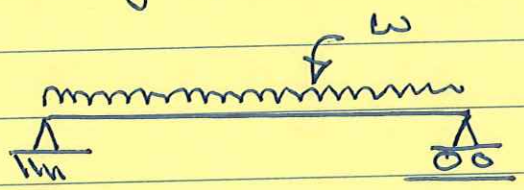
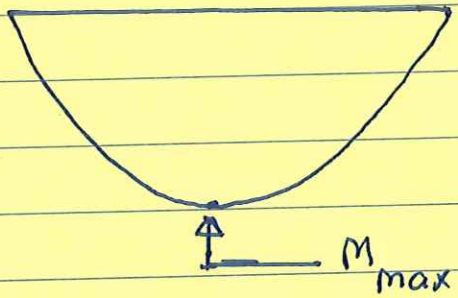
(A)

Arch Structures.

Motivation: Consider a simply supported beam carrying a uniform load.



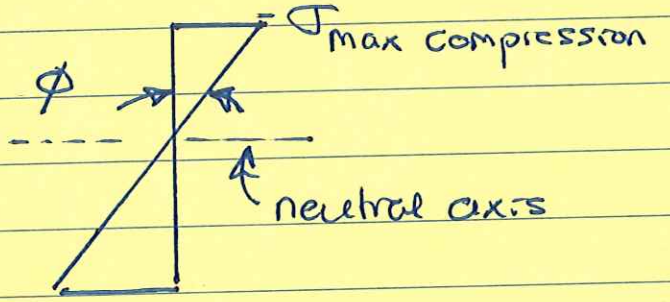
BMD



$$M_{max} = \frac{wL^2}{8}$$

From mechanics, $\phi = M/EI$.

ϕ ← beam curvature.

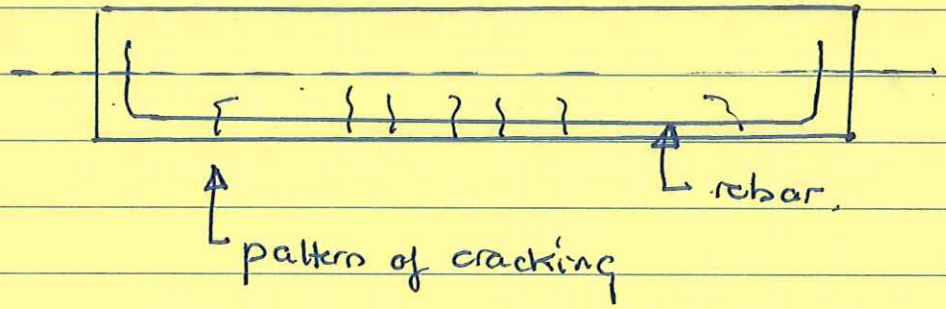


This pathway only works if we have a material that can resist stresses in the range

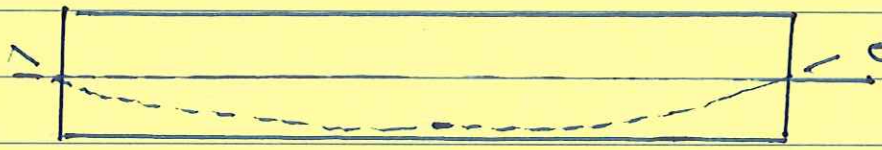
$\{ -\sigma_{max\ compression}, \sigma_{max\ tension} \}$

Materials such as concrete have an ability to carry loads in compression, but are very poor at carrying loads in tension.

(B). Sol'n 1: Augment concrete with a material that can carry tension -- steel! Reinforced concrete (1850-



Sol'n 2: Prestressed Concrete (1960- -).



- High strength tendons are threaded through the concrete and then pulled really tight! This puts the beam in compression (and reduces / eliminates the tensile stresses.

