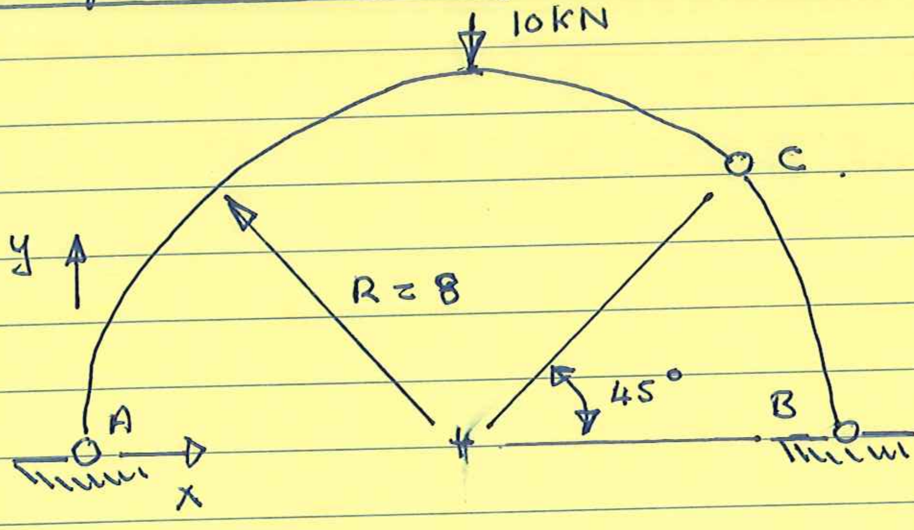
(A) **Analysis of Circular & Parabolic Arches.**

Analysis of a Circular Arch.



$\sum F_y = 0 \;\; \rightarrow \;\; V_A + V_B = 10$

$\sum M_A = 0 \;\; \Rightarrow \;\; 10R - V_B.(2R) = 0$

$V_A = 5\text{kN}$
$V_B = 5\text{kN}.$

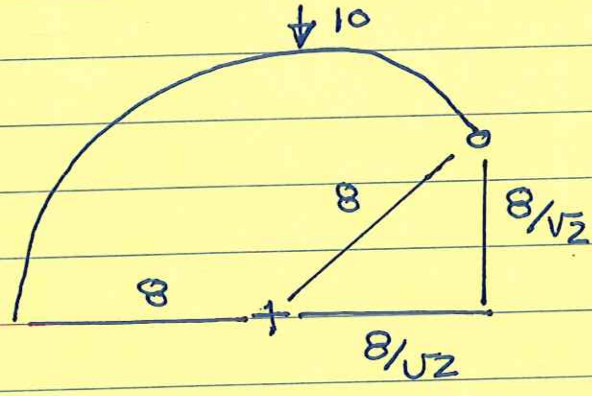
$\sum F_x = 0 \;\; \Rightarrow \;\; H_A = H_B$ (doesn't help).

