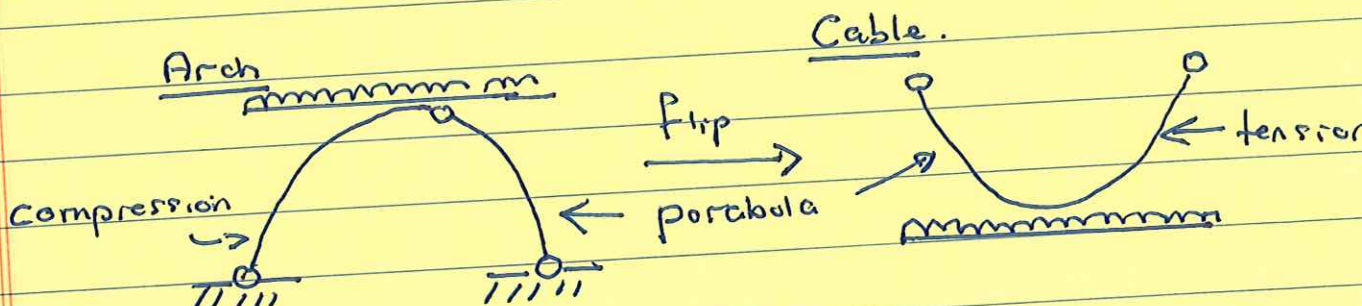
(A) Cable Structures.

Observation: In our analysis of arch structures we saw that a parabolic shape can carry a uniformly distributed load in compression alone. No shear, no bending moments.

If we take an arch and flip it upside down we would expect that the "cable elements" would carry a load in tension and form a parabolic shape, ie.



Cables.

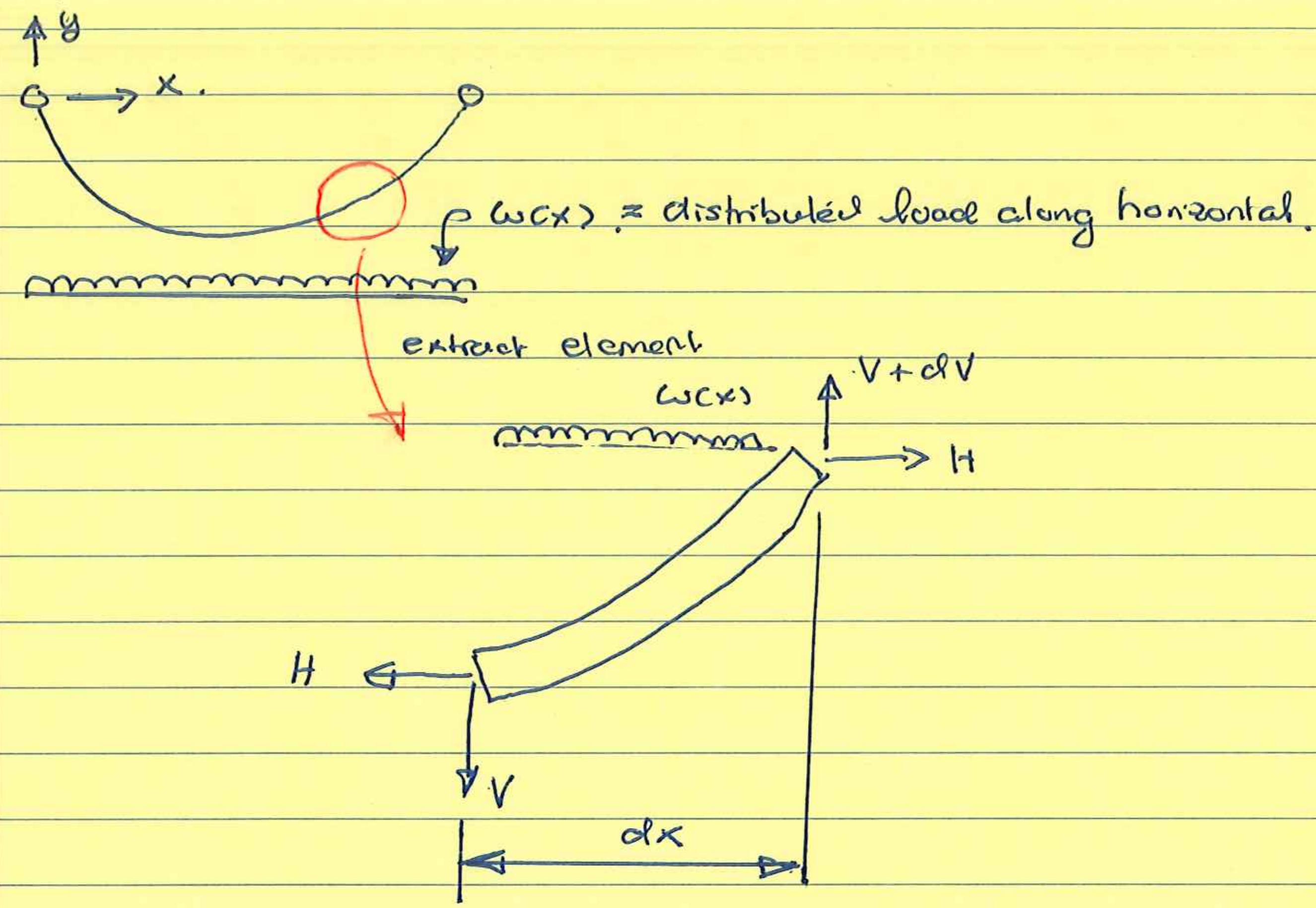
A cable is a flexible structure that cannot resist bending. The cable assumes a shape to carry the loads by tension alone. Thus, shape depends on loads.

