



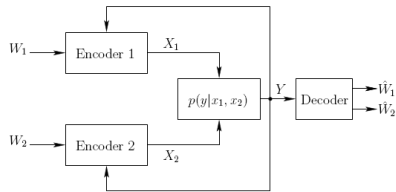
# A New Upper Bound for a Multiple Access Channel with Feedback

Ravi Tandon and Sennur Ulukus



## Multiple Access Channel with Feedback

### Introduction



- Channel described by  $p(y|x_1, x_2)$ .
- For a MAC, **feedback increases capacity** unlike a single user channel.
- Feedback capacity region not known in general.
- Only known for special cases, e.g., for a class of channels:  $X_1=f(X_2, Y)$ .
- Capacity region also known for the AWGN MAC with feedback.

### Cut-Set Outer Bound

$$CS = \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leq I(X_1; Y|X_2) \\ R_2 &\leq I(X_2; Y|X_1) \\ R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned} \right\}$$

over all input distributions  $p(x_1, x_2)$ .

Main idea behind our outer bound:

- Cut-set bound allows all correlations.
- Even with perfect feedback, **full correlation** between  $X_1$  and  $X_2$  **may not be possible** to achieve.
- Restricting the set of allowable  $p(x_1, x_2)$  and corresponding rates.

### New Outer Bounds

$$DB^{(1)} = \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leq \min(I(X_1; Y|X_2), H(X_1|T)) \\ R_2 &\leq I(X_2; Y|X_1, T) \\ R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned} \right\}$$

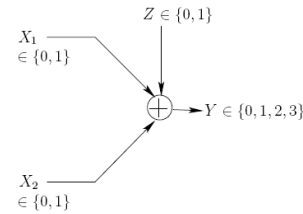
$$DB^{(2)} = \left\{ (R_1, R_2) : \begin{aligned} R_1 &\leq I(X_1; Y|X_2, T) \\ R_2 &\leq \min(I(X_2; Y|X_1), H(X_2|T)) \\ R_1 + R_2 &\leq I(X_1, X_2; Y) \end{aligned} \right\}$$

where both bounds are evaluated over all distributions of the conditionally independent form  $p(t)p(x_1|t)p(x_2|t)$ .

#### Remarks :

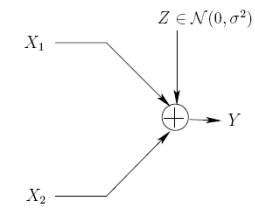
- For the case when  $X_1=f(X_2, Y)$ , our bounds are tight and yield the feedback capacity region.
- While evaluating the outer bounds, it is sufficient to restrict attention to auxiliary random variables  $T$  such that  $|T| \leq |X_1||X_2| + 3$ .
- For binary inputs, we have  $|T| \leq 7$ , making exhaustive search intractable.

## Binary Additive Noise MAC



- Channel given by  $Y=X_1+X_2+Z$ , where  $Z$  is binary and uniform on  $\{0,1\}$ .
- Capacity region not known.
- Our bounds show that the **cut-set bound is not tight**.

## AWGN MAC

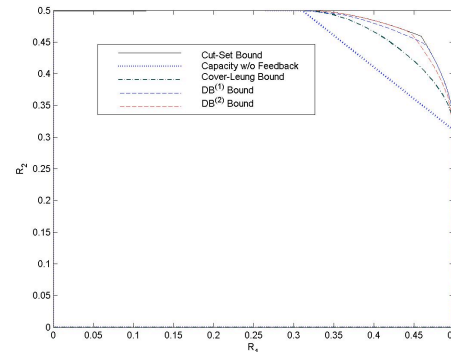


- Capacity region known [Ozarow, 1984]
- **Cut-set bound is tight**.
- An extension of Kailath-Schalkwijk scheme is capacity achieving..

For symmetric-rate point  $(R, R)$  in the feedback capacity region:

- Cut-set upper bound: 0.45915 bits/tx.
- Our upper bound: 0.45330 bits/tx.
- Cover-Leung's inner bound: 0.43621 bits/tx.
- Best known inner bound: 0.43879 bits/tx.

## Comparison of Our Bounds with the Cut-Set Bound



- $|T|=2$  is sufficient to evaluate our bounds by making use of composite functions.
- Our bounds improve upon the cut-set bound for all points where feedback increases capacity.

**Conclusion:** *Cut-set bound is in general not tight for the multiple access channel with feedback.*