Resource Allocation in Flow-Constrained Areas with Stochastic Termination Times

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Abstract

In this project we formulate an optimization problem for the assignment of dispositions to flights whose preferred flight plans pass through a flow-constrained area. For each flight, the disposition can be either to depart as scheduled but via a secondary route that avoids the flow-constrained area, or to use the originally intended route but to depart with a controlled departure time and accompanying ground delay. We anticipate that the capacity through the flow-constrained area will increase at some future time once the weather activity clears. The model is a two-stage stochastic program that represents the time of this capacity windfall as a random variable, and determines expected costs given a second-stage decision, conditioning on that time. The goal is to minimize the expected cost over the entire distribution of possible capacity increase times.

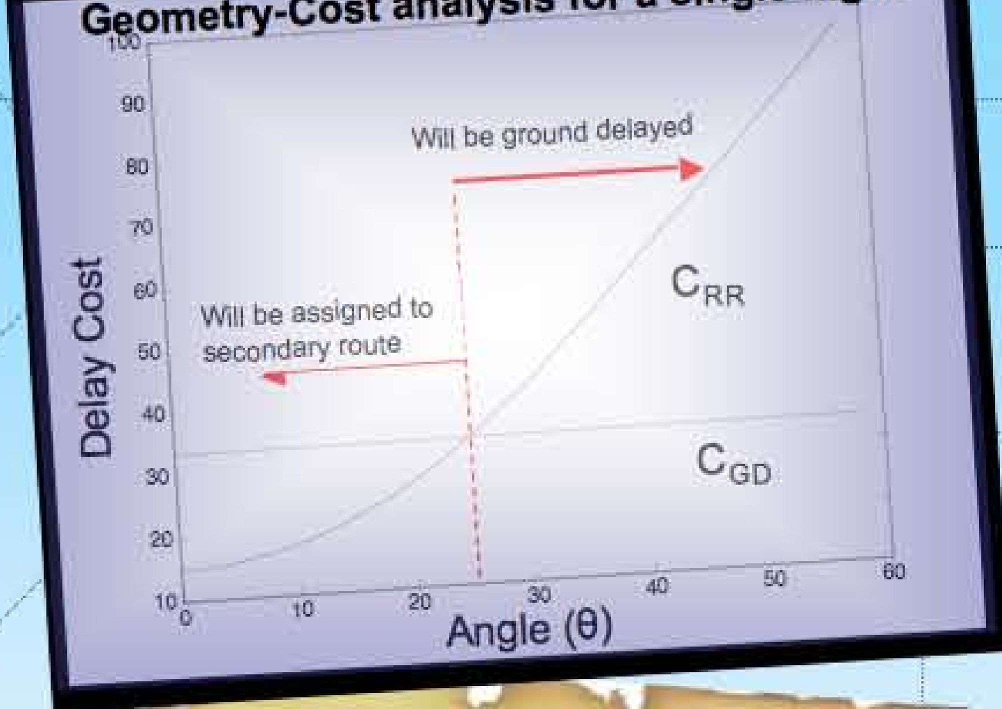
Model Inputs

- *Location of the FCA (Flow-Constrained Area)*
- *Nominal (good weather) capacity of the FCA*
- *Reduced FCA (bad weather) capacity of the FCA*
- *Start time of the AFP (Airspace Flow Program)*
- *Planned end time of the AFP*