Relationship between loads & deflections is no longer linear. Superposition does not apply.

Any section of the cable must be in equilibrium.

(B)

Equilibrium of a Cable Element.

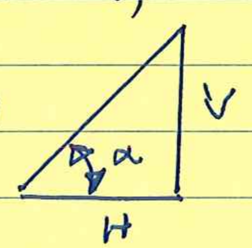


$$\sum F_y = 0 \Rightarrow V + dV = V + w(x).dx.$$

$$\Rightarrow \frac{dV}{dx} = w(x). \qquad \text{— (A)}$$

But, cable can only carry loads in tension, hence:

$$V = H \tan\alpha, \qquad \text{— (B)}$$



$$= H \frac{dy}{dx}.$$

Combine (A) & (B), $\dfrac{d^2y}{dx^2} = \left[\dfrac{w(x)}{H}\right]$ — (C)

(C)

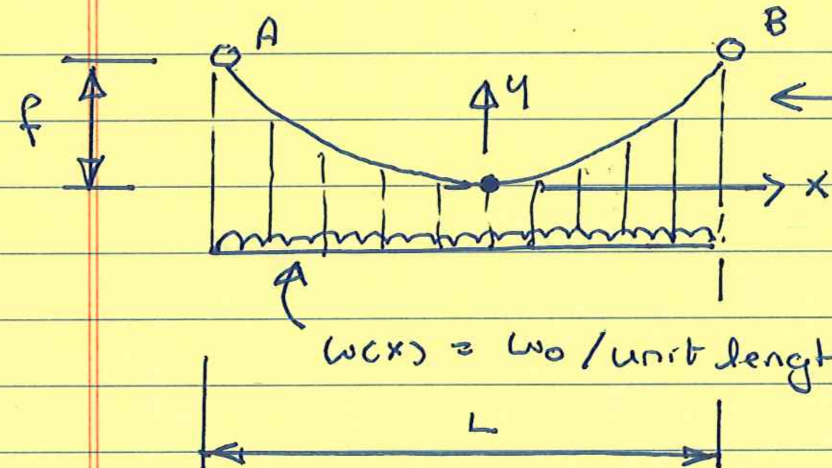
Procedure: Integrate equation (C) and apply boundary conditions

Note: If coordinates are defined $\overset{o}{\downarrow}{}_{y} \longrightarrow x$

then we have:

$$\frac{d^2y}{dx^2} = \left[\frac{-w(x)}{H}\right] \qquad \text{——— (D)}$$

Example 1: Constant uniform load along horizontal.



