Scheduling Around Weather Reduced Capacity

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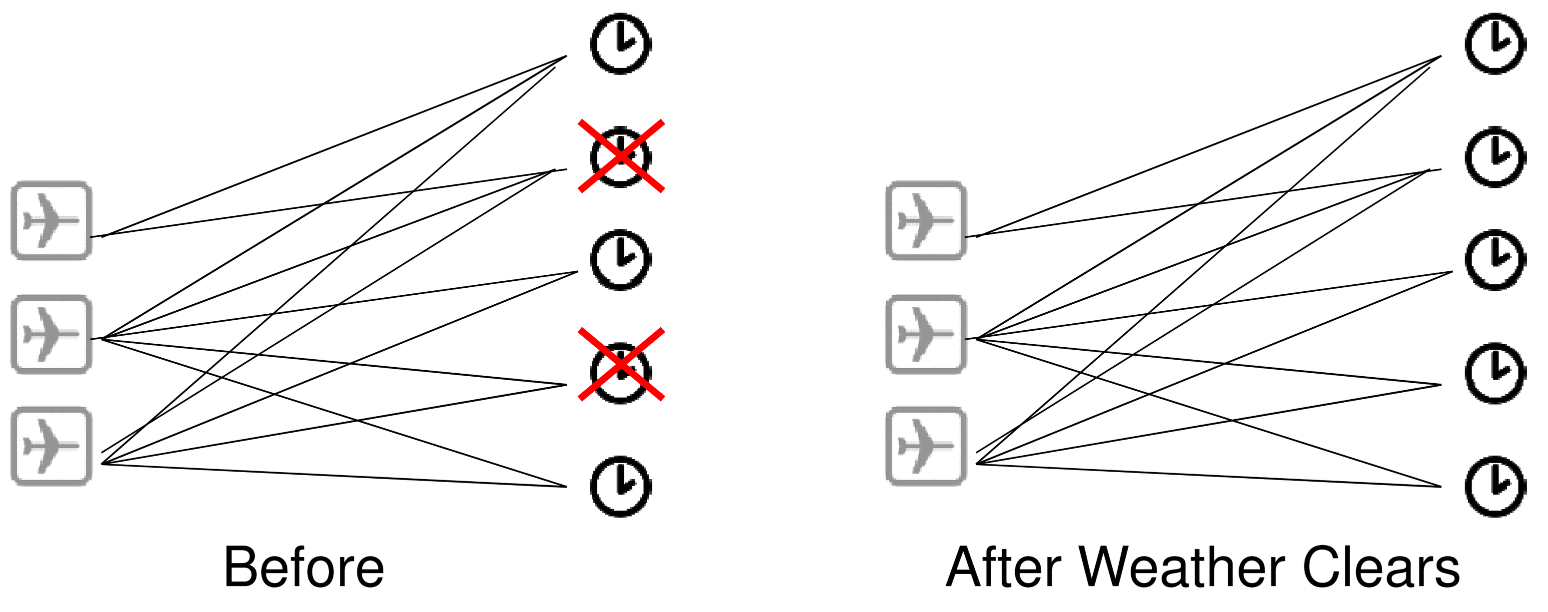
We need to assign flights to a reduced number of slots, with the knowledge that the number of slots will be increased at a later time, but this time of increased capacity is uncertain.



