Department of Civil and Environmental Engineering, Spring Semester, 2020

ENCE 353 Midterm 2, Open Notes and Open Book

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Exam Format and Grading. Attempt all three questions. Partial credit will be given for partially correct answers, so please **show all of your working**.

Question	Points	Score
1	15	
2	15	
3	10	
Total	40	

Question 1: 15 points

Analysis of a Supported Cantilever Beam Structure. Figure 1 is a front elevation view of a cantilever beam carrying two external loads P. EI is constant along the cantilever beam.

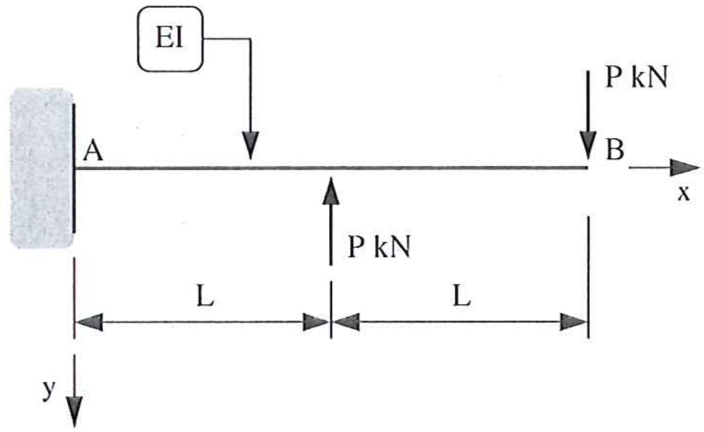
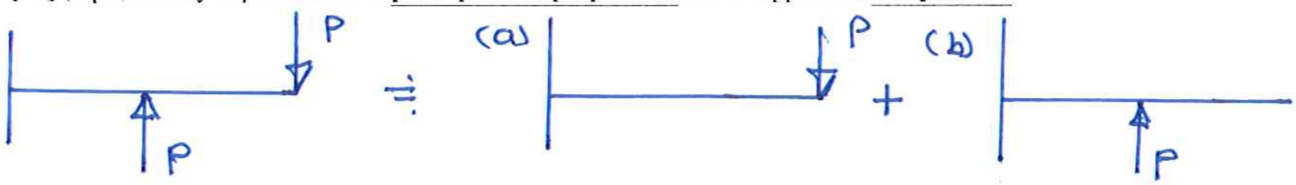


Figure 1: Cantilever beam carrying two applied loads P (kN).

[1a] (3 pts) Briefly explain how the **principle of superposition** can be applied to **this problem**.

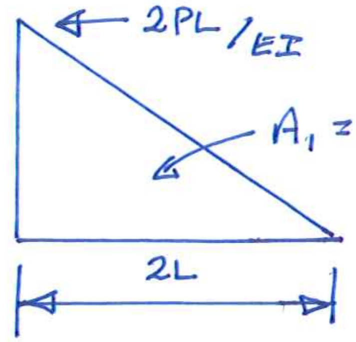


[1b] (5 pts) Use the method of **moment-area** to show that the clockwise rotation of point B is:

$$\theta_B = \left[\frac{3}{2}\right]\frac{PL^2}{EI}. \qquad (1)$$

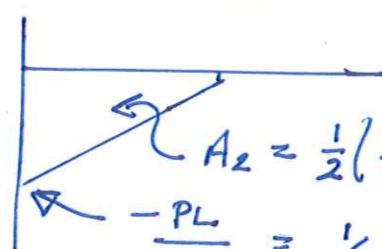
For system (a)

M/EI



$A_1 = \frac{1}{2}\left(\frac{2PL}{EI}\right)(2L) = \frac{2PL^2}{EI}$

For system (b)



$A_2 = \frac{1}{2}\left(\frac{PL}{EI}\right)L = \frac{1}{2}\frac{PL^2}{EI}$

Rotation $\theta_B = A_1 + (-A_2) = \frac{3}{2}\frac{PL^2}{EI}$. ✓

[1c] (7 pts) Use the method of **moment-area** to show that the vertical displacement at B is:

$$y(2L) = \left[\frac{11}{6}\right]\frac{PL^3}{EI}. \tag{2}$$

$y(2L)$ = first moment of area between B - A,

evaluated about B.

