Flight Cancellation Behavior and Aviation System Performance

Analytical Support to the FAA Office of Performance Analysis and Strategy

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Acknowledgments

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Executive Summary

To better understand and mitigate flight delays, the FAA's office of Performance Analysis and Strategy (PAS) forecasts delays at the nation's 30 major airports on a monthly basis. The purpose of this effort is to identify airports with significant delays, so that appropriate steps may be taken to prevent or mitigate such delays. The delay forecasts are performed six months in advance using demand projections based on published (look-ahead) airline schedules. The demand sets used for delay forecasts do not include flight cancellations made in response to operational contingencies. Therefore, in order to improve the fidelity of the FAA's delay model, it will be necessary to formulate a suitable model for cancellations.

This study focuses on two pieces of analysis to fulfill this goal. First, the factors that lead to flight cancellations need to be understood. We use econometric choice models applied to a large historical dataset to infer the preferences and behaviors of airlines with respect to flight cancellations. We have found that flight characteristics are an important factor for cancellations, even when accounting for adverse weather effects. Airline heterogeneity is large and reveals different behavior across the industry.

Second, we will use queueing simulation techniques similar to that used by the current FAA model to determine the effect flight cancellations have on delay estimates. The cancellation model estimates are used to predict flight cancellations and evaluate their impact on queueing delay forecasts used by the FAA.

We examine the effect of cancellations on delays using a sample of 160 airport-days. Our results indicate that delays are over-predicted by the FAA queueing models by 30% when not considering flight cancellations. Even a very naïve model of flight cancellations, where only time-of-day and distance effects are considered, will reduce the delay estimates by 9%.

Finally, a case study was performed based on the current operating environment at SFO, where runway closures in the summer of 2014 will lower the airport's capacity. Moreover, airlines are proposing schedules with 5% more demand. The increased schedule leads to an increase in queueing delay. We used a cancellation model with queueing delay as an explanatory variable to predict the increase in cancellations caused by the increase in demand. The results from our model indicate that departure cancellations will increase at an almost one-to-one ratio with the proposed demand increase, thus negating any benefit to airlines from a denser schedule.

1 Introduction

Flight delay is one of the primary performance metrics used in the aviation industry. Due to the scheduled nature of air transportation, small delays in the system can propagate to many other flights (Beatty, et. al., 1999), resulting in large delays for many passengers. On-time performance is a key metric airlines use to create a competitive advantage in the industry. In addition to being a slight on the reputation of airlines, flight delays are extremely costly, to both the airlines and the passengers. A recent study estimated the total flight delays in the year 2007 to be $32.9 billion (Ball, et.al. 2010).

Flight delays are a function of several factors, including the demand resulting from the flight schedule, and the capacity of the various components in the aviation system. One factor that greatly affects flight delays, but is not entirely understood is flight cancellations. Flight cancellations effectively cause a reduction in demand, which can in turn reduce delays for other flights in a queued system. Xiong (2010) has investigated this process during GDPs and found that airlines make tradeoffs between flight cancellations and flight delays.

To better be able to predict flight delays, we must also understand the factors leading to flight cancellations. Extreme weather is one of the most commonly attributed reasons for flight cancellations. Often, however, flights will be cancelled for strategic reasons. A flight could be cancelled to reduce delays on other flights for the same airline under periods of reduced capacity at a destination airport. Or a flight could be cancelled for reasons of safety, such as mechanical problems, or purely economic ones, such as low ridership. The exact factors that go into which flights are cancelled are not very well understood and likely vary across airlines.

Moreover, flight cancellations in their own right are a major source of delay and inconvenience to passengers. Bratu and Barnhart (2005) suggest that a majority of passenger delay was due to flight cancellations, despite cancellations making up a very small (2%) percentage of flight operations. Flight cancellations are much more onerous for passengers than flight delays for a number of reasons. First, rebooking the passengers requires finding empty seats on already crowded planes and can result in many hours or even days of delays for the passengers, particularly if the passengers have connecting flights. Second, flight operations are severely impacted because airlines typically use the same aircraft for several flight segments in a row. A flight cancellation will thus have an impact on downline segments ranging from a new aircraft assignment to additional cancellations.

There exists little work on the effect of flight cancellations on delay forecasts. Most of the work relating cancellations to delays is motivated by the goal of developing tactical decision-support tools for airlines (Cao and Kanafani, 1997; Argüello et. al. 1997; Yan and Yang, 1996) or assessing demand uncertainty during Ground Delay Programs (Ball et. al. 2001; Willemain 2002).

In this research, we will investigate the factors that contribute towards flight cancellations through the use of discrete choice models applied to historical flight data. From these models we can predict cancellation probabilities for each flight given certain characteristics of the flight. We will then use these cancellation probabilities in a queueing model to estimate the effect cancellations have on flight delays. We will incorporate the probabilistic cancellations into the queueing models using both Monte Carlo simulation and a first-order approximation and evaluate the differences between the two.

