Models for Geometric Composability of Engineered Physical Systems

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Thesis of this Colloquium

- Geometric **composability** is enabled by geometric **interchangeability** in engineered physical systems.
  - In fact, they are synonymous in this context.

- Engineered physical systems are fundamentally different from software systems.
  - You cannot manufacture identical copies of physical components.
  - Two physical components are interchangeable if they have the same ‘form, fit, and function.’
    - Geometry and materials are the two major deciding factors.
Dimensioning and Tolerancing an Industrial Part in an Engineering Drawing
Dimensioning and Tolerancing an Industrial Part in a 3D Geometric Model
Plato’s Theory of Forms
In Plato’s Seventh Letter (1/3)

- For everything that exists there are three instruments by which the knowledge of it is necessarily imparted; fourth, there is the knowledge itself, and, as fifth, we must count the thing itself which is known and truly exists. The first is the name, the second the definition, the third the image, and the fourth the knowledge.

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A circle is a thing spoken of, and its name is that very word which we have just uttered.

The second thing belonging to it is its definition, made up names and verbal forms. For that which has the name ‘round,’ ‘annular,’ or, ‘circle,’ might be defined as that which has the distance from its circumference to its centre everywhere equal.

Third, comes that which is drawn and rubbed out again, or turned on a lathe and broken up - none of which things can happen to the circle itself - to which the other things, mentioned have reference; for it is something of a different order from them.
Fourth, comes knowledge, intelligence and right opinion about these things. Under this one head we must group everything which has its existence, not in words nor in bodily shapes, but in souls—from which it is dear that it is something different from the nature of the circle itself and from the three things mentioned before.

Of these things intelligence comes closest in kinship and likeness to the fifth, and the others are farther distant.
Jeffersonian Assembly

“An improvement is made here in the construction of muskets, which it may be interesting to Congress to know, should they at any time propose to procure any. It consists in the making every part of them so exactly alike, that what belongs to any one, may be used for every other musket in the magazine…. Supposing it might be useful to the United States, I went to the workman. He presented me the parts of fifty locks taken to pieces, and arranged in compartments. I put several together myself, taking pieces at hazard as they came to hand, and they fitted in the most perfect manner. The advantages of this, when arms need repair, are evident.”
Interchangeable Manufacture

- 1765 Système Gribeauval
- 1801 Eli Whitney
- 1803 Marc Isambard Brunel
- 1820’s American System (Armory)
- Clocks, sewing machines, reapers, bicycles …
- Henry Ford’s automobiles …
American System

- A ‘rational’ jigs, fixtures, and gauging system; it is *rational* because it is based on a model.
  - Radical departure from ‘file & fit’ craftsmanship.
- All fixtures are designed with reference to the model; gauges (for inspection) were made based on the model.
- Henry Ford: “In mass production there are no fitters.”
Engineering Drawings

- First American national standard appeared in 1935; ISO standards after WW II.
- Drawings (with dimensions and tolerances) defined the models.
- 3D geometric models with dimensions and tolerances first appeared in mid-1990s.
Role of Congruence Theorems in Dimensioning
How would you dimension a triangle?

…and, by the way, how would you parameterize it?

Euclid’s Elements
Book I Prop. 4
(side-angle-side)

Euclid’s Elements
Book I Prop. 26
(angle-side-angle)

Euclid’s Elements
Book I Prop. 8
(side-side-side)

Aha! “Congruence theorems may provide the basis for a theory of dimensioning”
about the book . . . 

"Vijay Srinivasan’s *Theory of Dimensioning* is an important and needed book. It brings together in a unified setting the clumps of algebraic geometry, classical mechanics, and solid modeling that are used today by CAD-system designers and taught to (some) engineering and computer science graduate students, and it adds a major new component—a symmetry-group classification of surfaces that provides powerful tools for formulating key relational properties of geometric elements. The book grew out of Dr. Srinivasan’s early involvement in mechanical design and CAD at IBM, and his later immersion in the European and American efforts aimed at ‘mathematizing’ and generalizing mechanical tolerancing. The book could be viewed as a ‘status report’ from the research frontiers, but it actually offers much more than that. Because it is clearly and gracefully written, with many examples and exercises, it will almost certainly be used as a university text, and as a standard source for researchers.” — Professor Herbert B. Voelcker, Cornell University, U.S.A.