But we know, $M_c = 0$

$10 \times 8/\sqrt{2} + H_A(8/\sqrt{2})$

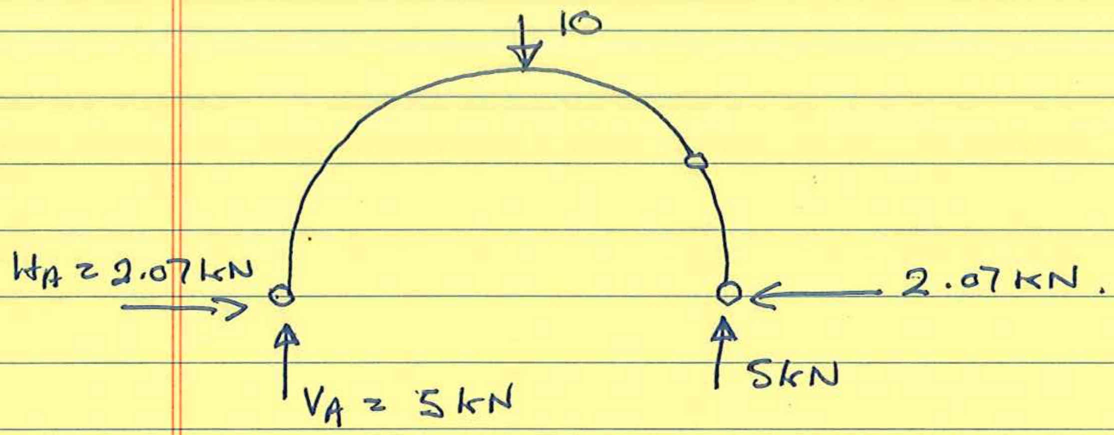
$= V_A(8 + 8/\sqrt{2})$

$\Rightarrow H_A = 2.07\text{kN}.$

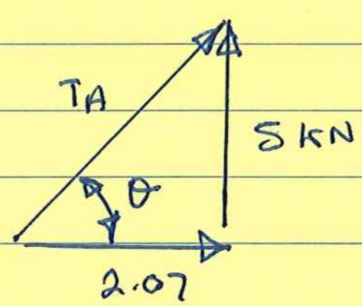


Note: We could have also looked at B – C $\Rightarrow H_B = 2.07$ k

(B) Total Reaction at A.



Total Reaction at A $\quad T_A = \left[H_A^2 + V_A^2 \right]^{1/2} = 5.41$ kN.

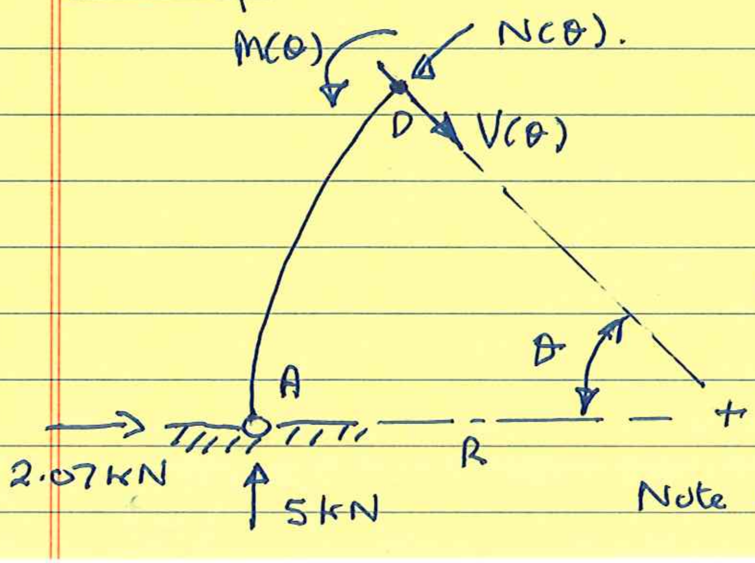


$\tan\theta = 5/2.07$

$\Rightarrow \theta_A = \tan^{-1}\left[5/2.07 \right] = 67.5^\circ$

Note: At supports A & B, the arch is vertical. But θ_A is not vertical $\Rightarrow$ <u>need to support moment internally</u>.

<u>Bending Moment & Shear Forces.</u> $\quad \sum M_D = 0$



$M(\theta) = 5R(1 - \cos(\theta)) - 2.07R\sin(\theta)$

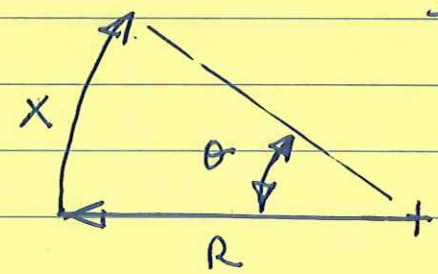
$V(\theta) = 5\sin(\theta) - 2.07\cos(\theta)$

$N(\theta) = -(5\cos(\theta) + 2.07\sin\theta)$

Note $N(\theta = 0) = -5$ kN ✓

(c) Does $V(x) = \frac{dM}{dx}$ still work?

Let's transform (x, y) coordinates to polar coordinates (R, θ).



In polar coordinates $x = \theta R$.

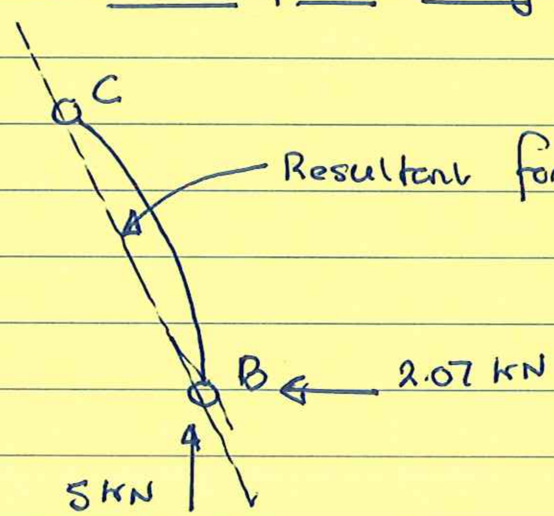
$$\Rightarrow \frac{dx}{d\theta} = R.$$

$$V(\theta) = \frac{dM}{dx} = \frac{dM}{dx} \cdot \frac{d\theta}{d\theta} = \frac{dM}{d\theta} \cdot \frac{d\theta}{dx} = \frac{dM}{d\theta} \cdot \frac{1}{R}$$