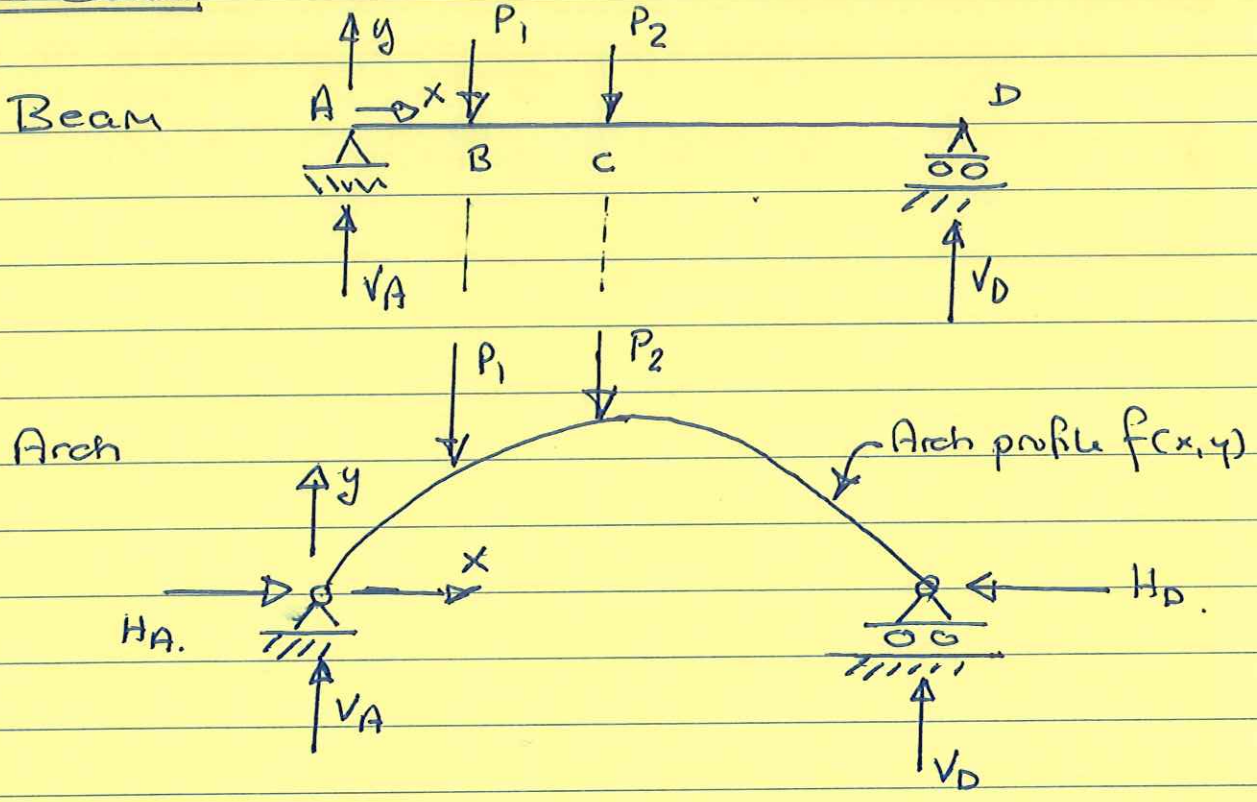
- Cable profiles can be designed to counter bending moments due to self weight.

Sol'n 3: Relax the constraint that beams need to be straight - i.e., consider curved beams.

(C)

Analysis of Arch Structures.

Basic Idea



In the segment A - B:

For the beam: $M(x) = V_A \cdot x$.

For the arch: $M(x,y) = V_A \cdot x - H_A \cdot y$.

V_A: vertical reaction at A

H_A: horizontal reaction due to arching mechanisms.

Takeaway:

- We can reduce bending moments by removing the constraint beams are straight. But now, we need to deal with H_A.

(D)

- Implicit assumption : arch supports will not move.

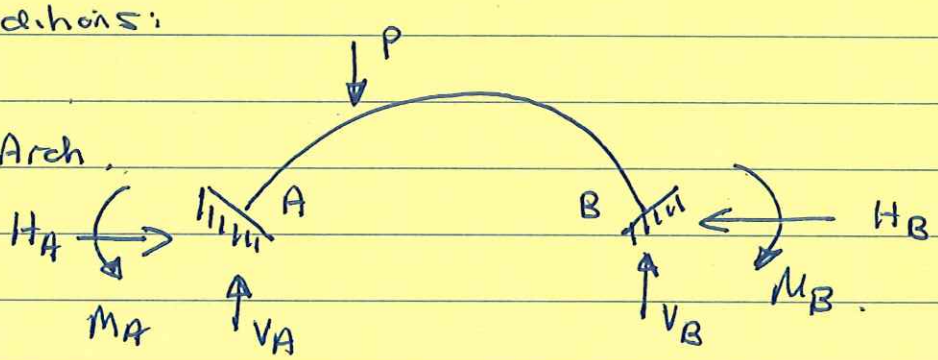
- To see what happens when things go wrong

Google : bridge collapse taiwan.

