Source Coding and Parallel Routing

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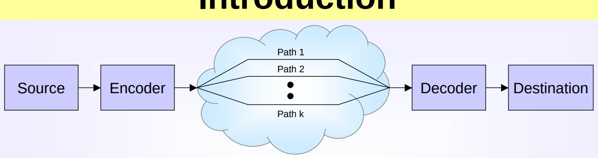
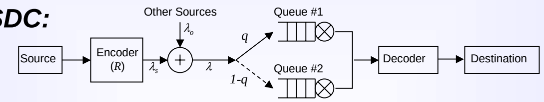
Introduction



- **Problem:** Choose the *routing parameters* together with the *coding parameters* to minimize average *end-to-end distortion*
- **Motivation:** Coupling of different network layers
  - Parallel routing (**layer 3**): uses diversity but introduces extra delays
  - Source coding (**layer 6**): compression
- **Trade-off:** Distortion vs. Delay
  - want less distortion ➔ need longer packets ➔ more delay

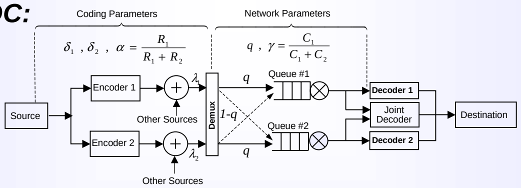
Model

- **Source:** Gaussian, Delay-sensitive: packets that arrive later than Δ sec are useless
- **Encoder:**
  - ***SDC:***



- Each source symbol encoded into one packet (description)
- Expected packet length: R bits
- greater R → smaller distortion → longer delay

  - ***DDC:***



- Each source symbol encoded into 2 packets
- expected description lengths: R_1, $R_2 < R$ bits
- Each DDC description carries less info than SDC packet
- If $R_1 + R_2 = R$, both descriptions can jointly carry as much info as an SDC packet
- DDC: more flexibility without adding extra traffic load

End-to-End Distortion

- **SDC:**

$$D_{SDC} = \begin{cases} 2^{-2R} & \text{if } T \le \Delta \\ 1 & \text{if } T > \Delta \end{cases}$$

$$\bar{D}_{SDC} = 2^{-2R}\Pr[T \le \Delta] + \Pr[T > \Delta]$$

  - Objective

$$(R^*, q^*) = \arg\min_{(R,q)} \bar{D}_{SDC}$$

- **DDC:**

$$D_{DDC} = \begin{cases} d_0 = 2^{-2(R_1+R_2)\frac{1}{1-([\sqrt{\Pi}-\sqrt{\Lambda}]^+)^2}} & \text{if } T^1 \le \Delta \;\&\; T^2 \le \Delta \\ d_1 = 2^{-2R_1(1-\delta_1)} & \text{if } T^1 \le \Delta \;\&\; T^2 > \Delta \\ d_2 = 2^{-2R_2(1-\delta_2)} & \text{if } T^1 > \Delta \;\&\; T^2 \le \Delta \\ 1 & \text{if } T^1 > \Delta \;\&\; T^2 > \Delta \end{cases}$$

$$0 \le \delta_1, \delta_2 \le 1 \quad , \quad \Pi = (1-d_1)(1-d_2) \quad \& \quad \Lambda = d_1 d_2 - 2^{-2(R_1+R_2)}$$

$$\bar{D}_{DDC} = d_0 P\left[T^1 \le \Delta, T^2 \le \Delta\right] + d_1 P\left[T^1 \le \Delta, T^2 > \Delta\right] + d_2 P\left[T^1 > \Delta, T^2 \le \Delta\right] + P\left[T^1 > \Delta, T^2 > \Delta\right]$$

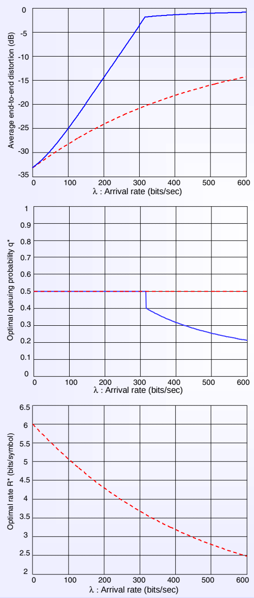
  - δ_i : redundancy of encoder i

small δ: good individual descriptions that jointly contribute little
large δ: not individually good descriptions but jointly can achieve same distortion as SDC

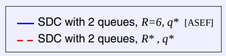
  - Objective

$$(R_1^*, R_2^*, q^*, \delta_1^*, \delta_2^*) = \arg\min_{(R_1, R_2, q, \delta_1, \delta_2)} \bar{D}_{DDC}$$

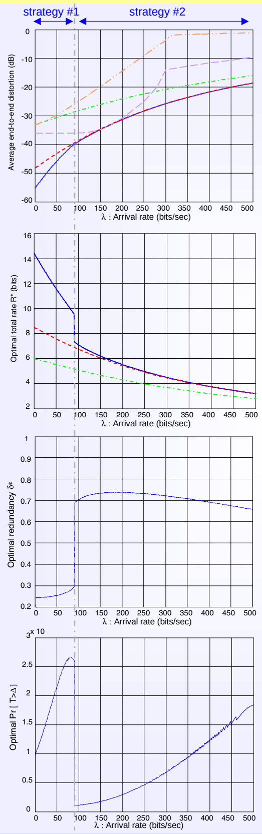
SDC Results



- **Problem setup:**
  - Two queues
  - Unlimited buffer size
  - Balanced capacities $C_1 = C_2$
- **Observations:**
  - Fixed R case:
    - One queue carries as much traffic it can handle. The rest of traffic is dumped to the other queue.
    - optimal routing helps save one of two queues instead loosing both
  - Optimal R case:
    - $q^* = 0.5$
    - R^* decreases with λ



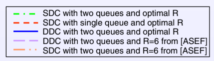
DDC Results



- **Problem setup:**
  - Two queues
  - Unlimited buffer size
  - Balanced capacities $C_1 = C_2 = 1000$ bits/s
  - Exponential packet length
  - $R = R_1 + R_2$
  - $\lambda_1 = \lambda_2 = \lambda$
- **Observations:**
  - $R_1^* = R_2^*$
  - $\delta_1^* = \delta_2^*$
  - $q^* = 0$, $q^* = 0$
  - R^* decreases with arrival rate λ
  - 2 strategies for R-optimal DDC

    strategy #1: long packets → high delay probability (both packets less likely to arrive) → low encoder redundancy

    strategy #2: shorter packets → low delay probability (both packets likely to arrive) → high redundancy
  - DDC outperforms SDC under similar conditions



[ASEF] M. Alasti, K. Sayrafian-Pour, A. Ephremides, N. Farvardin, "Multiple Description Coding in Networks with Congestion Problem," *IEEE Transaction on Information Theory, Vol. 47, No.3, March 2001*

Concluding Remarks

- Interesting Observation:
  - Known queuing theory result: *splitting the capacity between 2 links can only increase the delay and in our case increase the distortion.*
  - Known information theoretic result: *MDC does not reduce the distortion compared to SDC.*
  - Our result: *For a delay-sensitive source combining parallel routing and MDC in an optimal way can result in less distortion than an optimal SDC system with a single queue.*
- For a general memory-less source, explicit inner and outer bounds for the multiple description rate-distortion region have been found by Zamir. These bounds maintain the form of the distortion functions we discussed and so we expect our analysis to be applicable for any memory-less source.