Jointly Optimum Power and Signature Sequence Allocation for Fading CDMA

Onur Kaya Sennur Ulukus

Introduction

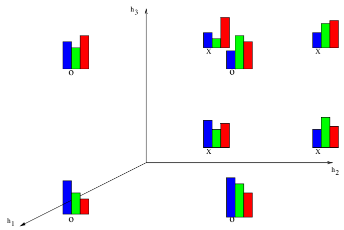
- Dynamic resource allocation – transmit powers, bandwidth, time slots; or in general waveforms – to combat fading and improve capacity.
- CDMA (Vector MAC): allocate transmit powers and signature sequences to users.

$$\mathbf{r} = \sum_{i=1}^{K} \sqrt{p_i h_i} b_i \mathbf{s}_i + \mathbf{n}$$

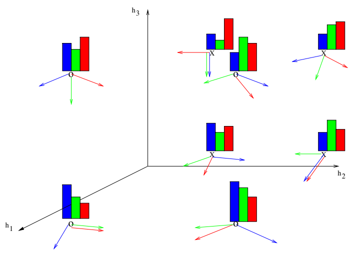
- Power control only: maximize ergodic sum capacity subject to average power constraints
  - **Fading channels**, waterfilling *in time*, users treat each other as noise,
  - More power to better channel states; no power to very poor channel states.
- Signature sequence allocation only: find sum-capacity maximizing set of sequences (waveforms) for a given set of (fixed) power constraints, and **no fading**.
  - Notion of oversized/non-oversized users according to power constraints,
  - Orthogonal sequences to oversized, GWBE sequences to non-oversized users.

Joint Power and Sequence Allocation

- We consider a CDMA system with perfect CSI at the transmitters.
- Then, both powers and sequences can be chosen as functions of channel states.
- First, fix an arbitrary valid power allocation over the fading states.



- For each fixed allocation, find the sequences that maximize the sum capacity at each state **h**.



- Define the signature sequence optimized sum capacity at **h**

$$C_{opt}(\mathbf{h},\mathbf{p}(\mathbf{h})) = \max_{\mathbf{S}(\mathbf{h})} C_{sum}(\mathbf{h},\mathbf{p}(\mathbf{h}),\mathbf{S}(\mathbf{h}))$$

- Then we can optimize only over power control policies, using optimum sequences computed for each policy.

$$\max_{\mathbf{p}(\mathbf{h})} E_{\mathbf{h}}\left[C_{opt}(\mathbf{h},\mathbf{p}(\mathbf{h}))\right]$$
$$\text{s.t. } E_{\mathbf{h}}\left[p_i(\mathbf{h})\right] = \bar{p}_i, \quad i=1,\dots,K$$
$$p_i(\mathbf{h}) \geq 0 \quad \forall \mathbf{h}, \quad i=1,\dots,K$$

Joint Power and Sequence Allocation – $K \leq N$

- Optimal signature sequences constitute an **orthogonal set** for any power alloc'n.
- Problem reduces to K independent single user Goldsmith-Varaiya problems, i.e.,

$$\max_{\mathbf{p}(\mathbf{h})} E_{\mathbf{h}}\left[\sum_{i=1}^{K}\log\left(1+\frac{p_i(\mathbf{h})h_i}{\sigma^2}\right)\right]$$
$$\text{s.t. } E_{\mathbf{h}}\left[p_i(\mathbf{h})\right] = \bar{p}_i, \quad i=1,\dots,K$$

- Concave maximization over an affine set of constraints, using KKT conditions,

$$p_k(\mathbf{h}) = \left(\frac{1}{\lambda_k}-\frac{\sigma^2}{h_k}\right)^+, \quad i=1,\dots,K$$

- Channel non-adaptive sequence selection performs as well as any channel adaptive selection.

