Power Control in Uplink and Downlink CDMA Systems with Multiple Flow Types

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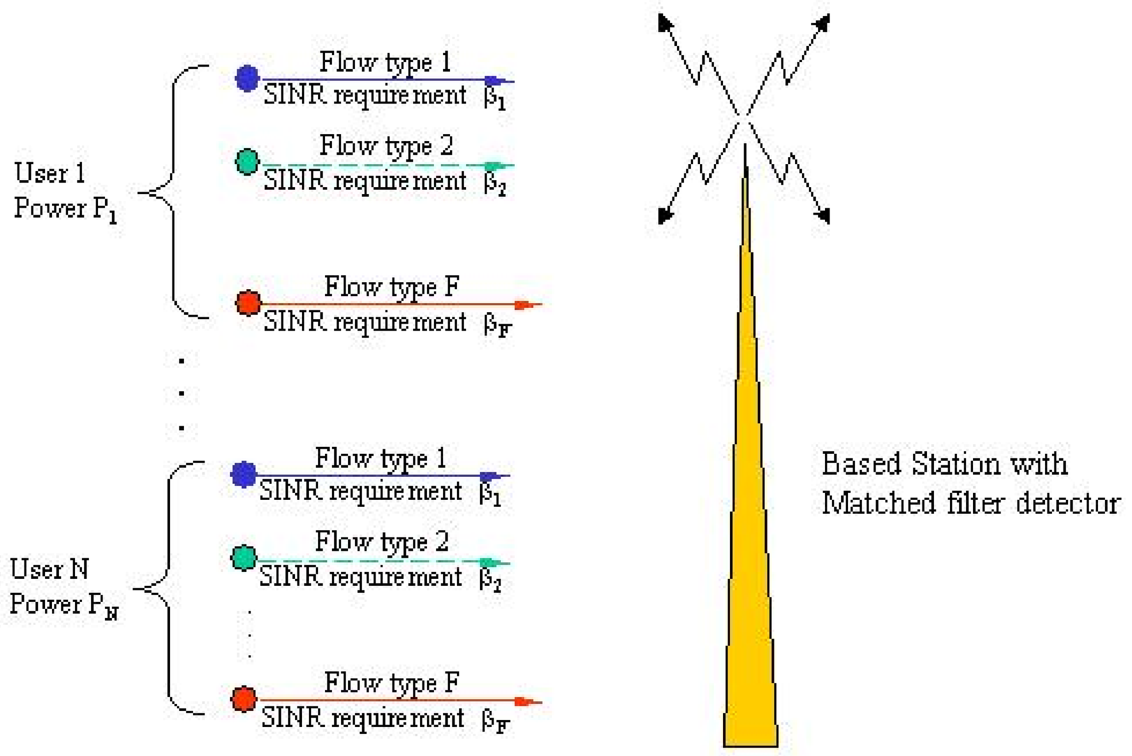
Power control is used to balance powers of the CDMA system users, so that no one creates too much interference to destroy the quality of communication of other users. In previous papers, the optimum power vector was found by inversion of a non-negative matrix related to channel gains and crosscorrelations. But all the models studied assume only one flow type at each node. In practice, users may have multiple flow types that have different QOS requirements. Each node has only one transmitter, i.e., only one power level is available in the uplink for all F flow types.

Model

- Power controlled synchronous CDMA system with total degrees of freedom L.
- A base station and N users (nodes).
- Each user has F flow types with SINR requirements $\beta_1 \le \beta_2 \le \cdots \le \beta_F$.
- Only one power level available at each user for all of its flow types.
- Base station has only one power level to transmit to all users and flow types.
- Users and base station can adjust power levels to satisfy SINR requirements.
- Matched filter receiver to detect signals at users and base station.



Questions

UPLINK

$$SIR_{if} = \frac{P_i}{\sigma^2 + \sum_{(j,g)\neq(i,f)} P_j \cdot \rho_{if,jg}^2} \ge \beta_f, \quad f = 1, 2, \cdots F, \; i = 1, 2, \cdots N.$$

$$\Rightarrow \quad P_i \ge \beta_f \sigma^2 + \beta_f \sum_{j=1}^{N} P_j \alpha_{ij}^f - \beta_f P_i, \quad \alpha_{ij}^f = \sum_{g=1}^{F} \rho_{if,jg}^2, \quad \mathbf{P} \ge \beta_f \mathbf{A}^{(f)} \cdot \mathbf{P} + \beta_f \sigma^2 \mathbf{1}, \; f=1,2,\ldots,F$$

- What is the conditions for the power control problem to have solution if fixed sequences are used ?
- What is the optimum (minimum) power vector if solution exists?
- What are the optimum sequences to minimize the total power of all users if optimum power vector is used?