← load supported by system of closely spaced hangers.

f = sag of cable

L = span of cable.

$$\frac{d^2y}{dx^2} = \left[\frac{w_0}{H}\right]$$

$$\Rightarrow \quad y(x) = \frac{w_0 x^2}{2H} + c_1 x + c_2.$$

Boundary conditions: $y(0) = 0 \Rightarrow c_2 = 0$

$$\left.\frac{dy}{dx}\right|_{x=0} = 0 \Rightarrow c_1 = 0$$

D

$$\Rightarrow \quad y(x) = \frac{w_0 x^2}{2H} \longleftarrow \text{parabola!!}$$

We also have:

$$y = f \quad \text{at} \quad x = \pm L/2 .$$

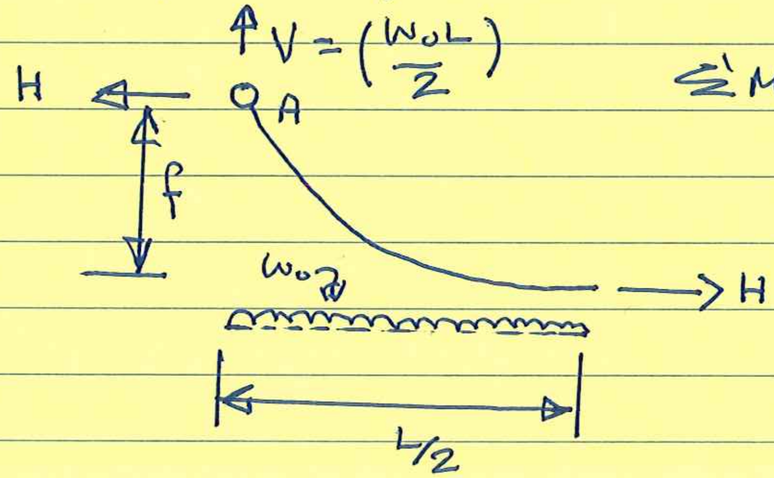
hence,
$$H = \frac{w_0 L^2}{8f} \qquad \text{—— (F)}$$