This report is organized as follows. Section 2 uses discrete choice methods applied to a large sample of flight on-time performance data to model the behavior of airlines regarding flight cancellations. Section 3 addresses some extensions to the basic discrete choice model that allow for heterogeneity in behavior across airlines and correlations between flight cancellations decisions across time. Section 4 evaluates the effect of cancellation prediction estimates from the choice models on flight delay forecasts using deterministic queueing models. Section 5 will provide a case study at SFO and the changes to capacity and demand that are scheduled to occur in the summer of 2014. Section 6 includes conclusions and final recommendations.

2 Cancellation Analysis

2.1 Cancellation Behavior

Flight cancellations are low probability events, and are inherently difficult to predict. However, when flight cancellations do occur, the impact is substantial. The passengers on the cancelled flight must be rebooked on other flights, often hours later. On the other hand, cancelled flights can reduce delays on later flights. Moreover, any delay that would be incurred by the cancelled flight will also be avoided. All of these effects must be considered when airlines decide to cancel flights, and their relative importance depends on many factors. Thus, when developing models to infer the preferences airlines have for deciding which flights to cancel, one must take into consideration many different variables.

Previous work on airline cancellation behavior has shown that flight cancellations are less likely on more competitive routes, flights into and out of hubs, and infrequently served routes (Rupp and Holmes, 2006). Fuller flights have been found to be less likely to be cancelled (Tien, et. al., 2009). During Ground Delay Programs (GDPs), airlines exhibit tradeoff behavior between flight cancellations and delays (GAO, 2011). This is partially due to the nature of GDPs, where airlines can keep ownership of the slots for flights they cancel. Such tradeoff behavior may be present to some degree even in flights not involved in GDPs, though. Distance and departure time heterogeneity has also been investigated (Xiong, 2009).

The exact factors that determine which flights are cancelled are not very well understood and likely vary across airlines. The first part of this report addresses this issue by using discrete choice models to infer airline preferences regarding flight cancellations. This analysis will allow airline cancellations to be predicted and incorporated into delay prediction models. The flight cancellation models presented here relate certain aircraft, flight, route, and airport characteristics to the probability of a flight being cancelled.

2.2 Econometric Model

For this analysis, airlines are viewed as decision makers that face an option to cancel or not cancel each flight in their schedule. For the purposes of this research, airlines are assumed to be utility maximizers. That is, airlines derive a certain amount of utility from each possible option for a flight (cancel or not cancel), and each choice is made because it maximizes the airline's utility for that possible choice situation. A set of observable factors that affect the airline's cancellation utility for a given flight are identified. These factors will enter into a random utility model in a linear fashion as follows:

$$U_{cancel} = V_{cancel} + \varepsilon_{cancel} = \sum_j \beta_j x_{cancel,j} + \varepsilon_{cancel} \qquad \text{(Equation 2-1)}$$

where U_{cancel} is the utility derived by the airlines for cancelling a particular flight, $x_{cancel,j}$ is the observable factor, j, corresponding to the flight in question, β_j are the coefficients corresponding to the observable factors, and ε_{cancel} represents the unobserved factors that influence the utility for the cancellation choice. V_{cancel} is called the deterministic utility because it contains the factors that are observable to the researcher, and ε_{cancel} is the random utility which contains factors that may be known to the choice maker, but cannot be observed by the researcher. Since we do not observe all the factors that influence the utility of cancellation, the remaining

influences that are unobservable to us appear random for each choice situation, hence the name and notation.

The type of discrete choice model used depends on the choice of distribution of the random utility, ε_i. One of the most popular models, which is used here for the initial model, is the logit model. This model assumes the random utilities, ε_{cancel}, are identically and independently distributed extreme value. The logit model is popular primarily because it results in a convenient, closed-form expression for the choice probabilities. The choice probabilities are estimated using maximum likelihood and the closed-form expression is shown below:

$$P_{cancel} = \frac{e^{V_{cancel}}}{1+e^{V_{cancel}}} = \frac{e^{\Sigma_j \beta_j x_{cancel,j}}}{1+e^{\Sigma_j \beta_j x_{cancel,j}}}$$
(Equation 2-2)

where P_{cancel} represents probability the flight in question is cancelled. We estimated the logit models using the Matlab software package, based on code provided by Professor Kenneth Train from UC Berkeley (Train, 2003).