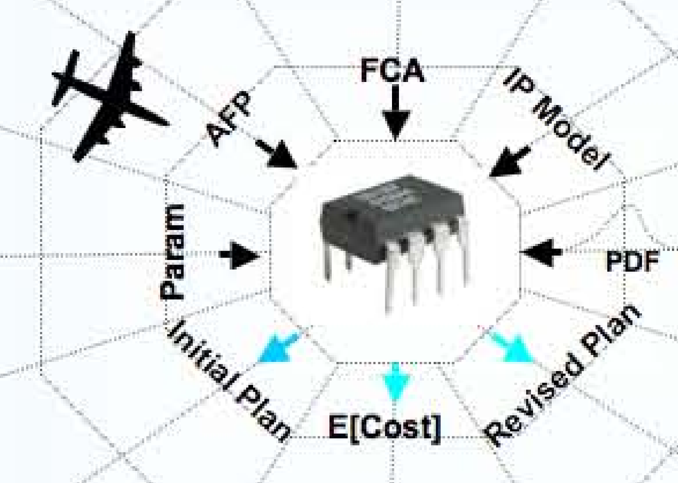
Model Outputs

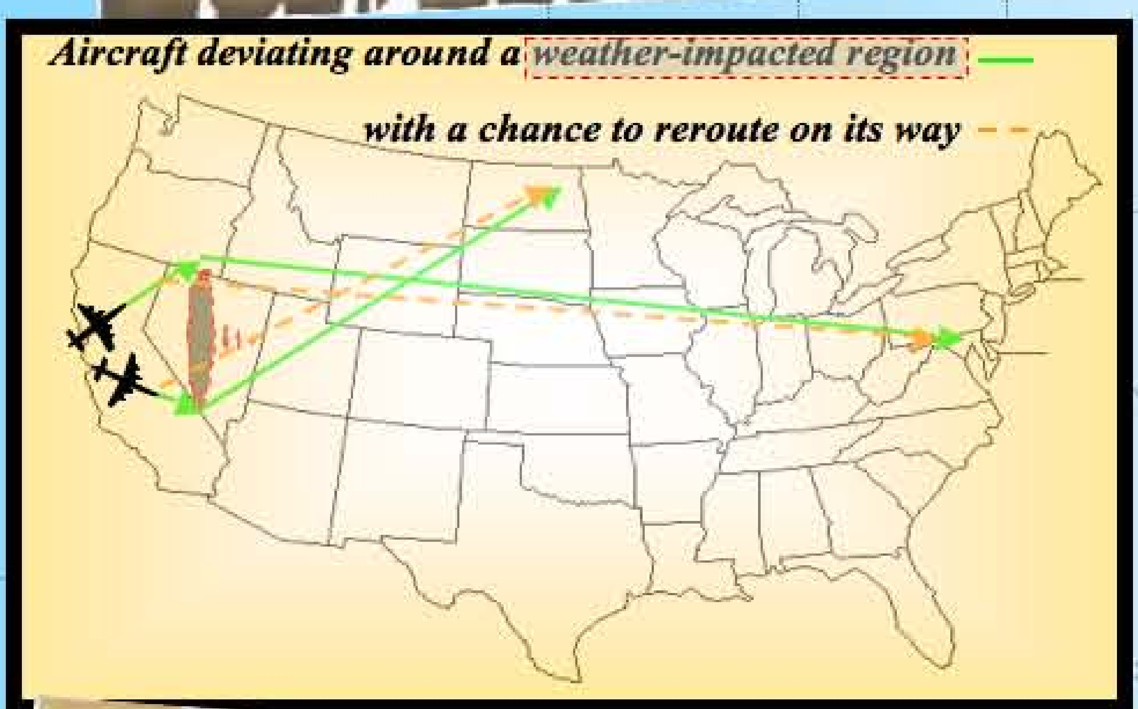
- *An initial plan that designates whether a flight is assigned to its primary route or secondary route;*
- *A recourse action for each flight under each possible early clearance time.*

