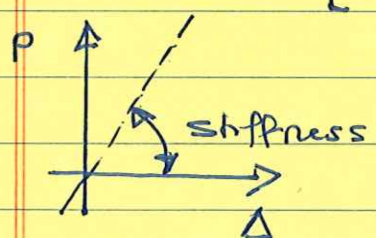
$$\left[y_B \right] = \left[\frac{L^3}{3EI} \right] \left[P \right] \qquad \text{—— (A)}$$

$$(1\times1) = (1\times1)\,(1\times1)$$

displacement vector. — flexibility matrix. — load vector.

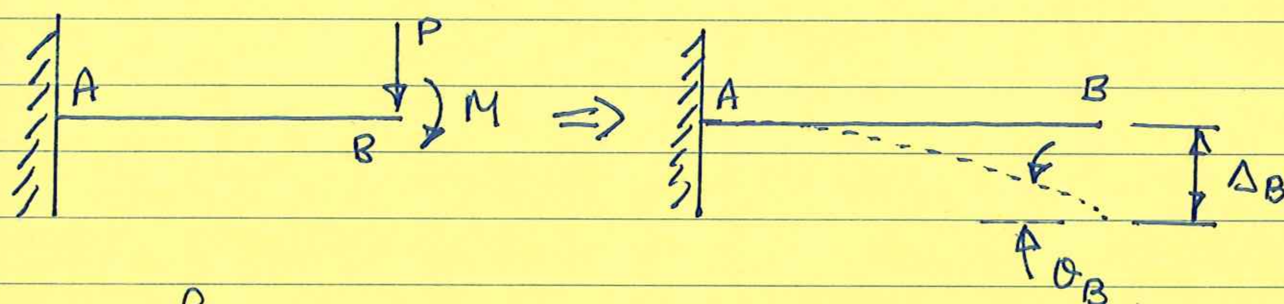
Using the displacement approach: rearrange (A).

$$\left[\frac{3EI}{L^3} \right] \left[y_B \right] = \left[P \right]$$



Stiffness matrix captures material & section properties, and geometry.

<u>Now consider problem with 2 degrees of freedom.</u>



Using flexibility method.

$$\begin{Bmatrix} \Delta_B \\ \theta_B \end{Bmatrix} = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{bmatrix} P \\ M \end{bmatrix} \leftarrow \text{load vector}$$

$$(2\times1) = (2\times2) \qquad (2\times1)$$

displacement vector

f_{ij} = flexibility matrix coefficient.

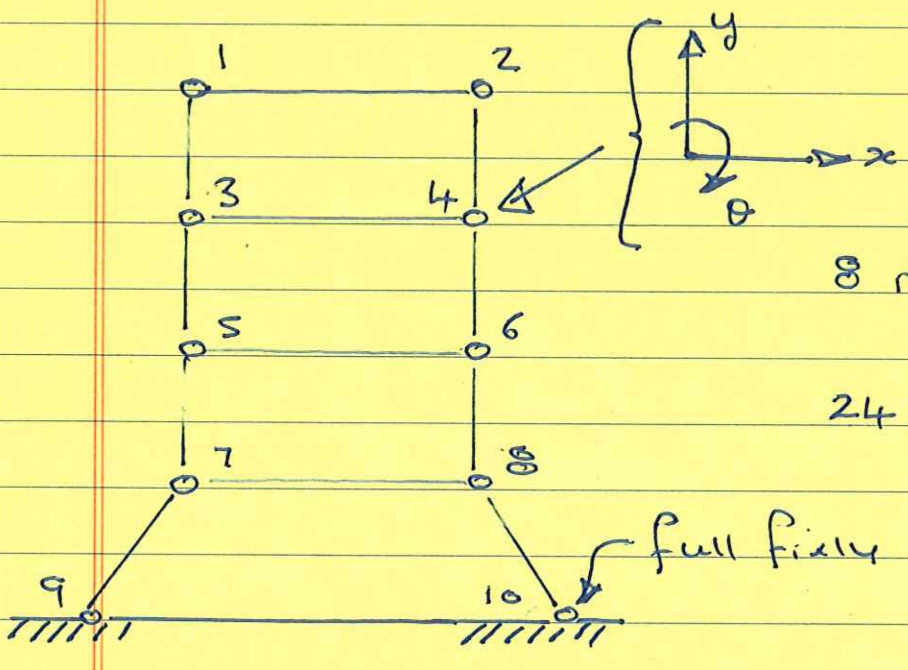
(c)

Using stiffness method.