2.3 Data

Historical airline on-time performance data will be used for this research. The primary reason for this is the abundant amount of on-time flight performance data available online. Survey data, while easily able to capture the exact tradeoffs of interest, would likely be very difficult to get. Airlines might not be interested in sharing their preferences for cancellations due to competitive advantages over other airlines, and in any case the information they provide may not be as reliable as their observed behavior. Historical flight data provides a large pool of cancellation decisions across many different airlines.

The flight cancellation data is taken from the on-time performance database obtained by the Bureau of Transportation Statistics (BTS) for all dates from 2010 to 2011, resulting in almost 12 million domestic flights. The data set includes on-time performance metrics for every flight scheduled by all airlines that have at least a 1% market share. Fare information is obtained from the BTS Airline Origin & Destination (O&D) survey that is a 10% sample of airline tickets from reporting carriers and includes quarterly average fare data for every major airport market pair. The aircraft information was obtained from the FAA Aircraft Registry database and paired with tail numbers from the on-time performance data. Finally, segment traffic information was obtained from the BTS T-100 database, and includes monthly averages for specific non-stop flight segments for each airline and aircraft type. Finally, hourly airport weather information was determined from the FAA Aviation System Performance Metrics (ASPM) database and the National Oceanic and Atmospheric Administration (NOAA).

After combining the data sources, the number of flights was reduced to about 8 million due to missing observations and differences in level of detail for each dataset. For example, some of the datasets only have information for flights corresponding to the ASPM77 airports. SAS software was used to aggregate and match the data from the different sources together. The data sources are shown below in Table 2-1.

Table 2-1: Data Sources

Data Source Summary		
Data Type	**Aggregation level**	**Source**
Cancellation / delays	Flight-by-flight	BTS Airline On-Time Performance Data
Market fares	Quarterly averages	BTS Airline O&D Survey
Aircraft information	Flight-by-flight	FAA Aircraft Registry
Market ridership	Monthly averages	BTS T-100 Database
Weather information	Hourly	ASPM and NOAA Databases

2.4 Model Specification

The explanatory variables used in the initial binary logit model are divided into several categories. The first group is flight characteristics, which include the average fare, number of seats, average load factor, and the flight frequency offered by the airline. The average fare is taken from the DB1B database and is aggregated over all flights in a quarter for the same airline, and non-stop segment. The number of seats is specific to the aircraft type and varies over each flight. The average load factor is aggregated over all flights in a month for the same airline, route, and aircraft type. Flight frequency is the average daily frequency of flight operations for an airline for a single route, averaged over a month.

The next category is airport congestion, which we capture by calculating queueing delay using the scheduled demand and realized capacities at the origin and destination airports. A deterministic queueing algorithm is used to simulate departures and arrivals at each airport separately (see section 4.2.1). The queueing delay is defined as the difference between the time when a flight can actually depart (or arrive) and the time that the flight was scheduled to depart (or arrive), assuming a first scheduled, first served queueing discipline. The queueing delay calculated in this way represents the level of congestion at an airport at the scheduled time of departure (departure delay) or the scheduled time of arrival (arrival delay).

We were also interested in capturing the effect of hub airports on an airline's cancellation behavior. Thus we included two dummy variables that are equal to one if the origin is a hub airport for the airline operating the flight (Hub_Origin) and if the destination is a hub airport for the airline operating the flight (Hub_Dest). We defined a hub airport to be one of the primary one of the hub airports listed by airline on their own website. If an airline did not list hubs online then we used the airports operated by that airline with at least 5% of their total operations. In general these airports corresponded to the ones that were listed internally as well. The hub airports used in our analysis are shown below in Table 2-2.

Table 2-2: Hub Airports

Airline	Hub Airports
AA (American)	DFW, ORD, MIA, JFK, LAX
AS (Alaska)	SEA, ANC, LAX, PDX
B6 (JetBlue)	JFK, BOS*
CO (Continental)	IAH, EWR, CLE
DL (Delta)	ATL, SLC, CVG, JFK, MSP, DTW, MEM
F9 (Frontier)	DEN
FL (AirTran)	ATL, MKE, MCO
UA (United)	ORD, SFO, IAD, DEN, LAX
US (US Airways)	PHX, CLT, PHL
WN (Southwest)	MDW*, LAS*, BWI*

* Airport not listed as hub, but with > 5% of total operations

In addition to airport congestion, we wanted to capture the effect of Ground Delay Programs (GDPs) on flight cancellation behavior. GDPs provide greater flexibility for airlines to change operations during a period of high delay, so we use a dummy variable to capture this. The GDP dummy variable is equal to one if the destination airport is involved in a GDP at the scheduled time of departure of a given flight. From the ASPM database, we have information regarding the number of Expected Departure Clearance Time (EDCT) flights that are scheduled to arrive at a destination airport for a particular quarter-hour time window. For a particular flight, we set the GDP variable equal to one if the destination airport has a non-zero number of EDCT flights scheduled for arrival during the quarter hour that corresponds to the flight's scheduled departure time. We use the departure time here rather than arrival time because we are trying to capture the conditions at the destination airport at the scheduled departure time. This time does not necessarily correspond to the time that the cancellation decision is made or the time in which the EDCT flight information is available to the airline (which is often much earlier), but since we do not know specific information about each EDCT flight we use this time in our analysis as a proxy for the cancellation decision time.