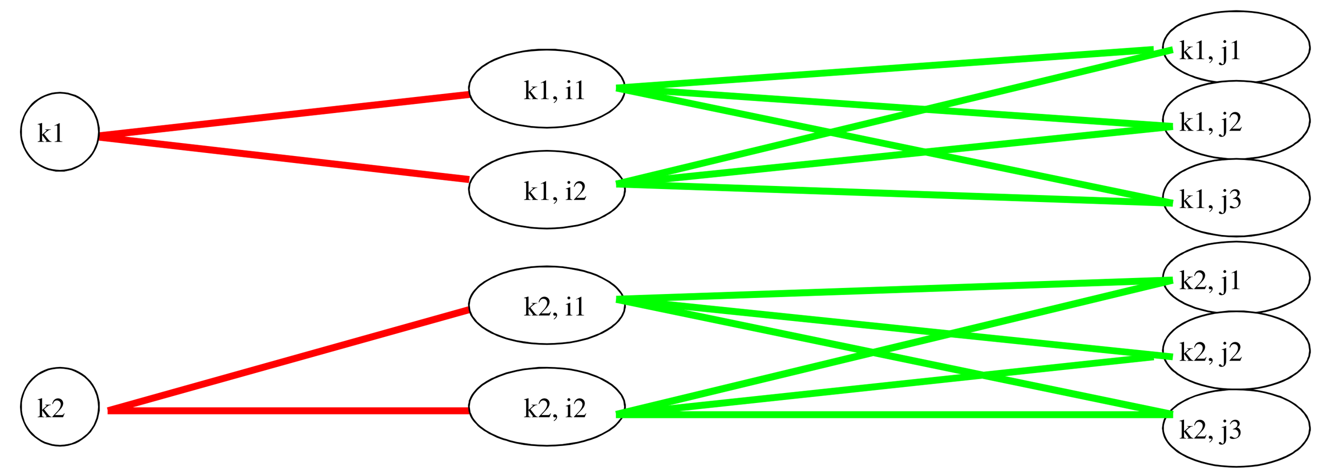
How can we formulate this problem as an integer program?

What if there are multiple ways of formulating this problem?

If we have different formulations, can we show that one is better than the other?

We can treat the time of capacity increase as a stochastic variable and formulate this as a two stage linear program.

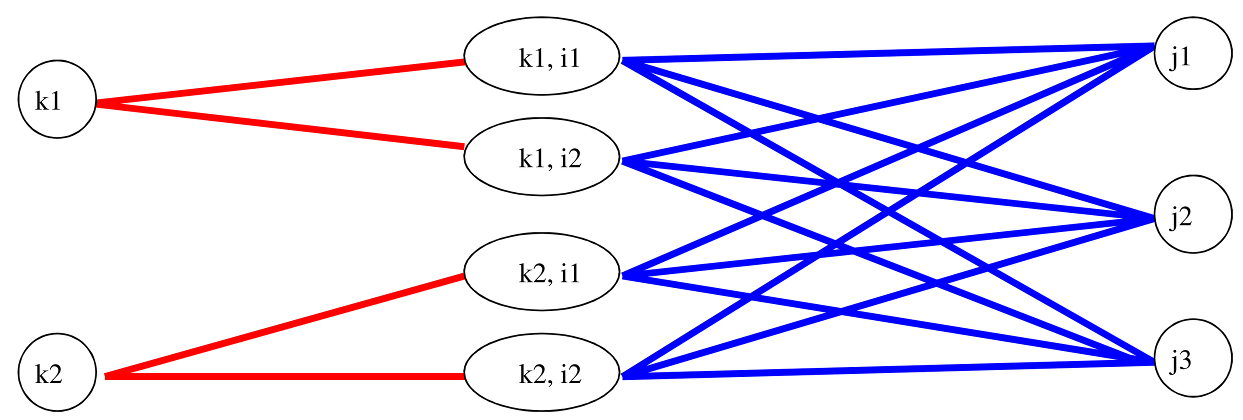
- The first stage is the initial assignment (before the weather clears)
- The second stage is the reassignments (given a probability distribution of weather clearance times).



Item Based Formulation 1

Each stage 1 flight can be reallocated in stage 2.

$$
\begin{aligned}
\sum_i x_{k,i} &= 1 \text{ for each k} \\
\sum_k x_{k,i} + x_{s,i} &= 1 \text{ for each i} \\
\sum_i x_{s,i} &= |I| - |K| \\
\sum_j y_{k,j,t} &= 1 \text{ for each k, t} \\
\sum_k y_{k,j,t} + y_{s,j,t} &= 1 \text{ for each j, t} \\
\sum_j y_{s,j,t} &= |J| - |K| \\
y_{k,j,t} - \sum_{i \in I_{k,j,t}} x_{k,i} &\leq 0 \text{ for each k, j, t}
\end{aligned}
$$