For system(a)

$y_a(2L) = A_1 \cdot \bar{x}_1$ $\bar{x}_1 = \frac{2}{3}(2L) = \frac{4L}{3}$

$A_1 = \frac{2PL^2}{EI}$

$= \frac{2PL^2}{EI}\left(\frac{4}{3}L\right) = \frac{8}{3}\frac{PL^3}{EI} \downarrow$

For system (b)

$y_b(2L) = A_2 \bar{x}_2$ $\bar{x}_2 = L + \frac{2}{3}L = \frac{5}{3}L.$

$A_2 = \frac{1}{2}\frac{PL^2}{EI}.$

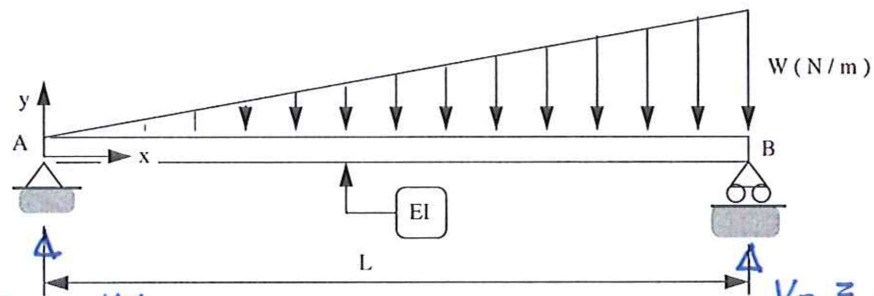
$= \frac{1}{2}\frac{PL^2}{EI}\left(\frac{5}{3}L\right)$

$= \frac{5}{6}\left(\frac{PL^3}{EI}\right) \uparrow$

Net displacement $y(2L) = \left(\frac{8}{3} - \frac{5}{6}\right)\frac{PL^3}{EI} = \frac{11}{6}\frac{PL^3}{EI}.$ ✓

Question 2: 15 points

Elastic Curve for Beam Deflections. Figure 2 is a front elevation view of a simply supported beam that carries a triangular load.



$V_A = \frac{WL}{6}$ $V_B = \frac{WL}{3}$

Figure 2: Simply supported beam carrying a triangular load.

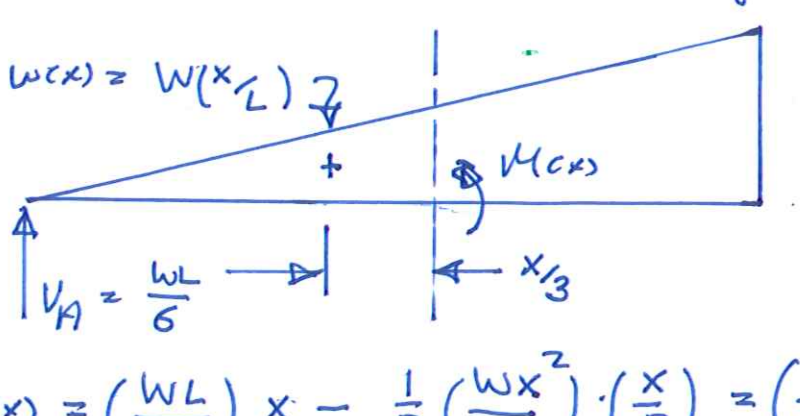
The load increases from zero at point A to W (N/m) at point B. Thus, the total beam loading is WL/2.

[2a] (5 pts). Starting from first principles of engineering (i.e., equilibrium of a substructure extracted from Figure 2), show that the bending moment at point x is:

$$M(x) = \left[\frac{W}{6L}\right] x\left(L^2 - x^2\right). \tag{3}$$

From statics $V_A = \frac{WL}{6}$ & $V_B = \frac{WL}{3}$

Now consider moment at an arbitrary position x.



$$M(x) = \left(\frac{WL}{6}\right) x - \frac{1}{2}\left(\frac{Wx^2}{L}\right)\cdot\left(\frac{x}{3}\right) = \left(\frac{W}{6L}\right) x\left(L^2 - x^2\right).$$

[2b] (5 pts). Show that the elastic curve for beam deflection is given by (notice that in Figure 2, the y axis is pointing upwards):

$$y(x) = \left[\frac{-W}{6LEI}\right]\left[\frac{L^2x^3}{6} - \frac{x^5}{20} - \frac{14L^4x}{120}\right] \tag{4}$$

$$\frac{d^2y}{dx^2} = \frac{-M(x)}{EI} \Rightarrow EI\frac{d^2y}{dx^2} = \frac{-W}{6L}(L^2x - x^3)$$

Integrating twice & applying boundary conditions.

$$y(0) = 0 \rightarrow B = 0$$

$$y(L) = 0 \rightarrow A = \left(\frac{L^5}{20} - \frac{L^5}{6}\right)\frac{1}{L} = \frac{-14}{120}L^4.$$

$$\Rightarrow y(x) = \left(\frac{-W}{6EIL}\right)\left[\frac{L^2x^3}{6} - \frac{x^5}{20} - \frac{14L^4}{120}\right].$$

✓

[2c] (5 pts). Show that the maximum beam curvature occurs at x = $L/\sqrt{3}$.

$$\phi = \frac{M(x)}{EI}. \qquad Max\ \phi \Rightarrow \frac{dM}{dx} = 0$$

$$\Rightarrow L^2 - 3x^2 = 0 \Rightarrow x = L/\sqrt{3}.$$

Question 3: 10 points

Simple Three-Pinned Arch. Figure 3 is a front elevation view of a simple three-pinned arch that carries a total snow loading of 3WL uniformly distributed over its upper section.

