Optimization Models for Off-loading Flow Constrained Areas

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Introduction

The Federal Aviation Administration (FAA) and the aviation community within the U.S. have recently raised new operational concepts for identifying, implementing and managing Flow Constrained Areas (FCA) based on Collaborative Routing (CR). The main purpose of the study is to determine how the FAA can meet its objectives while giving NAS users as much flexibility as possible when choosing routes.

FCA

<table>
<thead>
<tr>
<th>Weather block</th>
<th># of flights</th>
<th>Total number of involved flights</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>P</td>
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Given: $P$ total number of involved flights.

$\sum a_i = P$

Decision variables:

$x_i = 1$, if this flight stays in this area, 0 otherwise.

$y_i = \text{number of offloaded flights for airline company } i.$

$r_i = \text{target allocated number of flights for airline company } i.$

Objectives:

(1) Efficiency: Max $\sum x_i$ or Min $\sum (1 - x_i)$

(2) Equity: Min $\sum |y_i - r_i|$

Main Constraints:

Reduced capacity: $\sum \sum x_{j,k} \leq c_k$, $\forall j, \forall k$

Flight conservation: $\sum a_i x_i + y_i = a_i$, $\forall i$

Target allocation: $r_i = a_i \sum y_i / P$, $\forall i$

Multi-Objective Integer Programming Model

Computational Results

- FCA consists of 31 sectors, involving 45 airline companies/classes associated with 283 flights.
- Data has maximum simultaneous number of flights of 7, and the reduced capacity changes from 3 to 5.

Ongoing Research: Stochastic Model

Goal:

- Give flexibilities to airline companies
- Improve system efficiency and take into account the uncertainty and dynamics of demand on FCAs.

Two-stage decision procedure:

- Flexibility: give tokens to airline companies for some flights in the first stage.
- Utilization: assign the rest time resource to maximize system utilization.

Mathematical model:

Decision variables:

$X_f = 1$ if the flight is pre-allowed to use FCA, 0 otherwise

$Y_f = 1$ if the flight is adjusted to pass the FCA under scenario $\phi$ while $X_f = 0$; 0 no adjustment.

Objectives:

(1) Predictability: Max $\sum f x_f$

(2) Utilization: Max $\sum f w_f x_f + \sum f p^\phi y^\phi f$

Main Constraints:

Reduced capacity

\[ \sum_{f \in \phi} x_f + a^\phi y^\phi f \leq c_k \quad \forall k, \forall j, \forall \phi \]

Problem

Given:

- Starting and ending times of FCA
- A set of involved flights $F$
- Entrance and exit times for each flight at each sector
- The reduced sector capacity in this FCA area.

Goal:

Partition the involved flights into two groups:

1. maintain original routes,
2. reroute outside this area

What is a Flow Constrained Area (FCA)

- 3-dimensional volume of airspace associated with a time interval.
- FCAs are defined in response to
  1) severe weather
  2) capacity/demand imbalance due to excess demand.

Reduced capacity during FCA for a sector

Efficient frontiers

Normal

Sum of the deviations

Number of offloaded flights

Reduce by 2
Reduce by 3
Reduce by 4

Efficient frontiers

Alternative Network Flow Model

1. Single sector case

The underlying matrix is totally unimodular (TU) and problem can be efficiently solved as a pure network flow model.

2. Multiple sector case

The underlying matrix is not totally unimodular (TU) but problem can be solved as a network flow model with side constraints.