Generalized Washout Filters for Control of Nonlinear Systems

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Introduction and Motivation

- A washout filter is a high pass filter that washes out (rejects) steady state inputs, while passing transient inputs.
- Feedback through washout filters does not move the equilibrium points of the open-loop system.
- Washout filters facilitate automatic following of a targeted operating point.
- Although washout filters have been successfully used in many control applications, there is no systematic way for choosing the constants of the washout filters and the control parameters.

- We introduce generalizations of washout filter-aided feedback that overcome the limitations of washout filters and at the same time maintain their benefits.

Washout Filters

In discrete-time, the transfer function of a typical washout filter takes the form (Abed, Wang & Chen 1994)

\[ T(z) = \frac{Y(z)}{X(z)} = \frac{z-1}{z-(1-d)} \]

In the time domain, the dynamics of a washout filter can be written as

\[ z(k+1) = x(k) + (1-d)z(k), \]

along with the output equation

\[ y(k) = x(k) - dz(k). \]

\( d \) is the washout filter constant (at 0.2) for a stable filter.

The control input is taken to be a function of \( y \).

Linear Feedback through Washout Filters

Consider the nonlinear system

\[ x(k+1) = f(x(k), u(k)) \] (1)

Suppose that \( x_o \) is an unstable operating condition for system (1). In a small neighborhood of \( x_o \), system (1) can be rewritten as

\[ z(k+1) = Ax(k) + Bu(k) + h(x(k), u(k)) \]

where \( z \) now denotes the state vector referred to \( x_o \). \( u \) is a scalar input, \( A \) is the Jacobian matrix of \( f \) evaluated at \( x_o \), and \( h(.) \) represents higher order terms.

The dynamic equations of the washout filters can be written as

\[ z_i(k+1) = (1-d_i)z_i(k) + \sum_{j=1}^{m} c_{ij}x_j(k) \] (2)

where \( z_i \) is the state of the \( i \)-th washout filter, \( i=1, \ldots, m \), and \( m \) \( \leq n \).

There is no systematic way for choosing the constants of the washout filters and the control gain.

Generalization of Discrete-Time Washout Filter-Aided Feedback

- We introduce a generalization of washout filter-aided feedback in which the individual washout filters are coupled through a constant coupling matrix.
- Feedback through generalized washout filters is shown to overcome the limitations of washout filters and at the same time maintain their benefits.

The generalized washout filter-aided feedback results in the closed-loop system

\[
\begin{align*}
x(k+1) &= Ax(k) + Bu(k) + h(x(k), u(k)) \\
z(k+1) &= Px(k) + (I - P)z(k) \\
u(k) &= G(x(k) - z(k))
\end{align*}
\]

where \( P \) is a nonsingular matrix, and \( G \) is a feedback gain matrix. The linearization of the closed-loop system can be written as

\[
\begin{bmatrix}
x(k+1) \\
z(k+1)
\end{bmatrix} =
\begin{bmatrix}
A + BG & -BG \\
P & I - P
\end{bmatrix}
\begin{bmatrix}
x(k) \\
z(k)
\end{bmatrix}
\]

Proposition: Consider the closed-loop system with generalized washout filter-aided feedback. Suppose that the matrix \( I-A \) is nonsingular. Suppose also that the pair \((A,B)\) is stabilizable. Then there exists a nonsingular \( P \) and a \( G \) such that the eigenvalues of \( A_c = A + BG \) are in \( \mathbb{C} \) and \( \Re \lambda_1 > 1 \).

The goal is to stabilize the unstable fixed point \( x_o = (0.6314, 0.6314) \).

Example 1

Consider the two-dimensional example

\[
\begin{bmatrix}
x_1(k+1) \\
x_2(k+1)
\end{bmatrix} =
\begin{bmatrix}
1.9 & 0.5 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
x_1(k) \\
x_2(k)
\end{bmatrix} -
\begin{bmatrix}
x_2^2(k) \\
0
\end{bmatrix} +
\begin{bmatrix}
1 \\
0
\end{bmatrix} u(k)
\]

The unforced system \( (u=0) \) displays chaotic motion. The fixed point \( (0,0) \) is unstable: the eigenvalues of the linearization at the origin are \( \lambda_1 = 2.1343 \) and \( \lambda_2 = -0.2343 \). Since \( \lambda_1 > 1 \), the origin cannot be stabilized using stable washout filters nor by using delayed feedback control.

Feedback through generalized washout filters:

\[ G = \begin{bmatrix} -1.6343 & -0.7657 \end{bmatrix}, \quad P = \begin{bmatrix} 0.1(A - I)^{-1}(A + BG - I) \end{bmatrix} = \begin{bmatrix} -0.01674 & -0.05469 \\ -0.05837 & 0.07265 \end{bmatrix}. \]

The eigenvalues of \( A_c \) are \{-0.2343, 0.7277, 0.8164, 0.9\}.

Example 2: Henon Map

Consider the Henon map

\[
x_1(k+1) = 1 - 1.4x_1^2(k) + 0.3x_2(k) + u(k) \\
x_2(k+1) = x_1(k)
\]

The control is applied when the trajectory of the open-loop system enters the neighborhood \( \{|x-x_o| < 0.15\} \) of the origin.

Feedback through generalized washout filters:

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