A Game-Theoretic Look at Joint Multi-Access, Power and Rate Control

Yalin Evren Sagduyu, Anthony Ephremides

*Objective and Motivation **

- ***Objective:*** Analyze a cross-layering problem in ***wireless ad hoc networks*** from the perspective of stochastic games
  - Simple building block: ***Multi-Access Channel***
  - Joint Random Access (***MAC Layer***) & Power and Rate Control (***Physical Layer***)
- ***Motivation:*** Wireless ad hoc networks consist of selfish (non-cooperative) nodes with conflicting interests:
  - Intertwined conflicts of ***throughput***, ***energy*** and ***delay***
  - Selfish behavior prevents cheating for channel access
  - Inherently ***distributed*** and ***scalable***

Random Access as a Stochastic Game

- ***Single-cell*** system with classical uplink collision channel
  - Multiple blocked nodes contending for single channel
- Game with two ***actions*** : ***Transmitting or Waiting***
- Performance measures :
  - **(a)** ***throughput*** reward **1** for successful transmissions
  - **(b)** ***energy*** cost ***c*** for transmission attempts
  - **(c)** discount future payoffs by δ for each slot of ***delay***
- State of Game : ***n*** (number of backlogged nodes)
- Given ***n***, each user ***i*** selects transmitting probability $p_{i,n}$
  - **(a)** ***independently*** to maximize ***individual*** utility $u_{i,n}$
  - **(b)** ***cooperatively*** to maximize ***total*** system utility U_n
- ***Slotted collision channel*** model:
  - Each packet arrives at a "new" transmitter node
  - No new packet arrives until resolution of backlogged nodes
- ***Capture:*** Multiple successful transmissions per slot
  - Capture probability q_k **:** Probability that any transmitting node ***i*** is successful for total of ***k +1*** transmitting nodes
- Error-free feedback:
  - **(I)** ***Channel Outputs***: Success, Collision, Idle
  - **(II)** State of the Game ***n***

Expected Utilities for Random Access Game

$$u_{i,n} = \underbrace{p_{i,n}\, u_{i,n}(T)}_{\text{transmitting}} + \underbrace{(1-p_{i,n})\, u_{i,n}(W)}_{\text{waiting}}$$

$$u_{i,n}(T) = -c + \sum_{k=0}^{n-1} P(K_{i,n}=k)\,\left(1\cdot q_k + \delta\,(1-q_k)\sum_{j=0}^{k} u_{i,n-j}\binom{k}{j} q_k^{\,j}(1-q_k)^{k-j}\right)$$

$$u_{i,n}(W) = \delta \sum_{k=0}^{n-1} P(K_{i,n}=k) \sum_{j=0}^{k} u_{i,n-j}\binom{k}{j}(q_{k-1})^{j}(1-q_{k-1})^{k-j}$$

$K_{i,n}$ **:**number of ***transmitting*** nodes other than node ***i*** for state ***n***

Symmetric Random Access Games G (n)

- Conflicting nodes are ***identical***⇒***Symmetric*** Game
  - Users have the ***same*** strategies for the same game state ***n***

$$p_{i,n} = p_n \;\Rightarrow\; P(K_{i,n}=k) = P(K_n=k) = \binom{n-1}{k} p_n^{\,k}(1-p_n)^{n-1-k}$$
$$\Rightarrow\; u_{i,n} = u_n$$

- ***Physical layer***-based (symmetric) ***capture model :***
  - Success based on ***signal-to-interference-plus-noise-ratios***

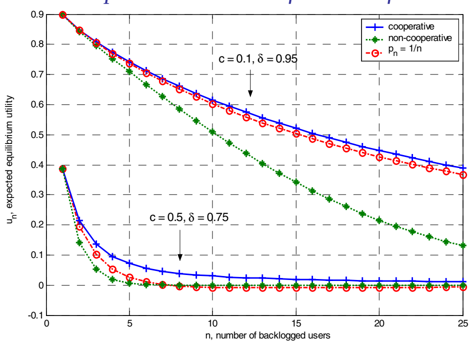
$$\frac{P_i}{\sigma^2 + \frac{1}{L}\sum_{k\neq i} P_k} \geq \beta$$

$P_i = P$: power of user ***i***,
σ^2: ***Gaussian noise*** power,
L: Processing gain, $L > 1$ if **CDMA** used

Equilibrium Strategies for Random Access Games

- Non-cooperative Nash Eq. **:** $u_n(T) = u_n(W) = u_n^{*}, \forall n$
- Cooperative Eq. **:** $\max_{p_n} U_n = \sum_{i=1}^{n} u_{i,n}, \forall n$
- ***Cooperation*** is ***better*** than ***selfish*** decisions:

Expected Utilities as function of ***n***

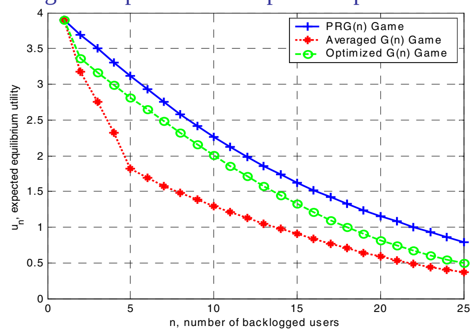


Power Control and Rate Adaptation Game PRG (n)

- ***Capture*** mechanism depends on ***powers*** & ***rates*** of nodes
- Given ***n***, node ***i*** selects power-rate pair $S_j \in J = E \times R$
  - E_j or R_j :Transmission ***power*** or ***rate*** of action S_j, $1 \leq j \leq |J|$
  - $ER_{i,n}(S_j)$: Prob. that node ***i*** chooses action S_j for state ***n***
  - ***Strategy*** : Given state ***n***, any node ***i*** selects $ER_{i,n}(S_j)$ to maximize its own utility $u_{i,n} = \sum_{S_j \in J} u_{i,n}(S_j)\, ER_{i,n}(S_j)$
  - Energy ***cost*** : $c\,E_j$, if $R_j > 0$
  - ***Reward*** for successful transmission : R_j
- ***M-ary Quadrature Amplitude Modulation*** as rate control
  - Selecting transmission ***rate*** *R* bits per slot
  - ⇒Employing *m*-ary QAM, ($m = 2^R$) or waiting ($m = 1$)
- ***Symmetric*** Nash Equilibrium for identical nodes:
  - For $n \geq 1$, find $ER_n(S_j)^*$, $S_j \in J$, such that $\{u_n((x,y)),(x,y)\in J\} = u_n^{*}$ given u^*_{n-k}, $n-1 \geq k \geq 1$

Superior Performance of Power and Rate Control Games

- ***Non-cooperative Eq.*** strategies of power & rate control
  - ***Power Set E*** = {0,1,2,3,4,5} unit power
  - ***Rate Set R*** = {0,1,2,3,4} (bits/packet)⇒2, 4, 8, 16-QAM
- ***Non-Cooperative Eq.*** strategies for random access ***G(n)***
  - Each user decides ***only*** to transmit or wait
  - Averaged or optimized over possible powers and rates



**** The material presented in this poster is based on paper "Power Control and Rate Adaptation as Stochastic Games for Random Access", Yalin Evren Sagduyu, Anthony Ephremides, in Proc. IEEE Control and Decision Conference, Dec. 2003.***