DOWNLINK

$$SIR_{if} = \frac{P}{\sigma^2 + \sum_{(j,g)\neq(i,f)} P \cdot \rho_{if,jg}^2} \ge \beta_f, \quad f = 1, 2, \cdots F, \; i = 1, 2, \cdots N.$$

$$\Rightarrow \quad \frac{\sigma^2}{P} \le 1 + \frac{1}{\beta_f} - \alpha_{if}, \quad \alpha_{if} = \sum_{j=1}^{N}\sum_{g=1}^{F} \rho_{if,jg}^2.$$

- What is the minimum power assignment of the base station to satisfy the SIR requirements if fixed sequences are used ?
- What are the optimum sequences to minimize that power assignment ?

Uplink Analysis

SPECIAL CASE OF F=1

- This goes back to the typical power control problem with well-known solution.
- If $\beta < 1/\rho_A$ is satisfied, then solution exists.
- The optimum power vector is $\mathbf{P}^* = \sigma^2 \beta (\mathbf{I} - \beta \mathbf{A})^{-1} \cdot \mathbf{1}$.

ρ_A is the Perron-Frobenius eigenvalue of non-negative matrix A.

Proposition: The optimum sequences to minimize the total power in an uplink power-controlled CDMA system with SINR requirement β are the Welch-Bound-Equality (WBE) sequences for N>L, and orthogonal sequences for $N \le L$. The corresponding optimum power vector is

$$N > L: \; \mathbf{P} = \frac{\beta \sigma^2}{1 + \beta - \frac{N}{L}\beta} \cdot \mathbf{1}, \quad N \le L: \; \mathbf{P} = \beta\sigma^2 \cdot \mathbf{1}.$$

- This implies that $1+1/\beta$ is the maximum number of users per degree of freedom.

SPECIAL CASE OF N=2

$$P_1 \ge a_f + b_f P_2, \text{ and } P_2 \ge c_f + d_f P_1, \; f=1,2,\ldots,F,$$

$$a_f = \frac{\beta_f \sigma^2}{1 - \beta_f(\alpha_{11}^f - 1)}, \; b_f = \frac{\beta_f \alpha_{12}^f}{1 - \beta_f(\alpha_{11}^f - 1)}, \; c_f = \frac{\beta_f \sigma^2}{1 - \beta_f(\alpha_{22}^f - 1)}, \; d_f = \frac{\beta_f \alpha_{21}^f}{1 - \beta_f(\alpha_{22}^f - 1)}.$$

Proposition: In the N=2 uplink power control problem with common SINR requirements, the solution exists if and only if

$$\frac{1}{\beta_f} > \alpha_{11}^f - 1, \; \frac{1}{\beta_f} > \alpha_{22}^f - 1, \; f=1,2,\ldots,F, \text{ and } \frac{1}{\max_{f=1,2,\ldots,F}(b_f)} > \max_{f=1,2,\ldots,F}(d_f).$$

The optimum power vector is the intersection of the 2 curves $P_1 = \max_{f=1,2,\ldots,F}\{a_f + b_f P_2\}$ and $P_2 = \max_{f=1,2,\ldots,F}\{c_f + d_f P_1\}$. Iterative algorithm can be used to find the minimum power solution in this fixed point problem.

$$P_1^{(0)} = P_2^{(0)} = 0, \; P_1^{(i+1)} = \max_{f=1,2,\ldots,F}\left\{a_f + b_f P_2^{(i)}\right\}, \; P_2^{(i+1)} = \max_{f=1,2,\ldots,F}\left\{c_f + d_f P_1^{(i)}\right\}.$$

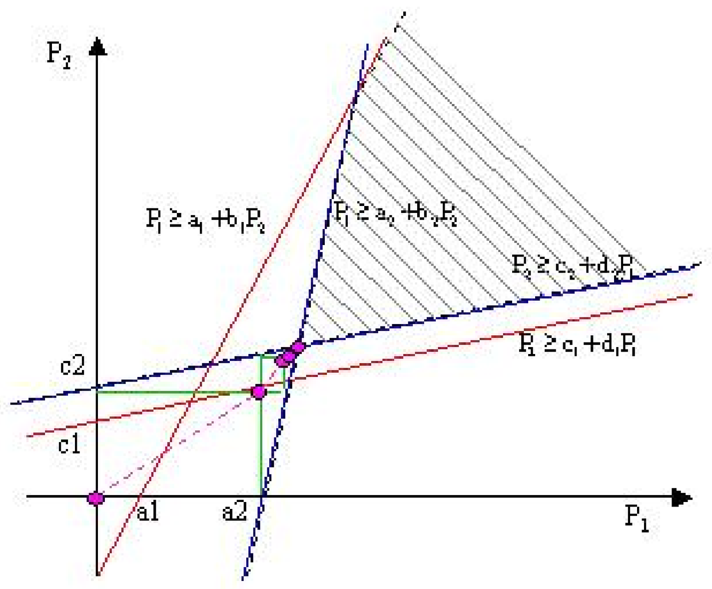
Example of N=2 and F=2

- The sufficient and necessary condition to have solution is

$$\frac{1}{\beta_1} > \rho_{A^{(1)}}, \; \frac{1}{\beta_2} > \rho_{A^{(2)}}, \; \frac{1}{b_2} > d_1, \text{ and } \frac{1}{b_1} > d_2.$$