Allocation Based Formulation

Each stage 1 arc can be reallocated in stage 2.

$$
\begin{aligned}
\sum_i x_{k,i} &= 1 \text{ for each flight } k \\
\sum_k x_{k,i} + x_{s,i} &= 1 \text{ for each slot } i \\
\sum_i x_{s,i} &= |I| - |K| \\
\sum_j w_{k,i,j,t} + w_{k,i,p,t} &= 1 \text{ for each } k, i, t \text{ (and } s, i, t) \\
\sum_k \sum_i w_{k,i,j,t} + w_{s_t,j,t} &= 1 \text{ for each } j, t \\
\sum_j w_{s_t,j,t} &= |J| - |K| \text{ for all } t \\
\sum_k \sum_i w_{k,i,p,t} &= |X| - |K| \\
w_{k,i,p,t} + x_{k,i} &= 1 \text{ for each } k, i, t
\end{aligned}
$$

Which is Better?

If we have different formulations, can we show that one is better than the other?

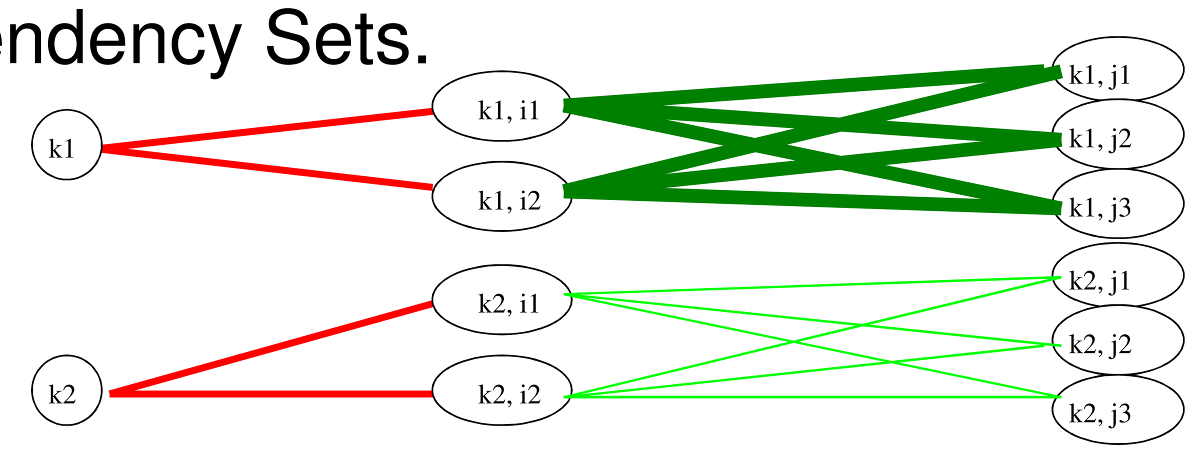
x_{ki}	$y_{k,j1,t}$	$y_{k,j2,t}$
-1	1	0
-1	0	1
0	1	1

This is a sub-matrix of a feasible solution to Item Based Formulation 1 that is not feasible to the Allocation Based formulation.

This shows that the Allocation based formulation is stronger than the Item Based Formulation 1.

Item Based Formulation 2

Similar to Item Based Formulation 1, but with Dependency Sets.



$$
\begin{aligned}
\sum_i x_{k,i} &= 1 \text{ for each k} \\
\sum_k x_{k,i} + x_{s,i} &= 1 \text{ for each i} \\
\sum_i x_{s,i} &= |I| - |K| \\
\sum_j y_{k,j,t} &= 1 \text{ for each k, t} \\
\sum_k y_{k,j,t} + y_{s,j,t} &= 1 \text{ for each j, t} \\
\sum_j y_{s,j,t} &= |J| - |K| \\
\sum_{D(y)=I_{k,j,t}} y_{k,j,t} - \sum_{i \in I_{k,j,t}} x_{k,i} &\leq 0 \text{ for each } I_{k,A,t}
\end{aligned}
$$