"This book fills a major void in the engineering literature. By contrast with other books on dimensioning, it is based on a recently developed elegant and unified theory. The treatment is logical and complete, requiring as prerequisites only some background in engineering design and a modest level of mathematical competence. The book is clearly written, and provides many illustrative examples. It is equally suitable for undergraduate and graduate students of engineering and computer science, and for professional engineers concerned with dimensions and tolerances.” — Dr. Michael J. Pratt, LMR Systems, U.K.

"Vijay Srinivasan has provided an extremely impressive unification of the conceptual and mathematical foundations of dimensioning. This book is indispensable for all the major disciplines that use dimensioning. Most importantly, Srinivasan’s exposition is deep, systematic, and thorough. I strongly recommend this book to both students and advanced experts. It is a true milestone.” — Professor Michael Leyton, Rutgers University, U.S.A.

about the author . . . 

Vijay Srinivasan is PLM Architect, IBM Corporation, White Plains, New York, and Adjunct Professor, Mechanical Engineering Department, Columbia University, New York, New York. He is a member of several national and international standards committees in the areas of product specification, verification, and data exchange, and has published extensively on these topics.

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Two Basic Axioms ...

- **Axiom of manufacturing imprecision:**
  - All manufacturing processes are inherently imprecise and produce parts that vary.

- **Axiom of measurement uncertainty:**
  - No measurement can be absolutely accurate, and with every measurement there is some finite uncertainty about the measured value or measured attribute.

These are independent axioms and both should be considered operative in any real situation.
Characteristics of Interchangeability

Even with the inevitable geometric variability, interchangeable parts belong to an ‘equivalence class’:

1. **reflexive**, i.e., A is interchangeable with A,

2. **symmetric**, i.e., if A is interchangeable with B, then B is interchangeable with A, and

3. **transitive**, i.e., if A is interchangeable with B and B is interchangeable with C, then A is interchangeable with C.
Dimensioning and Tolerancing an Industrial Part in a 3D Geometric Model
### FIG. 5-7 SPECIFYING FLATNESS

This on the drawing

Means this

The surface must lie between two parallel planes 0.25 apart. The surface must be within the specified limits of size.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition of the tolerance zone</th>
<th>Indication and explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.2 Flatness tolerance (see ISO/TS 12781-1 and ISO/TS 12781-2)</td>
<td>The tolerance zone is limited by two parallel planes a distance ( t ) apart.</td>
<td>The extracted (actual) surface shall be contained between two parallel planes 0.08 apart.</td>
</tr>
</tbody>
</table>

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**ISO 1101: 2004**

**ASME Y14.5 - 2009**
5.4.2.1 SPECIFYING FLATNESS

This on the drawing

\[ \exists r \in R^3 \times SO(3) \; \exists \; F \subset rT \]

\[ \iff \]

width(\(F\)) \(\leq t\)

FIG. 5-7 SPECIFYING FLATNESS

The surface must lie between two parallel planes 0.25 apart. The surface must be within the specified limits of size.

The extracted (actual) surface shall be contained between two parallel planes 0.08 apart.
Reflections on the role of science in the evolution of dimensioning and tolerancing standards
Composability & Interchangeability

Geometric interchangeability enables geometric composability:

- Modularity: Each part in the assembly is designed so that interchangeability is ensured.

- State-independence: Each part can be manufactured independently and then assembled.
Validation and Verification

For geometric interchangeability:

- **Validation** of models: Tolerance analysis and synthesis.
- **Verification** of manufactured piece for conformance to specifications: Dimensional and geometric metrology
Interchangeability – in general

Geometry is only one aspect of interchangeability of engineered physical systems.

Other aspects include materials, processes (e.g., heat treatment, annealing, anodizing, carburizing) etc.

Modern composite structures are a (complex) combination of geometry and materials.

Variability is inherent in all these aspects of interchangeability.
Product platforms (aka product families) are architected first.

Common components, subsystems, and their interfaces are defined in these architectures using simplified geometric models.

- Some standards arise at the (geometric) interfaces.
- Other standards characterize the interior bulk properties.

Physical systems are **not** architected as ‘stacks’.
Is there a Royal Road?

- **Story**: When King Ptolemy asked if there was a shorter path to learning geometry than Euclid's *Elements*, Euclid replied “there is no royal road to geometry.”

- There is no ‘royal road’ to geometric composability either – it has to be learned through years of diligent study and practice.
Inscription at the Entrance to Plato’s Academy in Athens

Αγεωμέτρητος Μηδείς Εισίτω

A moto worth adopting for Model-based Systems Engineering!
Thank You!