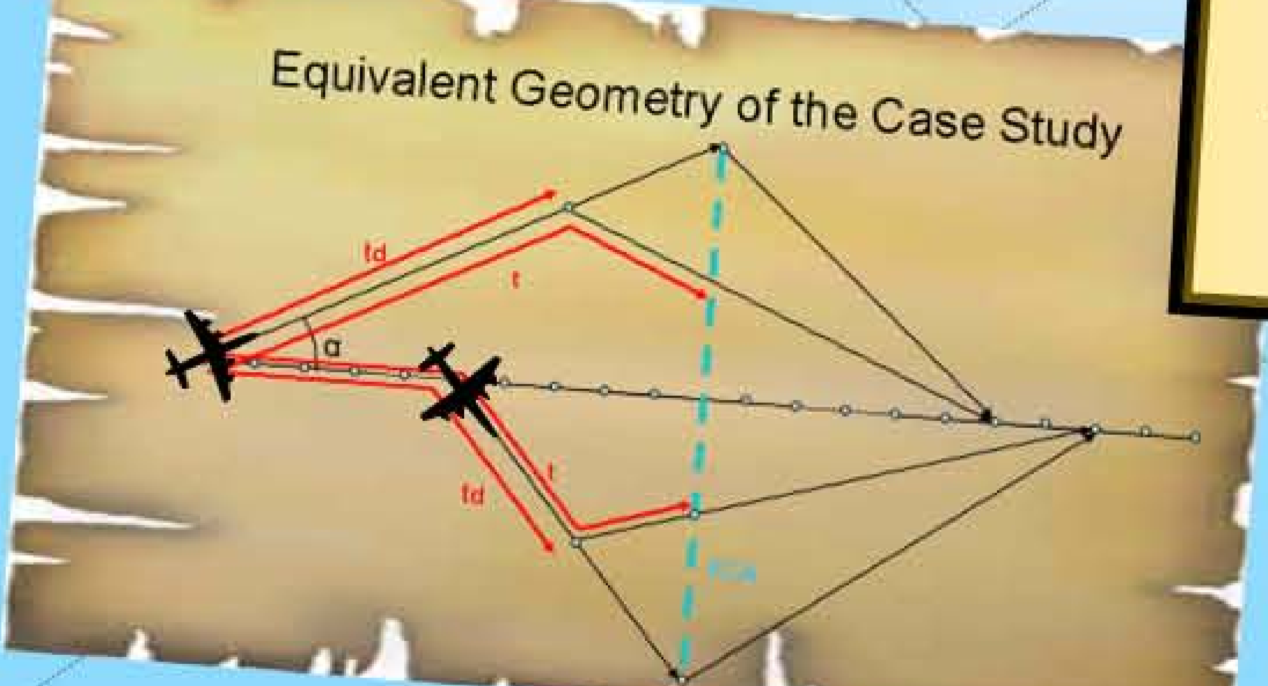
Geometry-Cost analysis for a single flight