We are also interested in time-of-day effects. At the beginning of the day, airlines have more resources available to handle flight disruptions. Cancellations affect flight operations later in the day, since aircraft, passengers, and crew need to be changed from their original schedule and flight delays build up throughout the day as small disturbances are propagated throughout the network. In addition, later flights have less flexibility for rescheduling passengers than earlier flights, while cancelling them has less impact on flight legs downstream.

We used a simple four level categorical variable based on the local departure time for a given flight. We divided the day into the following categories: (0300-0900, 0900-1500, 1500-2100, and 2100-0300). A dummy variable for each period was defined, with the 0300-0900 category set to zero as a base for comparison.

Distance effects are likely important as well. Longer flights must be cancelled well in advance of arrival time, so airlines do not have as much information about conditions at the destination for longer flights compared to shorter flights. Longer flights are also less frequent and larger, but since we are already capturing those effects explicitly using other variables, we will capture any distance effects that are independent of these other effects. We use five categories for distance, with the following ranges: 0-500 mi, 500-750 mi, 750-1000 mi, 1000-1500 mi, and 1500 mi or greater). A dummy variable is used for each category with the exception of the 500-750 mi category, which is set to zero as a base for comparison.

As mentioned earlier, one of the primary drivers of cancellations is weather. Thus, we capture weather at both the origin and destination through several different variables. The weather effects we measure are visibility, temperature, and wind speed, as well as indicators for the presence of Instrument Meteorological Conditions (IMC), Rain, Thunderstorms, and Snow. We record the weather at the origin airport at the scheduled time of departure and at the destination airport at the scheduled time of arrival.

Lastly, we wanted to capture airline heterogeneity through fixed effects for each airline. Thus, we have 11 dummy variables in total, with Mesa Airlines' low-cost Hawaiian carrier, go!Airlines arbitrarily chosen as the base. We also combined the regional affiliate flights with the mainline carrier and designated a dummy variable that is equal to one if the flight is not a mainline flight. For instance, if the flight is listed as a United flight, but operated by any of the regional affiliates under the United Express designation, then our regional carrier dummy will be set to 1 and the airline dummy for United will be set to 1 as well.

A list of explanatory variables used in our analysis is shown below in Table 2-3.

Table 2-3: Explanatory Variable Description

Category	Explanatory Variable	Variable Description
Flight Characteristics	Fare ($100)	Average quarterly fare for OD & equip
	Number of Seats (100s)	Aircraft size
	Load Factor	Average monthly load factor for OD & equip
	Flight Frequency (Daily)	Number of daily flights for OD & equip
Airport Congestion	Dep. Queueing Delay (10s min.)	Dep. queueing delay at origin (10s min.)
	Arr. Queueing Delay (10s min.)	Arr. queueing delay at destination (10s min.)
Hub Airport Effects	Airline Hub @ Origin	Hub airport at origin
	Airline Hub @ Dest	Hub airport at destination
Ground Delay	GDP (0 or 1)	Ground Delay Program in effect at destination
Time and Distance Fixed Effects	Dep. Time Categories	Four categories for time of day departure
	Distance Categories	Five categories for flight distance
	Day of Week	Dummy for each day
Weather Effects	Weather at Origin and Destination	IMC, Visibility, Wind Speed, Ceiling
		Rain, Snow, Thunderstorms
Airline Effects	Airline Fixed Effects	Dummy variables for 11 airlines

2.5 Summary Statistics

After combining all of our data sources, accounting for missing observations, missing data fields, and data matching issues, our full data set includes 8,857,952 flight observations over a two year period. Within this period there were 129,415 cancellations, or approximately 1.5% of all flights. The mean and standard deviation of some of the explanatory variables used in our model are shown below in Table 2-4. For the flight frequency, we calculated the mean value across all unique origin, destination, airline, and month combinations, so that the mean does not capture repeat observations that inherently have higher flight frequencies.

