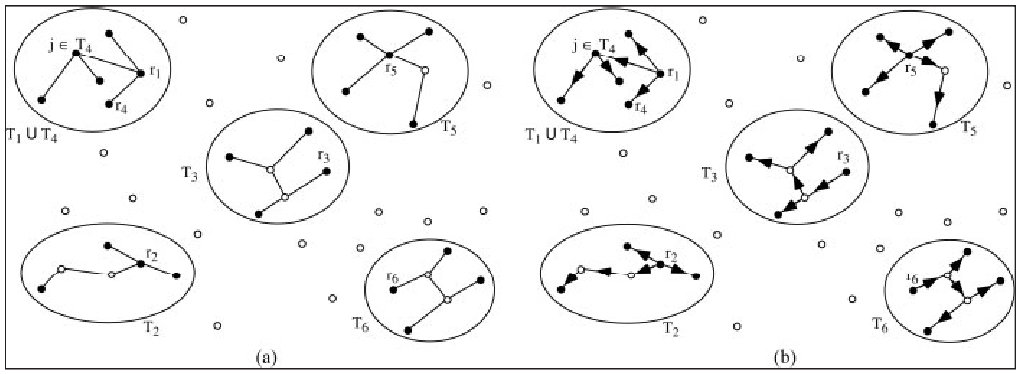
RESEARCH BRIEF

STRONG FORMULATIONS FOR NETWORK DESIGN PROBLEMS WITH CONNECTIVITY REQUIREMENTS

The challenge

Network Design Problems with Connectivity Requirements (NDC) arise in a wide variety of application domains including VLSI design and telecommunication network design. The increasing reliance on communication networks (and expectations of a digital future) places an enormous importance on the reliability of such networks. Given the enormous bandwidth capabilities of communication networks, and the increasing array of services provided over them, the failure of any link in such a network can have significant, perhaps even catastrophic consequences.



Example of the directing procedure in the Steiner forest problem. Filled nodes are required nodes, unfilled nodes are Steiner nodes. (a) Undirected forest. (b) Direct each forest away from the lowest indexed root node that it contains.

The potential

This research will enable organizations to design reliable networks more rapidly and reliably. The formulations developed in this research allow for the more efficient solution of large-scale problems and can also be used to benchmark heuristic procedures for the design of reliable networks.

The research

Dr. S. Raghavan and Dr. Thomas Magnanti studied the problem of designing a minimum cost network that enforces lower bounds on the number of edge-disjoint paths joining pairs of nodes in the network. Such a problem arises in important application areas where redundancy is crucial, such as the aforementioned VLSI and telecommunications network designs.

The researchers combined graph theory insights with robust modeling ideas to provide improved formulations for network design problems, either by reducing their size or by strengthening the quality of the associated linear programming relaxation.

NDC includes as special cases a wide variety of celebrated combinatorial optimization problems including the minimum spanning tree, Steiner tree, and survivable network design problems.

For "unitary" problems, in which solutions must be connected networks, Raghavan and Magnanti project out two classes of valid inequalities (partition inequalities and combinatorial design inequalities) that generalize known classes of valid inequalities for the Steiner tree problem. The result is a new directed formulation for the unitary NDC problem that is stronger than a natural undirected formulation.

They also provide a strengthened, directed model for "nonunitary" problems (such as the Steiner forest problem) and obtain an improved flow formulation for these more general problems.

Their results provide a unifying framework for strengthening formulations for NDC problems, and demonstrate the power of flow-based formulations for network design problems with connectivity requirements.

Research team

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Awards

This research won the Glover-Klingman prize from the journal *Networks* for the best paper published in the journal in 2005; the article appeared in the March 2005 issue. In part, the citation notes, "This paper stands out for its unifying perspective and its substantial contributions to understanding the role of flow formulations in the effective solution of an important class of combinatorial optimization problems."

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Links

"Strong formulations for network design problems with connectivity requirements," published in *Networks*, March 2005: *www3.interscience.wiley.com/cgi-bin/abstract/109863417/ABSTRACT.* Note: *Networks* charges a fee to download the article.