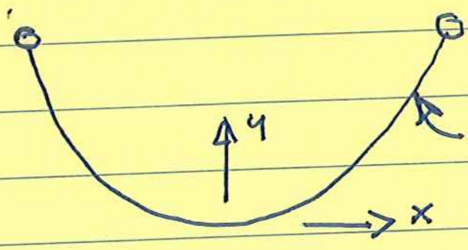
A

Analysis of a Cable hanging under its own weight.

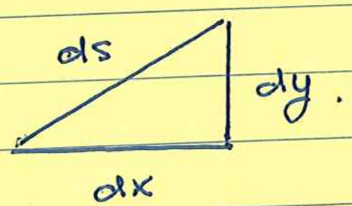
Anyone who has walked across the Golden Gate Bridge knows that the cables are huge. Self weight of the cables matters!

Analysis



Cable weight / unit length = q.

Consider weight of a small element.



Weight of small element = $q\,ds$.

Weight per unit length in the horizontal direction is

$$q\,ds = q_x \cdot dx$$

equivalent loading in horizontal directi

$$\Rightarrow q_x = q\left(\frac{ds}{dx}\right) \longrightarrow (A)$$

We also have $\dfrac{d^2y}{dx^2} = \left(\dfrac{q_x}{H}\right)$ — (B)

(B)

Combining (A) & (B) gives

$$\frac{d^2y}{dx^2} = \left(\frac{q}{H}\right)\left(\frac{ds}{dx}\right) \qquad \text{--- (C)}$$

From geometry

$$ds^2 = dx^2 + dy^2$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{q}{H}\left(\frac{ds}{dx}\right) = \frac{q}{H}\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2}$$

$$\Rightarrow y(x) = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \quad \text{--- (D)}$$

where $c = \left(q/H\right)$.

Equation (D) is a catenary curve -- mathematically, it is hyperbolic cosine function.

<u>When are the Parabula & Catenary similar?</u>

For large spans, the catenary & parabula are almost the same!!

We have:

$$y(x) = \frac{1}{2c}\left[e^{cx} + e^{-cx} - 2\right] \quad \text{where } c = \left(q/H\right)$$

Now expand e^{cx} & e^{-cx} as Taylor series:

(c) $$e^{cx} = 1 + (cx) + \frac{(cx)^2}{2!} + \frac{(cx)^3}{3!} + \cdots$$

$$e^{-cx} = 1 - (cx) + \frac{(cx)^2}{2!} - \frac{(cx)^3}{3!} + \cdots$$

Adding equations.

$$e^{cx} + e^{-cx} = 2 + (cx)^2 + \frac{2}{4!}(cx)^4 + \cdots$$

hence

$$y(x) = \frac{1}{2c}\left[(cx)^2 + \frac{2}{4!}(cx)^4 + \cdots\right]$$

$$= \left[\frac{cx^2}{2} + \frac{c^3x^4}{4!} + \cdots\right]$$

$$= \frac{q}{H}\left[\frac{x^2}{2} + \left(\frac{q}{H}\right)^2 \frac{x^4}{4!} + \cdots\right]$$

but $H = \dfrac{qL^2}{8f} \Rightarrow \left(\dfrac{q}{H}\right) = \left(\dfrac{8f}{L^2}\right)$

Second order terms will be very small when $8f/L^2 \to 0$, i,e, Large spans!