Table 2-4: Variable Summary Statistics

Variable	Mean	Std.
Avg. Fare ($)	183.69	63.62
Number of Seats	129.37	40.6
Load Factor	0.80	0.10
Daily Flights	3.06	2.73
Dep. Queueing Delay (min.)	2.99	1.89
Arr. Queueing Delay (min.)	2.43	1.96
Hub Origin	0.31	0.46
Hub Destination	0.31	0.46
Ground Delay Program	0.04	0.2
Distance (<500 mi)	0.32	0.47
Distance (750-1000 mi)	0.17	0.38
Distance (1000-1500 mi)	0.16	0.37
Distance (>1500 mi)	0.15	0.35
Dep. Time (9:00-15:00)	0.05	0.09
Dep. Time (15:00-21:00)	0.38	0.48
Dep. Time (21:00-3:00)	0.21	0.41
Regional Carrier	0.19	0.39

Some things to note are the average queueing delay of around 3 minutes for departures and 2.4 minutes for arrivals, and the 4% of flights that are involved in a GDP. The average number of flights per day between a given origin and destination for a given airline is 3.1. 31% of flights are departing from a hub, and 31% are arriving at a hub and almost 20% of all flights are operated by a regional carrier. The summary statistics (mean and std. deviation) for the weather effects used in our model are shown below in Table 2-5.

Table 2-5: Weather Summary Statistics

Variable	Mean	Std.
IMC Dest (0 or 1)	0.14	0.35
Temp Dest (deg F)	63.24	19.04
Vis Dest (mi)	9.33	1.9
WindSpd Dest (mph)	8.85	5.57
IMC Origin (0 or 1)	0.14	0.35
Temp Origin (deg F)	63.23	19.06
Vis Origin (mi.)	9.31	1.92
WindSpd Origin (mph)	8.82	5.55
Dest Rain (0 or 1)	0.060	0.230
Dest Snow (0 or 1)	0.010	0.120
Dest TStorm (0 or 1)	0.010	0.080
Origin Rain (0 or 1)	0.060	0.230
Origin Snow (0 or 1)	0.010	0.120
Origin TStorm (0 or 1)	0.010	0.080

The mean value and standard deviation is shown for each variable. For the indicator variables (with a 0 or 1 value), the mean is simply the percentage of flights with that weather condition. For instance, 14 % of flights faced IMC conditions at the destination, and 1% of flights had snow at the destination at the time of scheduled departure. The average visibility was 9 miles, with a significant standard deviation (1.9 mi.), and the average wind speed was 8.8 mph. Visibility ranged from 0 to 10 miles, with 84% of the observations having a visibility of 10 miles. Wind speed ranged from 0 to 47 mph, with 90% of observations having a wind speed of less than 12 mph.

The percentage of flights cancelled for each airline in our sample is shown in Table 2-6, along with the percentage of flights from our sample operated by each airline and the total number of cancellations during our sample period.

Table 2-6: Airline Summary Statistics

Airline	Cancellation %	% of Flights	Cancellations
DL (Delta)	1.80%	20%	31,889
UA (United)	1.50%	10%	13,287
US (US	1.40%	10%	12,401
AA (American)	2.40%	15%	31,889
CO (Continental)	1.40%	9%	11,161
WN (Southwest)	0.90%	22%	17,539
B6 (JetBlue)	1.50%	4%	5,315
F9 (Frontier)	0.40%	2%	709
FL (Air Tran)	1.00%	5%	4,429
AS (Alaska)	0.30%	3%	797
Overall	**1.48%**	**100%**	**129,416**

There is large variation in cancellation percentages across airlines in our sample, ranging from Alaska Airlines that only cancelled 0.3% of its flights during the two-year period of interest, to American Airlines, which cancelled 2.4% of its flights.

We have clear hypotheses about how many of the flight characteristics in the model should affect the likelihood of cancellation. Larger and fuller flights should be less likely to be cancelled in order to minimize the cost due to rescheduling passengers. Higher fare routes should be cancelled less frequently than lower fare routes because the airlines are seeking to maximize profits. Routes with higher fares are associated with the presence of high-value customers that represent a major source of revenue for the airline. Based on our discussions with airline employees, the airlines try to minimize the inconvenience of these passengers by cancelling their flights less than other flights with lower-value customers.

High flight frequency between two airports allows for easier rebooking of passengers, so these flights should be more likely to be cancelled than flights with low frequency. It is hypothesized that airlines seek to minimize their own network disruption through propagated delays, and thus flights into and out of hubs should be less likely to be cancelled than other flights. Poor weather makes cancellations more likely than fair weather. Airport capacities are reduced in times of poor weather, which can lead to large delays and cancellations. Lastly, congestion in the form of flight delays should make cancellations more likely. These hypotheses, summarized below in Table 2-7, will be referenced when discussing the results from the initial model.