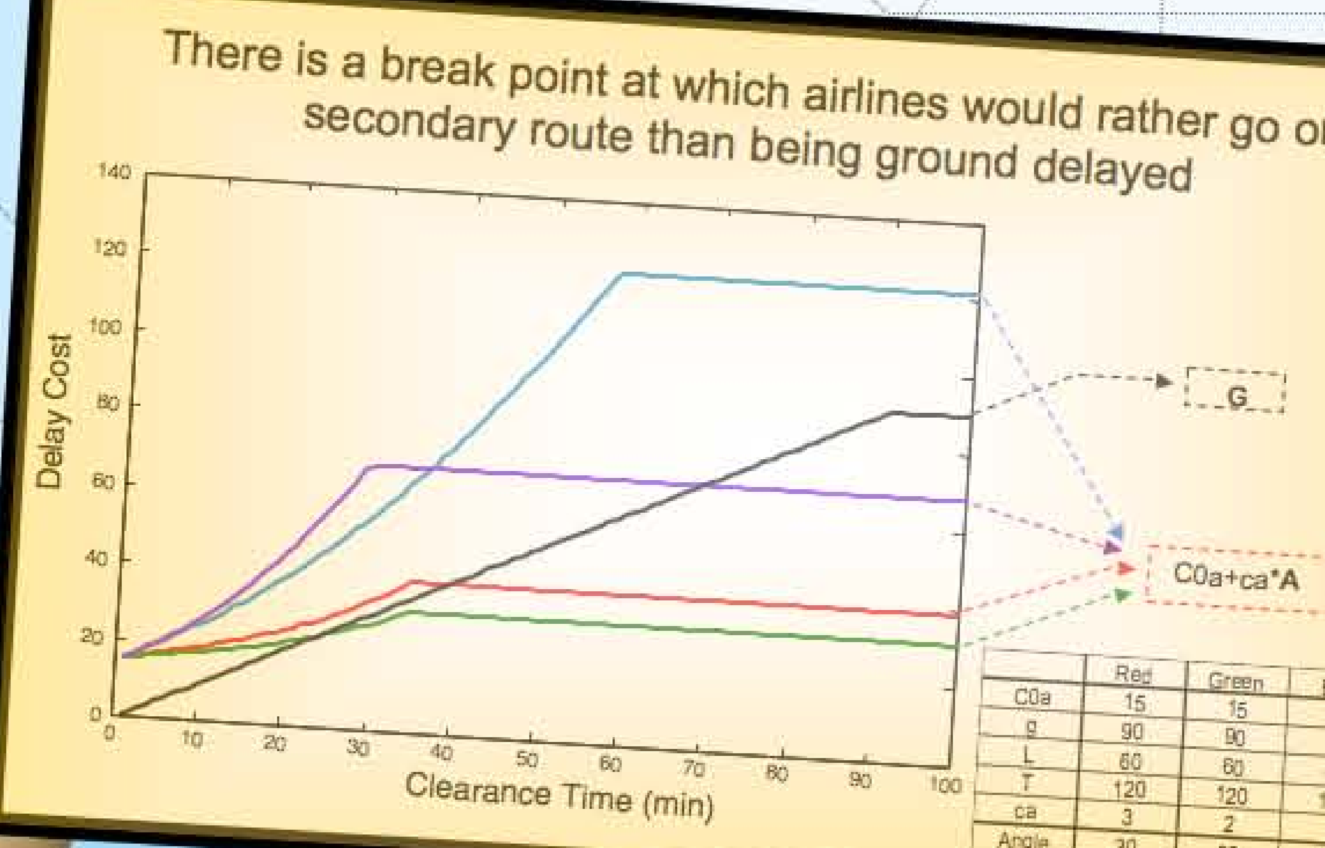


L = length of preferred route (in time)
θ = angle of reroute
T = length of problem time horizon
cg = cost per unit time of ground delay=1
ca = cost per unit time of air delay
coa = fixed cost for planning a reroute
CGD = expected cost of GD strategy
CRR = expected cost of RR strategy









	Red	Green	Blue	Purple
C0a	15	15	15	15
g	90	90	90	90
L	60	60	60	40
T	120	120	120	120
ca	3	2	2	2
Angle	30	30	60	60

Decision Variables for IP Model

$$x^p_{f,t} = \begin{cases} 1, & \text{if flight } f \text{ uses its primary route and has an appointment time } t \text{ at the FCA} \\ 0, & \text{otherwise} \end{cases}$$

$$x^s_f = \begin{cases} 1, & \text{if flight } f \text{ is assigned to its secondary route} \\ 0, & \text{otherwise} \end{cases}$$

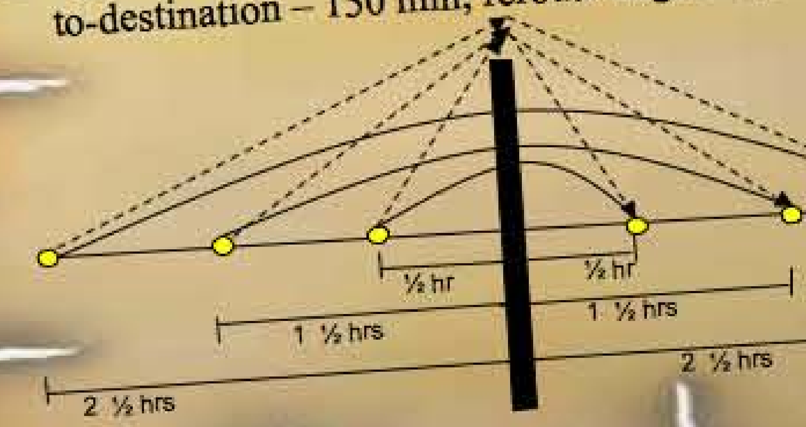
$$y^p_{f,t} \mid u = \begin{cases} 1, & \text{if at the time } U = u \text{ of the capacity windfall, flight } f \text{ is assigned to its primary route with appointment slot } t \text{ at the FCA} \\ 0, & \text{otherwise} \end{cases}$$

$$y^h_{f,t} \mid u = \begin{cases} 1, & \text{if flight } f \text{ was originally assigned to its secondary route, but under capacity clearing time } u \text{ has been assigned an FCA appointment slot } t \\ 0, & \text{otherwise} \end{cases}$$

$$y^s_f \mid u = \begin{cases} 1, & \text{if flight } f \text{ was originally assigned to its secondary route, and if, under AFP stop time } u\text{, that decision remains unchanged} \\ 0 & \text{otherwise} \end{cases}$$

Case Study

Short haul: length - 60 min: origin-to-FCA – 30 min, FCA-to-destination – 30 min; reroute angle – arctan(30/30). **Medium haul:** length – 180 min: origin-to-FCA – 90 min, FCA-to-destination – 90 min; reroute angle – arctan(30/90).
Long haul: length – 300 min: origin-to-FCA – 150 min, FCA-to-destination – 150 min; reroute angle – arctan(30/150).



