Optimal Power Control for Wireless Queueing Networks

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Abstract

- Optimally controlling transmit power for jointly minimizing:
  - Energy expenditure
  - Buffer overflow
- Considered System
  - Wireless Link,
  - Finite-capacity buffer,
  - Random packet arrivals,
  - Choosing from two alternative power levels.
- Result
  - Optimal policy is of threshold type.

Motivation

- Energy Efficiency
  - A key concern in wireless networks,
  - Limited and non renewable power supplies.
- Power Control, Traditional Work:
  - Mitigating effects of interference,
  - Satisfying some QoS constraints,
  - SINR, BER constraints (Foschini(93), Yates(95))

Energy Efficiency

- Bambos(96):Maximizing battery lifetime
  - Minimizing average power subject to average success rate constraint.
  - Interference has a known distribution.
  - No error control technique is specified.
- Zorzi(97):Trade-off between instantaneous performance and energy efficiency:
  - ARQ error control,
  - Gilbert-Elliott Channel,
  - Decreasing power in favor of increased retransmissions can be a more energy-efficient.

Random Arrivals

- Previous work generally considered variance of interference and noise.
- Random arrival of data is not considered generally.
- In a limited buffer system with random arrivals
  - An arrival burst can occur when queue is long.
  - Buffer overflow can be avoided by increasing power.

System Model

- Wireless Link
  - Finite buffer capacity (length= K)
  - Bernoulli packet arrivals (rate= λ)
  - Equal length packets (length= 1 bit)
  - Two alternative power Levels ($P_1<P_2$)
  - BPSK modulation
  - ARQ error control
  - Power decisions are made at every time slot.
  - Success probability = $1-Q(\sqrt{\frac{PT_s}{N_0}})$

State Transitions

$$d_{i,j}(u)=\begin{cases}1-\lambda, & i=0,\ j=0\\ \lambda, & i=0,\ j=1\\ \lambda Q_u+(1-\lambda)(1-Q_u), & K>i>0,\ j=i\\ \lambda(1-Q_u), & K>i>0,\ j=i+1\\ (1-\lambda)Q_u, & K>i>0,\ j=i-1\\ 1-Q_u, & i=K,\ j=i\\ Q_u, & i=K,\ j=i-1\\ 0 & |j-i|>1\end{cases}$$

MDP

- State, x_t: Buffer occupancy in the t^{th} slot ($x_t \varepsilon X=\{0,1,\cdots,K\}$)
- Control: u_t: Power control decision ($u_t\ \varepsilon\ \{0,1,2\}$)
- u_t is allowed only when $x_t=0$.
- Find the policy that minimizes the discounted cost:

$$V_\alpha = E_x \sum_{t=0}^{\infty} \alpha^t g(x_t,u_t) \quad \forall x \in X, u \in \{0,1,2\}$$

- Overflow cost:

$$O(x,u)=\begin{cases}0, & x<K\\ \lambda, & x=K\end{cases}$$

- Energy Expenditure cost:

$$E(x,u)=\begin{cases}P_u, & u=1,2\\ 0, & u=0\end{cases}$$

- Single Stage Cost:

$$g(x,u)=c_1E(x,u)+c_2O(x,u), \forall x \in X, u \in U$$

- Dynamic Programming Equations:

$$V^0(x)=\min_{u \in U(x)}\{g(x,u)\}$$

$$V^{n+1}(x)=\min_{u \in U(x)}\{g(x,u)+\alpha\sum_{x'} d_{x,x'}(u)V^n(x)\}$$

Optimality of Threshold Policy

- For $0<x<K$ optimal policy chooses P_2 if:

$$c_1(P_2-P_1) \le \alpha[\lambda(Q_2-Q_1)(V^n(x+1)-V^n(x))+(1-\lambda)(Q_2-Q_1)(V^n(x)-V^n(x-1))]$$

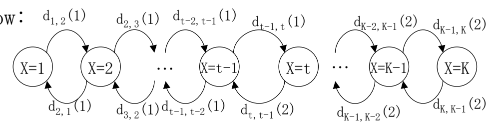
$$=\alpha[\lambda(Q_2-Q_1)h_x^n+(1-\lambda)(Q_2-Q_1)h_{x-1}^n] \quad (1)$$

- Here $h_x^n=V^n(x+1)-V^n(x)$
- Theorem: For P1<P2 the below inequalities hold in every iteration n:
  - For x greater than or equal to some x^n: $h_x^n \le h_{x+1}^n$ (2)
  - For $x<x^n$: $c_1(P_2-P_1)\ge\alpha(\mu_1-\mu_2)(\lambda h_x^n+(1-\lambda)h_{x-1}^n)$ (3)
  - For all x: $0\le h_x^n \le c_2$ (4)
  - (2) and (3) are useful for proving that the right hand side of inequality (1) crosses left side at most at one point.

Practical Determination of Thresholds

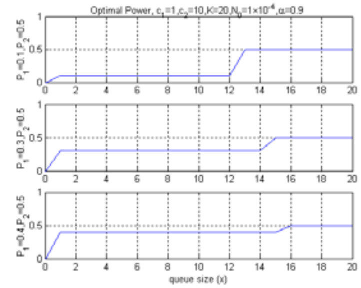
- Let $0\le t\le K$ be the threshold value.
- Queue state probabilities $\underline{\pi}=[\underline{\pi}_1\ \underline{\pi}_2,\cdots,\ \underline{\pi}_K]$ can be found by solving the Markov chain below:
- Find t* minimizing the cost below:

$$t^*=\arg\min_t\{\pi_0 g(0,0)+\sum_{x=1}^{t}\pi_x g(x,1)+\sum_{x=t}^{K}\pi_x g(x,2)\}$$

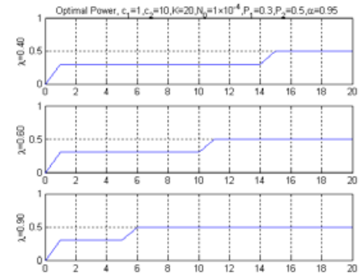


Computational Results

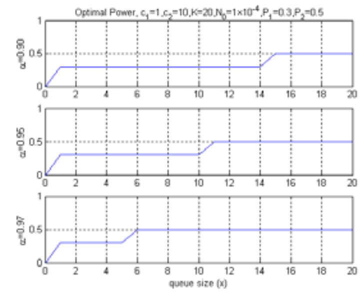
- Changing Power Levels



- Changing Arrival rate



- Changing Discount Factor



Results

- The optimal power control policy of threshold type.
- Optimal threshold
  - Increases as P_1 approaches to P_2,
  - Decreases as the arrival rate increases,
  - Decreases as the discount factor α increases.