Table 2-7: Cancelation Hypotheses

Flight Cancellation Hypotheses	
Flight Characteristic Trend	**Impact on Cancellation Likelihood**
Larger aircraft (vs smaller aircraft)	Less likely
High load factor (vs low load factor)	Less likely
Route with higher average fare (vs lower average fare)	Less Likely
Route with high flight frequency (vs low frequency)	More likely
Flight is into or out of airline hub	Less likely
Flights with poor weather at origin or destination	More likely
Flights with more queueing delay at origin or destination	More likely
Flight with GDP at destination	More likely
Flight operated by regional carrier	More likely
Longer flights (vs shorter flights)	Less likely
Evening departure times (vs morning departure times)	More likely

2.6 Estimation Results

The large amount of data in our sample prohibited us from estimating a single model for all flights across the two year time span. Thus, we created a simple random sample that is approximately 33% of the size of the full sample by selecting each flight for inclusion in the sub-sample with equal probability. The resulting subsample accounted for 3 million flights. The model estimates are shown below in Table 2-8 and Table 2-9.

Table 2-8: Logit Estimation Results 1

Variable	Estimate		Std. Err.	Variable	Estimate		Std. Err.
ASC(Cancel)	-2.422	**	0.118	IMCDest	-0.025		0.016
Fare($100)	0.071	**	0.011	TempDest (10s deg F)	-0.012	**	0.004
DepTime(9:00-15:00)	-0.064	**	0.014	VisDest (mi.)	-0.073	**	0.003
DepTime(15:00-21:00)	0.041	**	0.014	WindDest (mph)	0.019	**	0.001
DepTime(21:00-3:00)	0.159	**	0.041	IMCOrigin	0.031		0.016
Miles<500	-0.004		0.014	TempOrigin (10s deg F)	-0.029	**	0.004
Miles750-1000	-0.137	**	0.018	VisOrigin (mi.)	-0.097	**	0.003
Miles1000-1500	-0.105	**	0.020	WindOrigin (mph)	0.030	**	0.001
Miles1500more	-0.303	**	0.026	Hub Origin	-0.245	**	0.017
Num.Seats(100)	-0.276	**	0.037	Hub Dest	-0.086	**	0.017
LoadFactor	-2.142	**	0.051	GDP	0.359	**	0.022
FlightFrequency(flight/day)	0.039	**	0.002	Dest_Rain	0.047	*	0.019
Dep. Delay (10s min)	0.302	**	0.009	Dest_Snow	0.930	**	0.024
Arr. Delay (10s min)	0.319	**	0.010	Dest_TStorm	0.867	**	0.039
Dep. Delay Squared (100s min^2)	-0.014	**	0.001	Origin_Rain	0.200	**	0.019
Arr. Delay Squared (100s min^2)	-0.014	**	0.001	Origin_Snow	0.982	**	0.024
Sunday	-0.149	**	0.019	Origin_TStorm	0.949	**	0.039
Monday	-0.218	**	0.019	Regional_Carrier	0.124	**	0.036
Tuesday	-0.068	**	0.018				
Thursday	-0.080	**	0.018				
Friday	-0.149	**	0.018	** Significant at 1% level			
Saturday	-0.260	**	0.021	* Significant at 5% level			

Table 2-9: Logit Estimation Results 2

Variable	Estimate		Std. Err.
DL (Delta)	1.132	**	0.096
UA (United)	1.156	**	0.097
US (US Airways)	0.914	**	0.096
AA (American)	1.423	**	0.096
CO (Continental)	0.764	**	0.098
WN (Southwest)	0.820	**	0.095
B6 (JetBlue)	1.199	**	0.098
F9 (Frontier)	0.262	*	0.117
FL (Air Tran)	0.625	**	0.100
AS (Alaska)	-0.010		0.115

** Significant at 5% level

As shown in Table 2-8, above, the vast majority of the variables are significant. With one exception, results match our expectations. Fare appears to have a positive and significant effect, which is contrary to our hypothesis. The estimated coefficients on other flight characteristic variables are consistent with our expectations. Load factor has a negative and large sign. Higher load factors make a flight much less likely to be cancelled. Similarly, aircraft size has a negative effect as well. Departure time of day shows an increasing likelihood of cancellation as the day progresses. The baseline departure time category is 3:00-9:00, so the signs of the coefficients of the other categories are measured relative to the baseline category. There is a small negative sign for 9:00 – 15:00 and a small positive sign for 15:00 – 21:00. The coefficient for the last group, 21:00 – 3:00 is much larger than the other coefficients and positive. This indicates that late night flights are more likely to be cancelled than earlier flights. We expect later flights to be cancelled more than earlier flights at least partially due to higher delays that build up over the course of the day. Although we are capturing queueing delays explicitly, these do not reflect the cumulative effect of earlier delays on a flight.