	All Options			Reroute but No Recourse			No Reroute		
air/ground delay cost	4			4			4		
Objective Function	2379.4			2622.8			6808		
C(xp)	187			455			11381		
C(xs)	764.7			722.0			0		
U:Clearance time	3:00 PM	4:20 PM	5:40 PM	3:00 PM	4:20 PM	5:40 PM	3:00 PM	4:20 PM	5:40 PM
Probability of clearance	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
SV(yp)	61	114	0	345	364	110	10117	6247	1928
SV(yh)+SV(xs->yp)	595.1	207.8	19.7	446.6	68.7	0.0	0.0	0.0	0.0
GD(u)	126	73	187	110	91	345	1264	5134	9453
AD(u)	169.6	556.9	745.0	275.4	653.3	722.0	0.0	0.0	0.0
C(u)	804.4	2300.6	3166.9	1211.6	2704.0	3232.8	1264.0	5134.0	9453.0
E[cost]	2379.4			2622.8			6808.0		
E[GD]	143.3			250.3			6808.0		
E[AD]	559.0			593.1			0.0		
n(xp=1)	65			73			200		
n(yp=1)	148	77	65	150	84	73	200	200	200
n(xs=1)	135			127			0		
n(yh=1)	41	55	10	0	0	0	0	0	0
n(ys=1)	11	68	125	50	116	127	0	0	0
n(xs->yp)	83	12	0	77	11	0	0	0	0
Running Time (sec)	283			226			428		

Objective Function

$$Min\left[C(X) - \sum_u P_u S(Y_u)\right]$$

Initial plan cost –possible saving under each scenario U

Main Constraints

Under original plan each flight must either be assigned to a secondary route or to its primary routes – possibly with a delay

$$\sum_t x^p_{f,t} + x^s_f = 1 \quad \forall f$$

FCA capacity constraints under original plan:

$$\sum_f x^p_{f,t} \le C^0_t \quad \forall t$$

under revision, flight can only arrive earlier than u + E(f) if it has already departed

$$y^p_{f,t} \mid u = x^p_{f,t} \quad \forall f,u, \quad \forall t \in \{1,\dots,u+E_f\}$$

Under revision, flight originally assigned to its secondary route can go on primary route if it has not departed yet

$$\sum_t y^p_{f,t} \mid u \le 1 - x^s_f \quad \forall u, \forall f \ni D_f < u$$

Under revision flight on primary route can only arrive earlier at FCA

$$y^p_{f,t} \mid u \le \sum_{q \ge t} x^p_{f,q} + x^s_f \quad \forall u,f,t$$

Flight on hybrid route cannot arrive at FCA at time t unless u plus the revision time is less than t

$$y^h_{f,t} \mid u = 0 \quad \forall f,u, \quad \forall t \mid t^d_{f,t} \le u$$

Flight cannot use a hybrid route unless it was originally assigned to its secondary route

$$y^h_{f,t} \mid u + y^s_f \mid u \le x^s_f \quad \forall u,f,t$$

Under all scenarios each flight must either be assigned to a time slot at FCA or complete its secondary route:

$$\sum_t y^p_{f,t} \mid u + \sum_t y^h_{f,t} \mid u + y^s_f \mid u = 1 \quad \forall u,f$$

FCA capacity constraints under scenario u:

$$\sum_f y^p_{f,t} \mid u + \sum_f y^h_{f,t} \mid u \le C^u_t \quad \forall u,t$$

Conclusions and future work

we have defined the basics of a stochastic optimization model for simultaneously making ground delay and reroute decisions in response to en route airspace congestion. We have also given the results of an initial computational experiment.

Future steps should include more computational experiments and model refinements aimed at improving the computational performance of the integer program and at exploring the changes in airspace planning the model provides.

We anticipate the need to provide many refinements and extensions to this model to better address practical problem solving.

NEXTOR Research team at UMD

Design:Moein