Solutions to Arch Problems

Department of Civil and Environmental Engineering, Fall Semester, 2017

ENCE 353 Midterm 2, Open Notes and Open Book

Name : AUSTIN

Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please show all of your working.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch. A vertical load P is applied at node D.

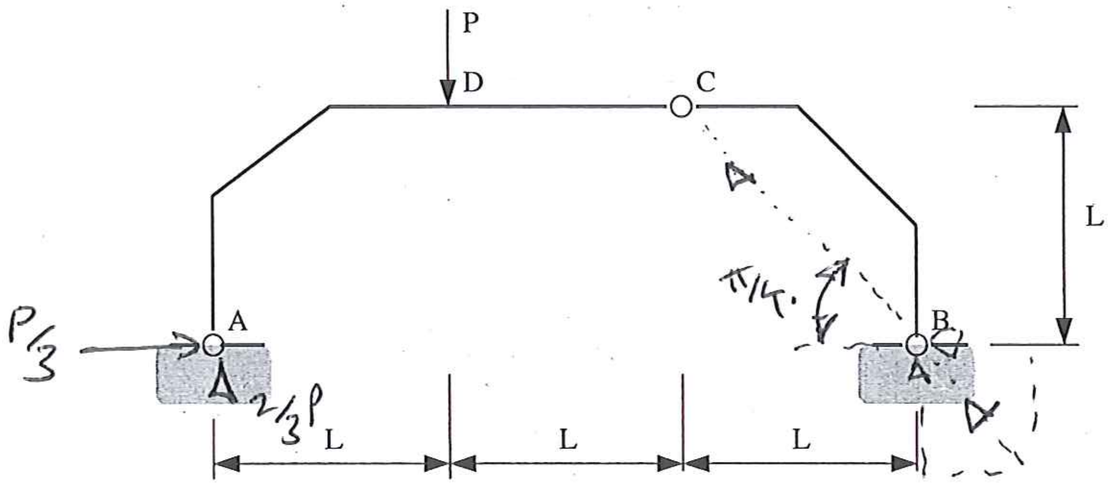


Figure 3: Front elevation view of a simple three-pinned arch.

[3a] (5 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of L and P.

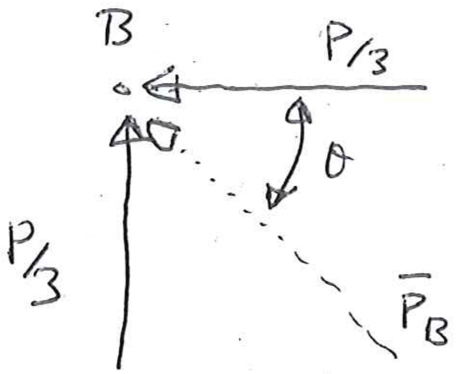
$$\Sigma M_A = 0 \Rightarrow PL = V_B 3L \Rightarrow V_B = P/3$$

$$\Sigma V = 0 \Rightarrow V_A + V_B = P \Rightarrow V_A = \tfrac{2}{3}P.$$

$$\Sigma' M_C = 0 \Rightarrow V_B \cdot L + H_B L = 0 \Rightarrow H_B = -\tfrac{P}{3}$$

$$\Sigma H = 0 \Rightarrow H_A + H_B = 0 \Rightarrow H_A = P/3.$$

[3b] (3 pts) Compute the magnitude and orientation of the **total reaction force vector** at support B. Show that it passes through the hinge at C. You can annotate Figure 3 if you think it will help to explain your solution.



Magnitude $P_B = \frac{\sqrt{2}}{3} P$

$\tan(\theta) = \left(\frac{P}{3} / \frac{P}{3} \right) = 1 \Rightarrow \theta = \pi/4$.

[3c] (2 pts) Suppose that your calculations indicated that the "total reaction force at support B" **did not** pass through the hinge at C. What would that mean?

- Structure not in equilibrium.
- Calculations wrong...

Department of Civil and Environmental Engineering,　　　　Spring Semester, 2020

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2	15	
3	10	
Total	40	

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch that carries a total snow loading of 3WL uniformly distributed over its upper section.

