A Convex Parameterization of All Stabilizing Controllers for Non-strongly Stabilizable Plants, under Quadratically Invariant Sparsity Constraints

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Decentralized Control via Sparsity Constraints

We are dealing with the case of Linear, Time Invariant, Dynamical Systems for which we propose a convex–programming based, design method of Optimal, Decentralized Controllers. The decentralized setting is modeled via the sparsity constraints paradigm, meaning that the decentralized controller consists of interconnected blocks having access only to certain measurements.

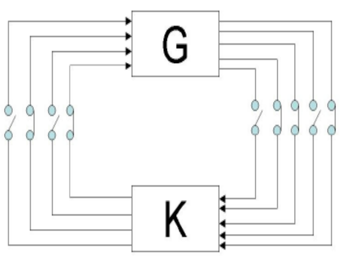

Figure 1: The Closed Feedback Loop of Plant G and Controller K

What Has Been Done

What has been done: For a large class of decentralized configurations that have a particular structural property called *quadratic invariance* the problem is readily proved tractable via convex programming ([2]) under the restrictive hypothesis that the plant is *strongly stabilizable*.

The Decentralized, Optimal Control Problem

The Decentralized, Optimal Control Problem: Minimize a (given) norm of the lower linear fractional transformation of some generalized plant, subject to internal stability and a subspace constraint on the allowable controllers (denoted here by $\mathcal{S}$), on the account that $K \in \mathcal{S}$ is a compact way to formally describe the decentralized constraint on the controllers.

$$\min_{\substack{K \text{ stabilizes } P \\ K \in \mathcal{S}}} \left\| P_{11} + P_{12}K\left(I - GK\right)^{-1}P_{21} \right\| \qquad (1)$$

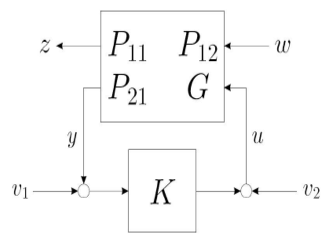

Figure 2: Feedback Interconnection of a Generalized Plant and Its Controller

The set $\mathcal{S}$ is completely defined by the sparsity pattern of the controller, that is by which of the sub–blocks of G are imposed to be identically zero.

What's New?

What's new: We have extended the techniques from ([2]) to the most general case of arbitrary stabilizable (not necessary strongly stabilizable) plants.

The Idea

The Idea: We make use of an alternative Youla–like parametrization which doesn't directly provide the set of all stabilizing controllers, but instead gives a parametrization of all the closed loop transfer–functions achievable with LTI, stabilizing controllers.

Theorem([1]. The set of all stable closed loop transfer-functions achievable via a LTI controller is given by

$$\Omega(Q) \stackrel{def}{=} \left(H(P,K_0) - \begin{bmatrix} I_{n_y} & O \\ O & O \end{bmatrix}\right) Q \left(H(P,K_0) - \begin{bmatrix} O & O \\ O & I_{n_u} \end{bmatrix}\right) + H(P,K_0). \qquad (2)$$

where K_0 is *any* stabilizing controller, while the Youla–like parameter Q, runs through the set of stable transfer function matrices.

The alternative parametrization above, is the gist in coming up with a change of variables that allows to recast problem (1) as

$$\min_{Q \in \mathbb{Q}} \left\| T_1 - T_2 Q T_3 \right\|. \qquad (3)$$

where the set $\mathbb{Q}$ is solely defined by a certain sparsity pattern.

The equivalent convex problem (3) can be solved with the already available tools developed for solving the strongly–stabilizable case ([2]).

References

[1] K. Mori "Relationship Between Standard Control Problem and Model–Matching Problem Without Coprime Factorizability", *IEEE Trans. Aut. Control*, Vol. 49, No.2, 2004. (pp. 230-233)

[2] M. Rotkowitz, S. Lall "A Characterization of Convex Problems in Decentralized Control", *IEEE Trans. Aut. Control*, Vol.51, No.2, 2006. (pp. 274-286)