Advanced Stochastic Network Queuing Models of the Impact of 4D Aircraft Trajectory Precision

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Background-Motivation

Research Objectives

- Develop Queuing Models that Predict Benefit of Increased Aircraft Trajectory Precision
  - Reduced inter-arrival time
  - Reduced variation in inter-arrival time
  - Reduced service time
  - Reduced variation in service time
  - Increased number of servers

- Develop Modeling and Visualization Environment to Allow
  - Validation of Queuing Model Results Against Simulation
  - Visualization of Benefit Mechanisms
  - Validate Proposed Queuing Models
  - Apply Validated Models to Next Generation Air Transportation System (NGATS) Concepts

- Visualization Interactive Tool to Visually Evaluate the Queue Size and Average Delay for Aircraft in Queue

- Delay Savings Use of Queuing Models to Estimate Delay Savings From 4D Trajectory Precision

Modeling the Levels of Aircraft Trajectory Uncertainty

- Low Precision Case: Stochastic Queuing Models
  - Arrivals are time-dependent Poisson process
  - Service times are time-dependent Erlang k process
  - Assume n servers
  - Kendall notation: (M(t)/E(k)(t)/n)

- High Precision Case: Deterministic Queuing Models
  - Given
    - Arrival schedule (aggregate or disaggregate)
    - Capacity or deterministic minimum headways
  - Construct cumulative arrival and departure curves to obtain
    - Delay and queue length by time of day
    - Average and total delay

- Intermediate Case: Diffusion Approximation
  - Dynamics of joint probability density functions are analogous to dynamics of physical flows or other density problems
  - Continuous approximations using systems of coupled partial differential equations
  - Because derivatives of probability density functions are modeled, they can be integrated to produce moment estimates

Model Development

- Necessary assumptions made:
  - Continuity; the queue length measurement at any time needs not be integer
  - Markovian; the changes of queue lengths are independent of the prior queue states
  - 2nd order approximation; the transition density function g can be captured in its first and second moments

- Diffusion Approximation to a Single Airport

  \[ f(x,t) = \text{density of length of queue at time } t \]

  \[ g(\delta x, x, t) = \text{probability density of change from } x \text{ to } x + \delta x \text{ at time } t \]

  \[ g(x,t) = \text{mean of transitions at } x \text{ from } t \text{ to } t + \Delta t \]

  \[ \frac{\partial}{\partial t} f(x,t) - \frac{1}{2} \frac{\partial^2}{\partial x^2} f(x,t) + \int_0^x (x,t,\lambda) f(x,t) \, dx = \frac{d}{dx} M(x,t) f(x,t) \]

  \[ \text{Kolmogorov forward equation} \]

- Initial and Boundary Conditions
  - Queue starts empty at the beginning and ends up empty at the end of the day
  - Prevent negative queue lengths with a reflecting barrier:

  \[ f(0,t)M(0,t)M(t) = 0, \quad t > 0 \]

- Examples of Capacity Scenarios at San Francisco Airport

  - Computed delays with the stochastic and deterministic models for each airport, under each capacity scenario, for various peak and non-peak days

  - Diffusion Approximation to a Single Airport

  - The comparison of the stochastic and the deterministic models quantifies the resulting decrease of delays due to the Increase of 4DT Trajectory precision: Approximately 10% reduction from stochastic