<u>Points to note:</u>

1. We have positioned the coordinate system to simplify the math. Other locations will also work.

2. The horizontal component of cable force, H, is constant along the structure.

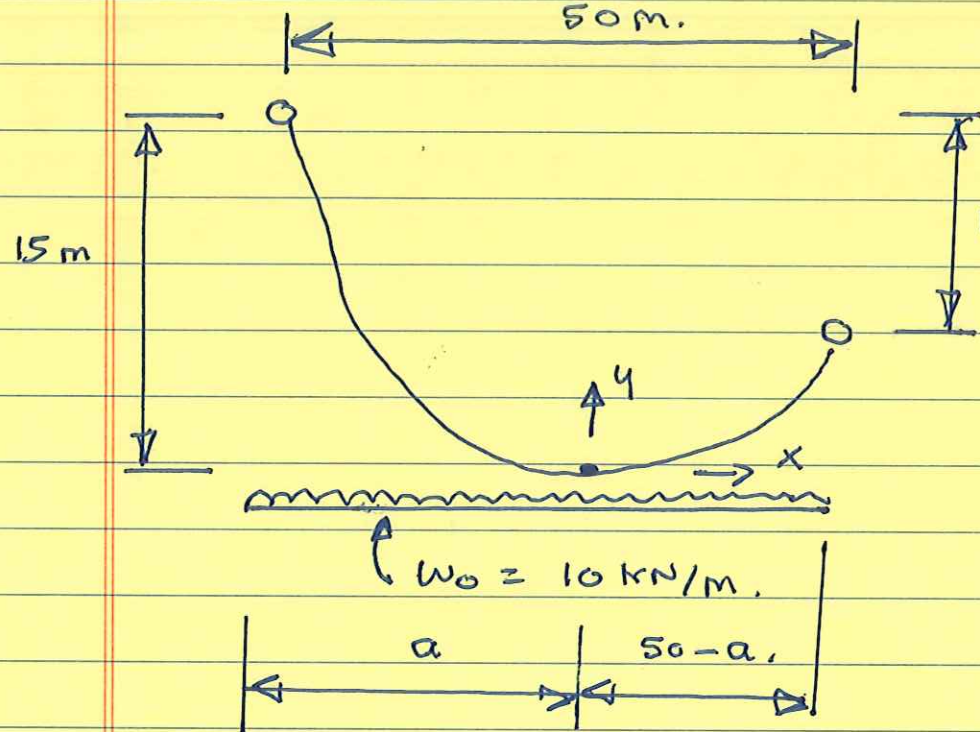
3. Equation (F) can be verified by looking at equilibrium of ½ structure.



$$\sum M_A = 0 \quad Hf = \frac{w_0 L}{2} \cdot \frac{L}{4}$$

$$\Rightarrow H = \frac{w_0 L^2}{8f} \quad \text{— (G)}$$

(E) Example 2: Find Location of Cable Profile Minimum.



From previous section, we know that cable shape will be a parabola.

$$\Rightarrow \quad 15 = k a^2 \quad \text{(left-hand side)}$$

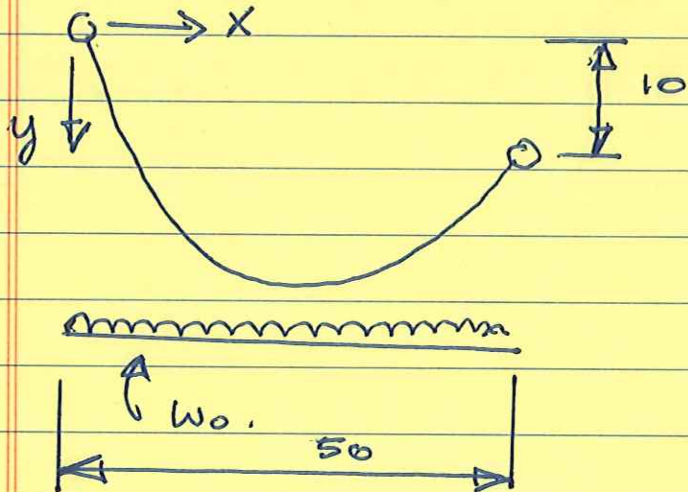
$$5 = k(50-a)^2 \quad \text{(right-hand side)}$$

$$\Rightarrow \quad \left(\frac{15}{5}\right) = \frac{a^2}{(50-a)^2} \quad \Rightarrow \text{find } a.$$

then use statics to solve for vertical reactions

& horizontal cable force.

(F) Alternate Method: Suppose origin is at upper left-hand support.



$$\frac{d^2y}{dx^2} = \left[\frac{-w_0}{H}\right]$$

Integrate twice and apply boundary conditions.

$y(0) = 0$

$y(50) = 10$

Then, use extra information on max dip to find H.

Note on loadings.

When $w(x) = w_0$ $\xrightarrow[\text{twice}]{\text{integrate}}$ quadratic cable shape.

If $w(x) = kx$ (linear function).

$$\Rightarrow \frac{d^2y}{dx^2} = \left[\frac{kx}{H}\right] \xrightarrow{\text{integrate}} \text{cable shape is a cubic!}$$