Types of Arch.

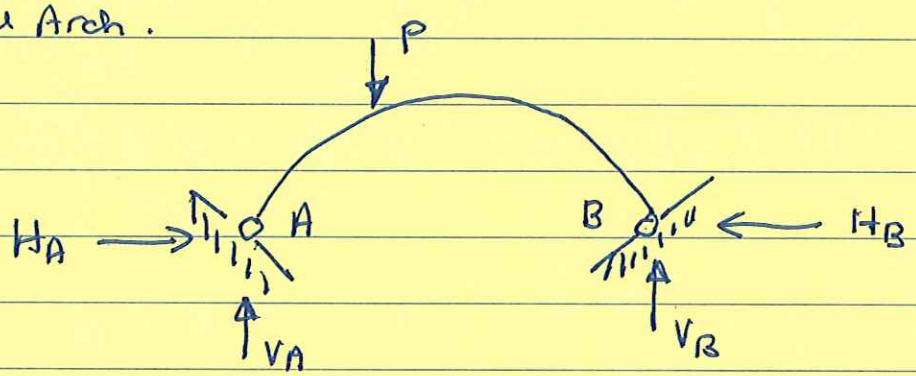
Support conditions:

(1) Fixed Arch.



Statically indeterminate : $i = 3$.

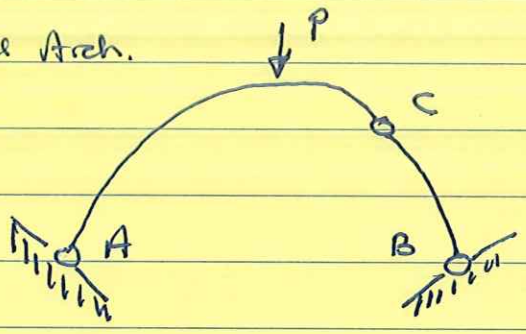
(2) Pinned Arch.



Statically indeterminate : $i = 1$.

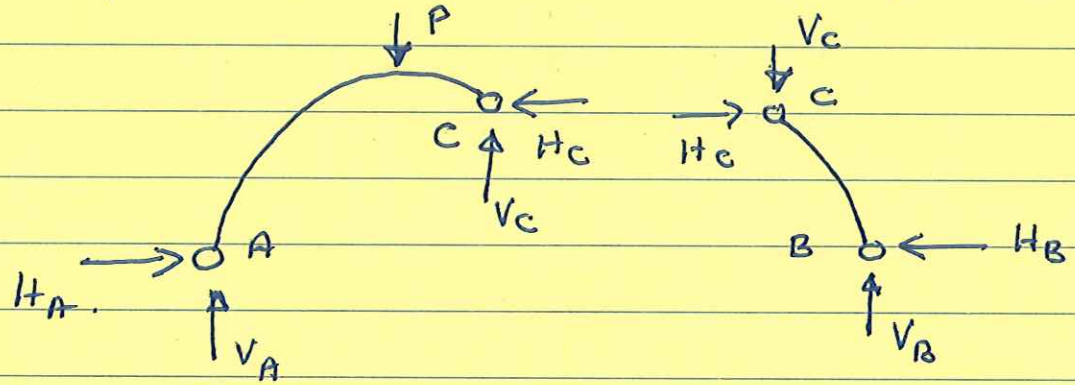
(E)

(3). Three-pinned Arch.



statically determinate!

We can pull arch apart & examine equilibrium of individual substructures, i.e.,



Basic Questions:

1. What are the support reactions?
2. What is the force transferred across the internal hinge?
3. What is $M(x)$ & $U(x)$.
4. Does $U(x) = dM/dx$ still work?

Harder Questions:

5. Does position of the internal hinge matter?
6. What is the optimal shape?