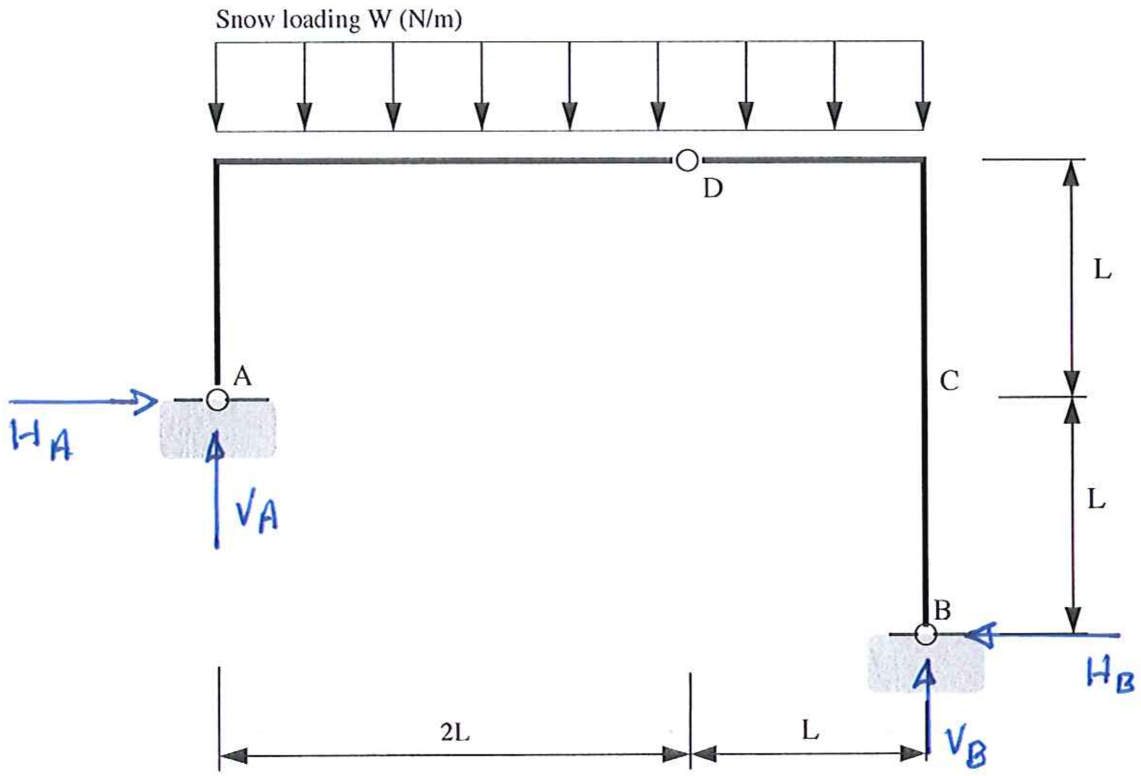


Figure 3: Front elevation view of a three-pinned arch that supports a snow loading.

[3a] (6 pts) Compute the vertical and horizontal components of reaction force at supports A and B as a function of W and L.

$\sum V = 0 \rightarrow V_A + V_B = 3WL.$ —— (A)

$\sum H = 0 \rightarrow H_A = H_B$ (not useful). —— (B)

$\sum M_D = 0$ (LHS). $(2WL)L + H_A \cdot L = 2L\,V_A$

$\rightarrow 2WL + H_A = V_A$ —— (C)

$\sum M_D = 0$ (RHS) $(WL)(\frac{L}{2}) + H_B(2L) = V_B L$

$\rightarrow WL + 4H_B = 2V_B.$ —— (D).

Queation 3a continued:

Add (C) + (D), insert (B)

$$2wL + H_A + wL + 4H_B = 2(V_A + V_B) = 6wL$$

$$\Rightarrow 3wL + 5H_A = 6wL$$

$$\Rightarrow H_A = H_B = \frac{3}{5}wL. \quad \text{— (E)}$$

Plug (E) into (C) & (D)

$$V_A = \frac{13}{10}wL, \quad V_B = \frac{17}{10}wL.$$

<u>Check equilibrium</u>.

$$V_A + V_B = \left(\frac{13}{10} + \frac{17}{10}\right)wL = 3wL \checkmark$$

[3b] (4 pts) Draw and label the bending moment diagram.