Resource Allocation in Flow-Constrained Areas with Stochastic Termination Times

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Abstract
In this project we formulate an optimization problem for the assignment of dispositions to flights whose preferred flight plan pass through a flow-constrained area. For each but via a secondary route that avoids the flow-constrained area, or to use the original intended route but with a controlled departure time and accompanying ground delay. We anticipate that the capacity through the flow-constrained area will increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that event. The goal is to minimize the expected cost over the entire distribution of possible capacity increase times.

Model Inputs
- Location of the FCA (Flow-Constrained Area)
- Nominal (good weather) capacity of the FCA
- Reduced FCA (bad weather) capacity of the FCA
- Start time of the AFP (Airspace Flow Program)
- Planned end time of the AFP

Model Outputs
- An initial plan that designates whether a flight is assigned to its primary route or secondary route.
- A recourse action for each flight under each possible early clearance time.

Geometry-Cost analysis for a single flight

Decision Variables for IP Model

- \( L \): length of preferred route (in time)
- \( \theta \): angle of reroute
- \( T \): length of problem time horizon
- \( c_{\text{g}} \): cost per unit time of ground delay
- \( c_{\text{a}} \): cost per unit time of air delay
- \( \alpha \): fixed cost for planning a reroute (CGD)
- \( \beta \): expected cost of GD strategy
- \( \gamma \): cost of CR strategy

Objective Function

Initial plan cost – possible saving under each scenario \( U \)

Main Constraints

FCA capacity constraints under original plan:

- \( \sum \alpha_{	ext{f}} \leq C_{\text{f}} \)
- \( \forall f \)

Under revision, flight can only arrive earlier than \( u \) if it has already departed:

- \( \sum \alpha_{	ext{f}} \leq C_{\text{f}} - u \)
- \( \forall f, v \in \{ \text{FCA} \} \)

Flight on hybrid route cannot arrive at FCA at time \( t \) unless it plus the revision time is less than \( t \):

- \( \sum \alpha_{	ext{f}} \leq C_{\text{f}} - u \)
- \( \forall f, v \in \{ \text{FCA} \} \)

Case Study

- **Short haul**: length - 60 min; origin-to-FCA – 30 min; FCA-to-destination – 30 min; reroute angle – 45° (30/50).
- **Medium haul**: length - 160 min; origin-to-FCA – 90 min; FCA-to-destination – 90 min; reroute angle – 45° (30/50).
- **Long haul**: length - 300 min; origin-to-FCA – 150 min; FCA-to-destination – 150 min; reroute angle – 45° (30/50).

Conclusions and future work

We have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to en route airspace congestion. We have also given the results of an initial computational experiment.

Future steps should include more computational experiments and model refinements aimed at improving the computational performance of the solver program and at exploring the changes in airspace planning the model predicts.

We anticipate the need to provide many refinements and extensions to this model to better address practical problem solving.