Thus, we need to check $V(\theta) = \frac{1}{R} \cdot \frac{dM}{d\theta}$.

It works!!

Note: There are no external loads acting on segment B-C of the arch. Therefore, resultant force at B must pass through the hinge at C.

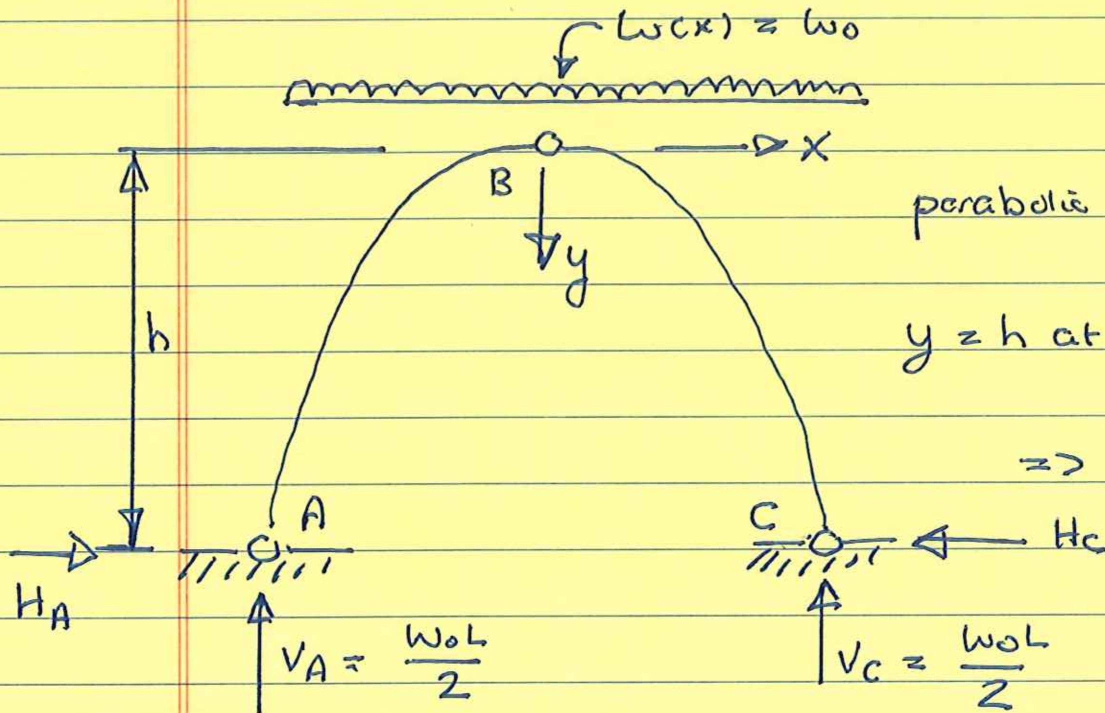


(D)

Analysis of a Parabolic Arch.

Basic Question: Can we do better than a circular arch shape?

Analysis of a Parabolic Arch.