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} \Delta_B \\ \theta_B \end{bmatrix} = \begin{bmatrix} P \\ M \end{bmatrix}$$

k_{ij} = stiffness matrix coefficient..

Scaling things up (Matrix methods with MATLAB).



Each node has 3 degrees of freedom (dof).

8 nodes are free to move

↓

24 dof in system model.

flexibility matrix is symmetric, fully populated.

Flexibility Method.
$$\begin{bmatrix} \Delta \end{bmatrix} = \begin{bmatrix} \times & \times & \times & \times & \times \\ \times & \times & & & \\ \times & & & & \\ \times & & & & \times \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$$

(24×1) $\quad (24 \times 24)$ $\quad (24 \times 1)$

Fully populated, symmetric $\Rightarrow$ Requires $\frac{n}{2}(n+1)$ storage $= O(n^2)$ storage.

(D)

Stiffness Method.

$$\begin{bmatrix} \times & \times & \times & & 0 \\ \times & \times & \times & & \\ \times & \times & \times & \times & \\ & \times & \times & \times & \times \\ 0 & & & \times & \times \end{bmatrix} \begin{bmatrix} \Delta \end{bmatrix} = \begin{bmatrix} P \end{bmatrix}$$

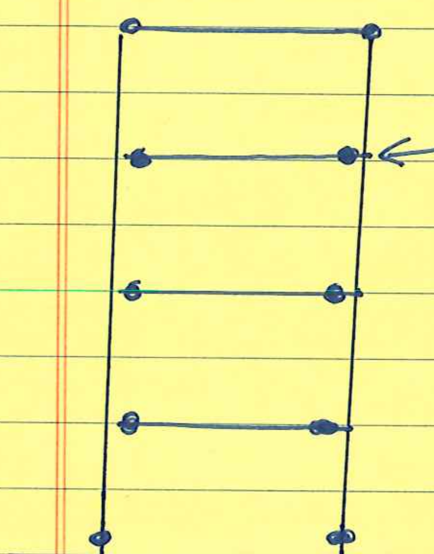
(24×24) (24×1) (24×1)

stiffness matrix is symmetric & banded.

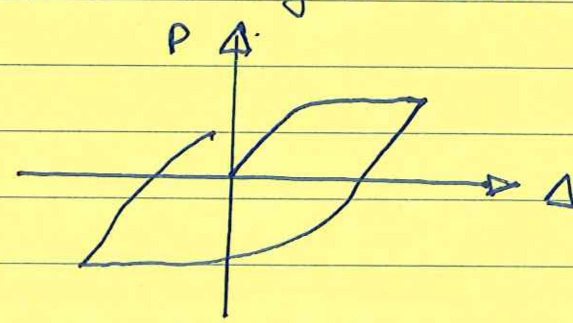
=> Storage requirements $O(n)$.

Historically, stiffness method has been more popular than flexibility method because computational requirement (storage) are lower.

Benefits of Flexibility Method. Great convergence properties for nonlinear problems.

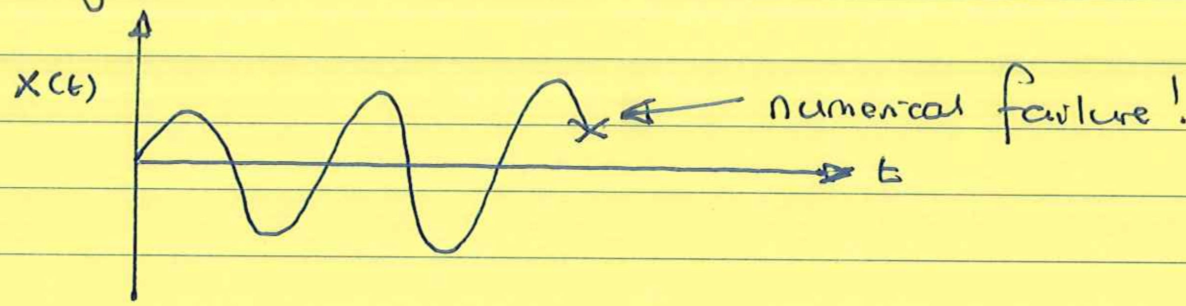


- plastic hinge
- nonlinear cyclic behavior



(E)

Simulation Programs based on displacement method can jam.



<u>Solution:</u> Embed flexibility inside stiffness!

displacement method

At section level

flexibility method