The distance effects generally match our expectations. The baseline category is the 500-750 mile group, so the coefficients are interpreted with respect to that category. The distance effect decreases roughly monotonically with distance. Thus we see that, in general, longer flights are less likely to be cancelled than shorter flights, even when accounting for the effects of aircraft size, load factor, and frequency separately. This is consistent with our expectations and conversations with flight dispatchers. Airlines can wait longer to make cancellation decisions for shorter flights so that they have better information about conditions at the destination. Thus they tend to allow longer flights to proceed on the assumption that conditions at the destination at the time of arrival will be fairly normal. This behavior is further encouraged under GDPs when longer flights are often exempted from ground delays.

Flight frequency is positive and significant, which also matches our expectations. We would think that the more flights that are offered by an airline on a particular route makes accommodating passenger routing changes easier when a cancellation is necessary. Thus, a flight on a route with higher frequency is most likely to be cancelled than a flight on a route with lower frequency, all

else equal. These effects together illustrate the tradeoffs made by airlines to minimize the disruption of passengers.

Both of the queueing delay variables, which represent the level of congestion at the origin and destination airport, are statistically significant and positive, with similar magnitudes. This indicates that larger queueing delays, caused by an imbalance between demand and capacity, highly influence cancellations. We suspected that there was a non-linear effect of queueing delay on cancellation utility, so we included the square of departure and arrival delay as well. These two coefficients are both negative and significant, which suggests that there is a diminishing marginal effect on cancellation utility as the queueing delays become very large.

Next, we consider the day-of-week effects. Wednesday is set to zero as a baseline for comparison. The results suggest that flights on weekend days are less likely to be cancelled than flights in the middle of the week. Based on conversations with flight dispatchers, aircraft maintenance is often scheduled for the middle of the week, which makes aircraft substitutions more difficult in the event of a mechanical issue. This could be one reason for this trend in cancellations throughout the week.

The weather effects are mostly significant and consistent with our expectations. The only non-significant weather variables are the IMC variables, which indicate that we are explicitly capturing all of the factors contributing to IMC conditions directly in the other weather variables. Higher temperatures are generally an indication of better weather, and these result in flights being less likely to be cancelled. High winds and low visibility increase the chances of a flight being cancelled. We see a similar effect of weather at both the origin and destination. Recall that we measured the weather for the origin at the scheduled time of departure and for the destination at the scheduled time of arrival. The cancellation decision has be made prior to departure, so there is inherently less certainty associated with the weather conditions at the destination. It appears that forecasts are reliable enough at the time of these decisions to overcome this.

The other weather variables were entered as indicators, taking a value of one if the condition was present. The conditions we measured were rain, snow, and thunderstorms at the origin and destination. Not surprisingly, the presence of snow and thunderstorms increase the chance of cancellation more than rain. To gauge the magnitude of the effect of snow and thunderstorms on the cancellation likelihood, note that the presence of thunderstorms at the origin is equivalent to almost 30 mph winds, while the presence of snow has an even stronger effect. Snow and thunderstorm have impacts of roughly equal magnitude whether they are at the origin or destination, similar to what we saw for visibility and wind.

Next we look at the hub variables. These are indicator variables that are equal to one if the flight departs from a hub airport of the airline operating the flight (Hub_Origin) or arrives at a hub airport of the airline operating the flight (Hub_Dest). Both of these coefficients are negative and significant, with the origin variable having a larger magnitude. These results suggest that airlines do not like to cancel flights into or out of their hub airports. These flights are important to airlines due to the large number of connecting passengers at hub airports, so this result is not surprising. This also may explain why the Hub_Origin effect is the stronger one, since a cancellation of a flight from a hub "strands" passengers at the connecting hub, rather than their origin or destination.

Now we will consider the airline fixed effects, including the dummy variable for regional carriers. We see that the regional carrier dummy is positive and significant. Regional carrier flights are more likely to be cancelled than mainline flights operated by the same airline, all else equal. This is consistent with what we've seen in practice. Regional carrier flights typically have other characteristics that are favorable for flight cancellations (i.e. short flight distance, smaller planes,

operating out of hubs), so the cancellation effect for these flights is even stronger than what are suggested by the coefficient for the regional carriers.