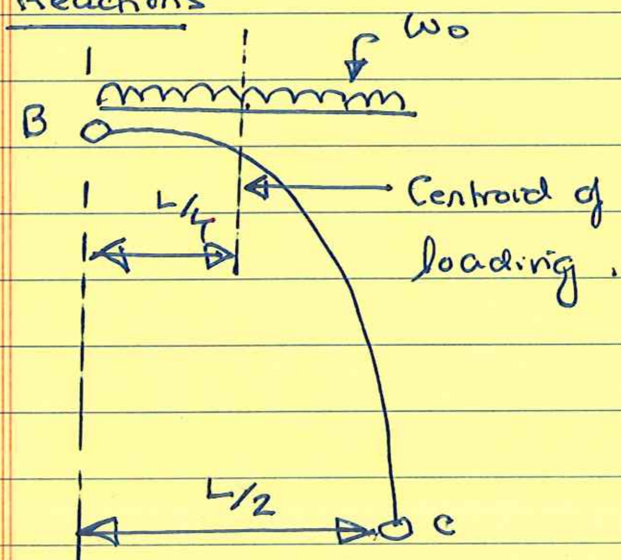
parabolic shape $y = kx^2$

$y = h$ at $x = \pm L/2 \Rightarrow k = \frac{4h}{L^2}$

$\Rightarrow y(x) = \left(\frac{4h}{L^2}\right) x^2$

Parabolic shape $y(x) = \left[\frac{4h}{L^2}\right] x^2$; $\frac{dy}{dx} = \left(\frac{8h}{L^2}\right) x$

Reactions



$\sum V = 0 \Rightarrow V_A = V_C = \frac{W_oL}{2}$

$\sum M_B = 0$

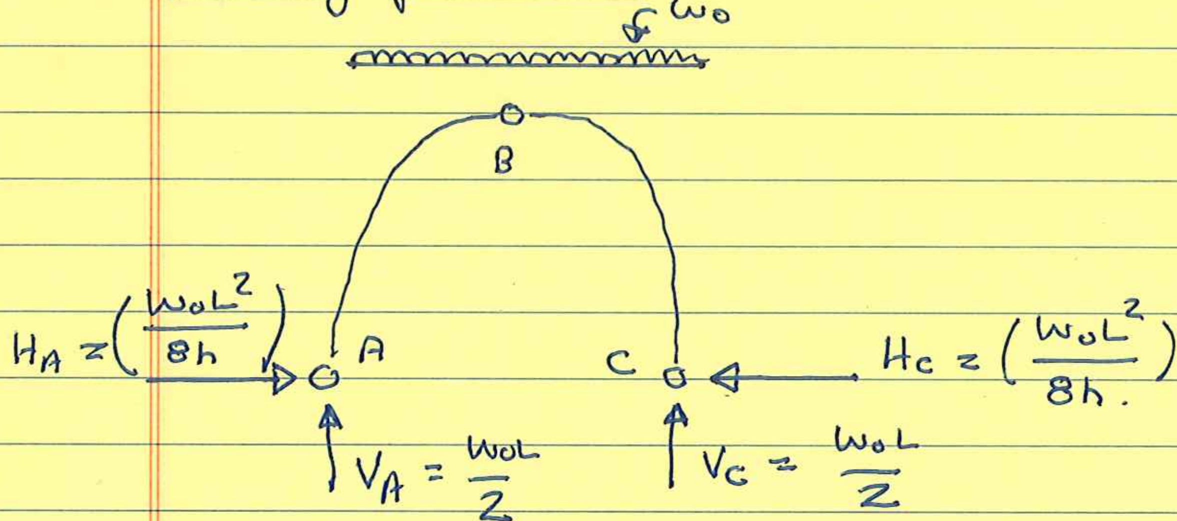
$$\underbrace{\frac{W_oL}{2} \cdot \frac{L}{4}}_{\text{loading}} + H_c \cdot h - \underbrace{\frac{W_oL}{2}}_{V_C} \cdot \frac{L}{2} = 0$$

(E)

$$\Rightarrow H_c \cdot h = \frac{w_o L^2}{8}$$

$$\Rightarrow H_c = \left[\frac{w_o L^2}{8h}\right]$$

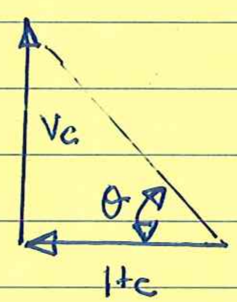
<u>Summary of Reactions</u>



<u>Total Reactions.</u>

$$\text{Magnitude} = \left\{V_c^2 + H_c^2\right\}^{1/2} = \frac{w_o L}{2}\left[1 + \frac{L^2}{16h^2}\right]^{1/2}$$

$$\text{Direction.} \quad \tan(\theta) = \left(\frac{V_c}{H_c}\right) = \left(\frac{4h}{L}\right) \quad \text{— (A)}$$



From geometry $y = kx^2 = \left(\frac{4h}{L^2}\right)x^2$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{8h}{L^2}\right)x$$