- The following iterative algorithm converges to the optimum vector.

$$P_1^{(i+1)} = \max\left\{a_1 + b_1 P_2^{(i)}, a_2 + b_2 P_2^{(i)}\right\},$$
$$P_2^{(i+1)} = \max\left\{c_1 + d_1 P_1^{(i)}, c_2 + d_2 P_1^{(i)}\right\}.$$



GENERAL CASE OF N>1 and F>1

$$\mathbf{P} \ge \mathbf{I}(\mathbf{P}), \quad I_i(\mathbf{P}) = \max_{f=1,2,\ldots,F}\left\{\beta_f \left(\mathbf{A}^{(f)}\mathbf{P}\right)_i + \beta_f \sigma^2\right\}, \; i=1,2,\ldots,N.$$

Proposition: If $\mathbf{P} \ge \mathbf{I}(\mathbf{P})$ with $\mathbf{I}(\mathbf{P})$ defined above has solution, then the iterative algorithm $\mathbf{P}^{(i+1)} = \mathbf{I}(\mathbf{P}^{(i)})$ converges to the optimum power vector .

Downlink Analysis

- The minimum power level of base station, P^*, satisfies $\frac{\sigma^2}{P^*} = \min_f\left(1 + \frac{1}{\beta_f} - \max_i \alpha_{if}\right)$.
- Feasible β_f should satisfy $\frac{1}{\beta_f} > \max_i \alpha_{if} - 1$.

SPECIAL CASE OF F=1

$$\frac{\sigma^2}{P^*} = 1 + \frac{1}{\beta} - \max_i \sum_j \rho_{ij}^2.$$

Proposition: The optimum sequences to minimize the power in downlink CDMA system with SINR requirement β are WBE sequences for N>L, and orthogonal sequences for $N \le L$. the minimum power assignment is

$$N > L: P^* = \frac{\beta\sigma^2}{1 + \beta - \frac{N}{L}\beta}, \quad N \le L: P^* = \beta\sigma^2.$$

- This implies that $1+1/\beta$ is the maximum number of users per degree of freedom.

SPECIAL CASE OF N=1

$$\frac{\sigma^2}{P^*} = \min_f\left(1 + \frac{1}{\beta_f} - \alpha_f\right), \; \alpha_f = \sum_g \rho_{gf}^2.$$

- For N=1 special case, both downlink and uplink are point-to-point link.
- If $F \le L$, then orthogonal sequence is the optimal sequence.
- If F>L, the problem is now $\max_S \min_f \left(1 + \frac{1}{\beta_f} - \alpha_f\right)$.

Proposition: The set of solutions for $\max_S \min_f \left(1 + \frac{1}{\beta_f} - \alpha_f\right)$ with $\beta_1 \le \beta_2 \le \cdots \le \beta_F$ includes solutions that satisfy $\alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_F$.

GENERAL CASE OF N>1 and F>1

Proposition: The set of optimal solutions of $\max_S \min_f \left(1 + \frac{1}{\beta_f} - \max_i \alpha_{if}\right)$ with $\beta_1 \le \beta_2 \le \cdots \le \beta_F$ includes solutions that satisfy $\alpha_{if} = \alpha_f$ and $\alpha_1 \ge \alpha_2 \ge \cdots \ge \alpha_F$.

SUMMARY

We studied a power controlled CDMA system with N nodes and F flow types with the constraint that each node uses the same power level for all flows.

- For the F=1 case, we found that for both the uplink and the downlink if N>L, the optimum sequences are WBE sequences, and the user capacity is $1+1/\beta$ users per degree of freedom. Also if $N \le L$, the optimum sequences are orthogonal.
- For the uplink problem with N=2 and F arbitrary, the necessary and sufficient condition to have a solution was found and proved. We extended the results to N=2 with different flow types at the two nodes by minor modification of the notation.
- For the general N>1 uplink problem, we provided an iterative algorithm to find the optimal solution and proved its convergence. Also, the results were extended to the case with different flow types at nodes.
- For the downlink case with F>1, the power assignment problem was solved and some properties of the optimum sequences were proved.