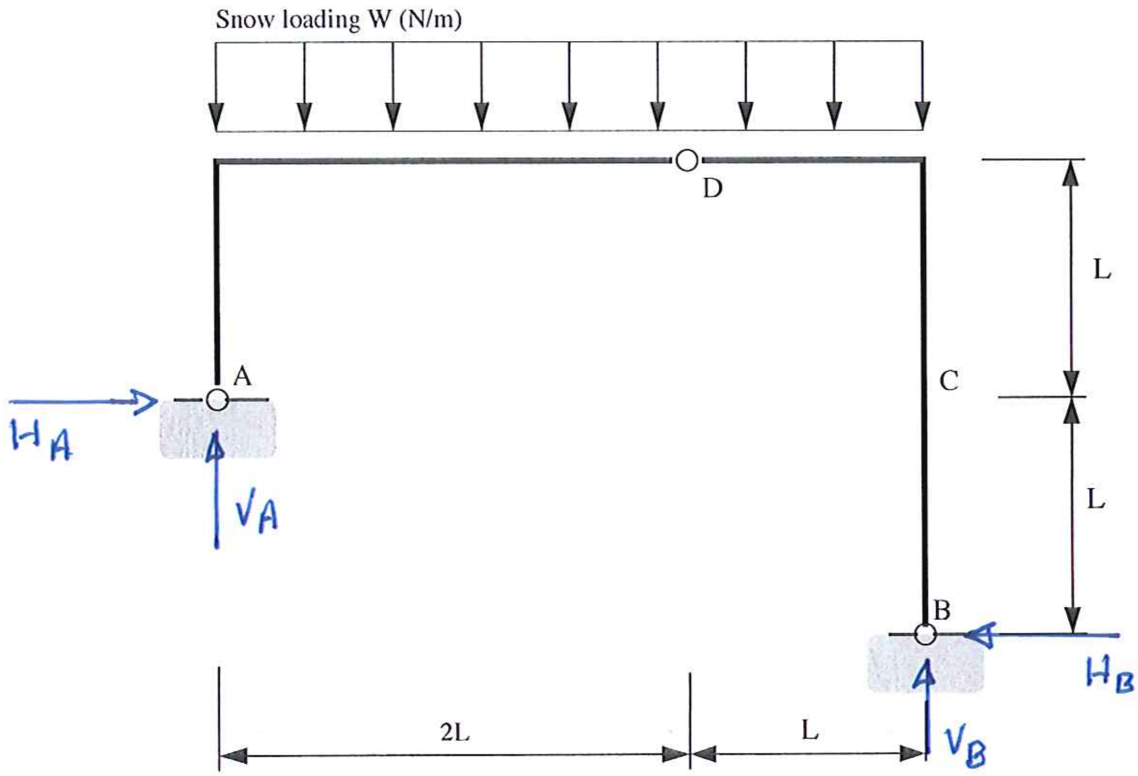

Figure 3: Front elevation view of a three-pinned arch that supports a snow loading.

[3a] (6 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of W and L.

$\sum V = 0 \rightarrow V_A + V_B = 3WL.$ —— (A)

$\sum H = 0 \rightarrow H_A = H_B$ (not useful). —— (B)

$\sum M_D = 0$ (LHS). $(2WL)L + H_A \cdot L = 2L V_A$

$\rightarrow 2WL + H_A = V_A$ —— (C)

$\sum M_D = 0$ (RHS) $(WL)(\frac{L}{2}) + H_B(2L) = V_B L$

$\rightarrow WL + 4H_B = 2V_B.$ —— (D).

Queation 3a continued:

Add Ⓒ + Ⓓ, insert Ⓑ

$$2wL + H_A + wL + 4H_B = 2(V_A + V_B) = 6wL$$

$$\Rightarrow 3wL + 5H_A = 6wL$$

$$\Rightarrow H_A = H_B = \frac{3}{5}wL. \quad \text{— Ⓔ}$$

Plug Ⓔ into Ⓒ & Ⓓ

$$V_A = \frac{13}{10}wL, \quad V_B = \frac{17}{10}wL.$$

Check equilibrium.

$$V_A + V_B = \left(\frac{13}{10} + \frac{17}{10}\right)wL = 3wL \quad ✓$$

[3b] (4 pts) Draw and label the bending moment diagram.