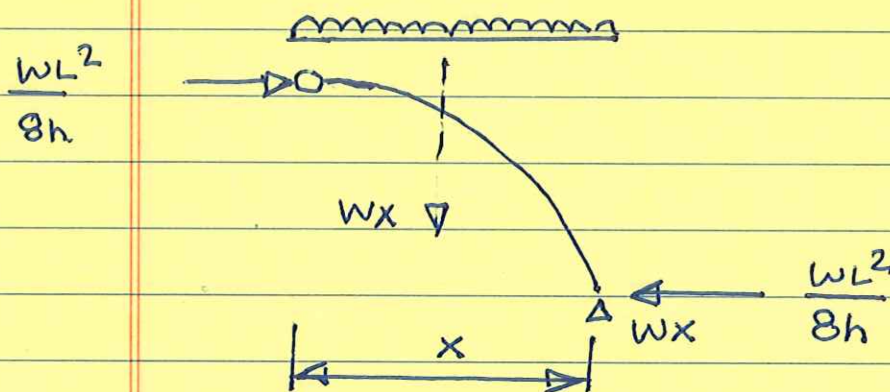
<u>Key Observation.</u> At $x = L/2$, $\frac{dy}{dx} = \left(\frac{8h}{L^2}\right)\frac{L}{2} = \left(\frac{4h}{L}\right)$ — (B)

Equations (A) & (B) are identical → Pure compression, no bending, no shear at foundation!

(F) Arbitrary Section.

So loads are transferred to the foundation by pure compression -- no bending, no shear! This is perfect!

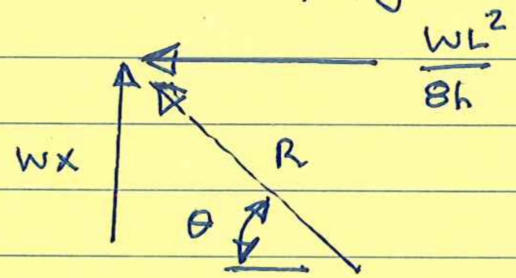
What about an arbitrary point in the arch?



Bending moment at x. $M(x) = (wx)\left(\frac{x}{2}\right) - \frac{wL^2}{8h} \cdot y$

$$= \frac{wx^2}{2} - \frac{wx^2}{2} = 0$$

Resultant of ground reaction forces.



$$R = w\left[x^2 + \left(\frac{L^2}{8h}\right)^2\right]^{1/2}$$

$$\tan(\theta) = \left(\frac{8xh}{L^2}\right) \text{ ——— } (C)$$

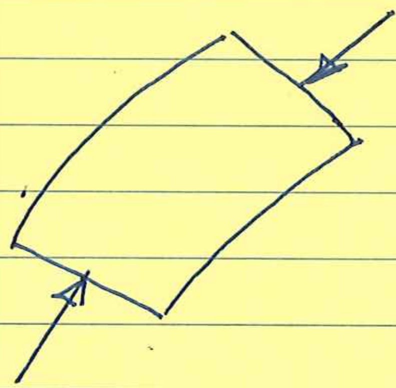
Slope of curve section $\frac{dy}{dx} = 2kx = \left(\frac{8xh}{L^2}\right)$ — (D)

Equation (C) equals (D) $\rightarrow$ pure compression throughout!

(G) Note: No bending, no shear throughout a parabolic arch carrying uniform load.

Amazing!

Furthermore, this result is independent of the hinge position.



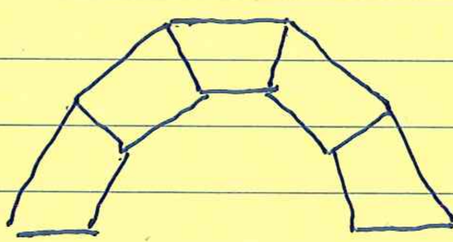
Compressive load only!

No bending → no induced tension effects (only axial compression).

Hence, use parabolic arches in masonry construction.

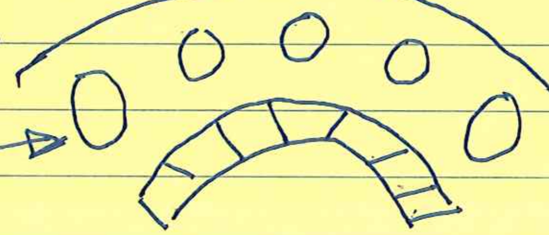
Applications: Medieval cathederals, Roman aqueducts.

Venetian Gothic Architecture



→ humpbacked bridges.

Most of load is self weight.



Holes left to give uniform loading }