Joint Power and Sequence Allocation – $K > N$

- For a given power control policy $\mathcal{P}(\mathbf{h})$, let $L(\mathbf{h})$ and $\bar{L}(\mathbf{h})$ be sets of oversized and non-oversized users respectively, for a given **h**.
- Define **D**=diag$(p_1h_1, \dots, p_Kh_K)$. Optimum signature sequences satisfy,

$$\mathbf{SDS}^T\mathbf{s}_i(\mathbf{h}) = \mu_i(\mathbf{h})\mathbf{s}_i(\mathbf{h})$$

$$\mu_i(\mathbf{h}) = \begin{cases} \dfrac{\sum_{j\in\bar{L}(\mathbf{h})} p_j h_j}{N-|L(\mathbf{h})|}, & i \in \bar{L}(\mathbf{h}) \\ p_i h_i, & i \in L(\mathbf{h}) \end{cases}$$

- The sequence optimized ergodic sum-capacity is then

$$E_{\mathbf{h}}\left[\sum_{i\in L(\mathbf{h})}\log\left(1+\frac{p_i(\mathbf{h})h_i}{\sigma^2}\right)+\left(N-|L(\mathbf{h})|\right)\log\left(1+\frac{\sum_{i\in\bar{L}(\mathbf{h})}p_i(\mathbf{h})h_i}{\sigma^2\left(N-|L(\mathbf{h})|\right)}\right)\right]$$

Theorem (Number of simultaneously transmitting users): *Let $\bar{K}(\mathbf{h})$ be a subset of $\{1,\dots,K\}$, such that $\forall i \in \bar{K}(\mathbf{h})$, $p_i^*(\mathbf{h}) > 0$, where $p_i^*(\mathbf{h})$ is the maximizer of $E_{\mathbf{h}}[C_{opt}(\mathbf{h},\mathbf{p}(\mathbf{h}))]$. Then, with probability 1, $|\bar{K}(\mathbf{h})| \leq N$.*

- At most N users transmit: assign orthogonal sequences to those users.
- Optimum power allocation is similar to single user waterfilling

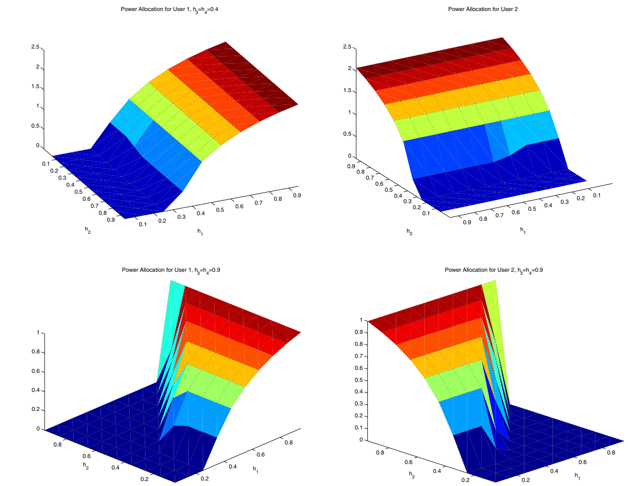
$$p_i(\mathbf{h}) = \begin{cases} \left(\dfrac{1}{\lambda_i}-\dfrac{\sigma^2}{h_i}\right), & i \in \bar{K}(\mathbf{h}) \\ 0, & \textit{otherwise} \end{cases}$$

- Here, a channel adaptive allocation of orthogonal sequences is necessary.
- Define $\gamma_i = h_i/\lambda_i$, and let $\gamma_{[i]}$ be the order statistics for γ_is, and let for given **h**

$$\gamma_{[1]} \geq \dots \geq \gamma_{[n]} > \sigma^2 \geq \gamma_{[n+1]} \geq \dots \geq \gamma_{[K+1]} = 0$$

- If $n \leq N$, the users with highest $n\,\gamma_i$s transmit with powers $p_i^*(\mathbf{h})$.
- If $n > N$, by Theorem, the users with highest $N\gamma i$'s transmit with positive powers.

Optimum Power Allocation: $K = 4$, $N = 3$



Iterative Power and Sequence Optimization

- Instead of simultaneously solving for all powers, which in turn requires solving for λ_i, we propose the following one-user-at-a-time algorithm:

repeat
  for i = 1 to K and for all **h**
    -find oversized users
    -compute signature sequences for all users
    -update i^{th} user's power using waterfilling keeping other powers fixed
  end
until **p(h)** *converges.*

Convergence of the Iterative Algorithm

- The algorithm corresponds to iteration of the best sequence-only update for all users and best power-only update for one user, so sum capacity values are non-decreasing.
- The sum capacity is bounded from above, so this algorithm converges to a limit.
- The fixed point $\mathbf{p}^{n+1}(\mathbf{h}) = \mathbf{p}^n(\mathbf{h})$ satisfies the KKT conditions.
- Algorithm converges to jointly optimum power and signature sequence allocation.