The airline dummy variables (Table 2-9) are all positive and significant, with the lone exception of Alaska Airlines. Recall that the airline used as the base is the low-cost carrier of Mesa Airlines, go! Airlines. All the coefficients can be interpreted relative to this base carrier. To better infer the meaning of the coefficients, we present a scatter plot of the overall cancellation percentage for the airline on the x-axis and the coefficient fixed effect for the airline on the y-axis. This allows us to observe airlines' proclivity to cancel relative to others when controlling for the properties of the flights the airline operates, as compared to the "raw" cancellation percentages. This plot is shown below in Figure 3-1.
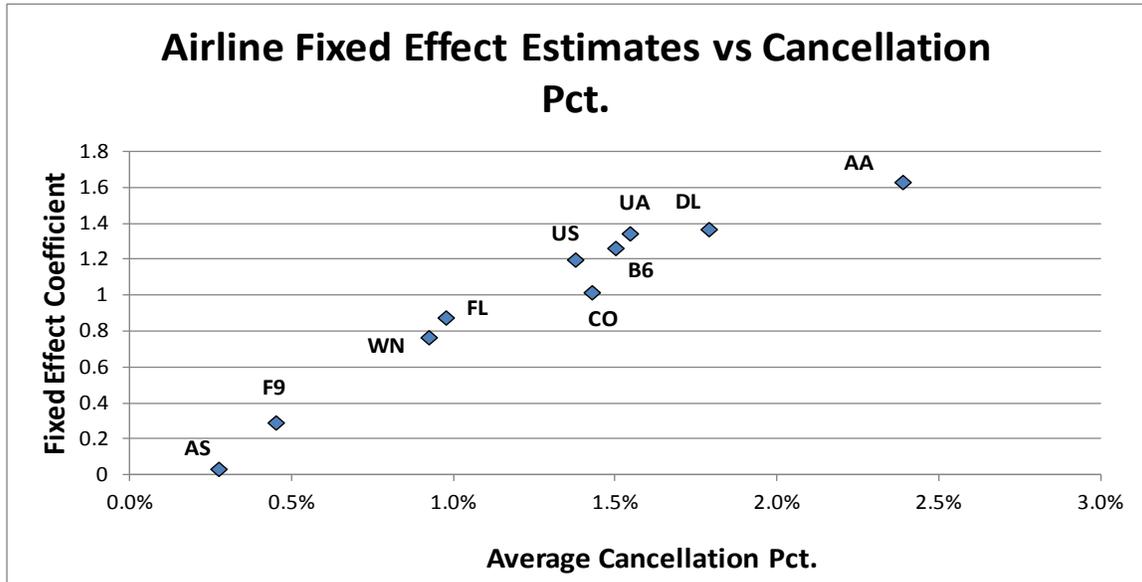
Figure 2-1: Airline Fixed Effects and Cancellation Pct.

From Figure 2-1, we can see that the fixed effect coefficients and cancellation percentage are highly correlated. We can conclude from this that there are large differences in the cancellation rates across airlines and the differences are not caused by differences in the characteristics of the flights, airports, and operating conditions. Some airlines just cancel more than others. The former group consists largely of network, legacy carriers and the latter of low cost carriers. The one exception to this pattern is Jet Blue (B6), which has about the same cancellation proclivity as United, Delta, and US Airways.

We tried various model specifications, including one with airport fixed effects. Specifically, we used dummy variables for flights with an origin or destination at the 16 largest airports. The improvement in model fit was very small compared to the improvement gained by the airline fixed effects. Thus, for our final model we use only airline fixed effects and leave out any airport fixed effects.

We can quantify the effects for each variable by calculating the odds ratio for a given change in a parameter. We define the odds of cancellation as the ratio of the probability of cancelling a flight and the probability of not cancelling a flight:

$$Odds = \frac{\hat{p}_{cancel}}{1 - \hat{p}_{cancel}}$$ (Equation 2-3)

The cancellation probabilities have a closed form solution, as shown in Equation 2-2. We can use the analytical expression from that equation to re-write the odds ratio as follows:

$$Odds = \frac{\frac{e^{\widehat{V}_{cancel}}}{1+e^{\widehat{V}_{cancel}}}}{1-\frac{e^{\widehat{V}_{cancel}}}{1+e^{\widehat{V}_{cancel}}}} = \frac{\frac{e^{\widehat{V}_{cancel}}}{1+e^{\widehat{V}_{cancel}}}}{\frac{1}{1+e^{\widehat{V}_{cancel}}}} = e^{\widehat{V}_{cancel}} = e^{\sum_j \widehat{\beta}_j x_{cancel,j}} \qquad \text{(Equation 2-4)}$$

The odds ratio is simply the ratio of the odds for two different sets of explanatory variables. For example, we can increase the value of one explanatory variable by 1 unit and calculate the odds ratio based on the increase. For this example, we will assume that $x_{cancel,1}$ is increased by 1 unit:

$$OR_1 = \frac{e^{\sum_j \widehat{\beta}_j x_{cancel,j}+\widehat{\beta}_1}}{e^{\sum_j \widehat{\beta}_j x_{cancel,j}}} = \frac{e^{\sum_j \widehat{\beta}_j x_{cancel,j}} e^{\widehat{\beta}_1}}{e^{\sum_j \widehat{\beta}_j x_{cancel,j}}} = e^{\widehat{\beta}_1} \qquad \text{(Equation 2-5)}$$

The odds ratio for a one unit change in an explanatory variable is simply the exponential function of the coefficient for that explanatory variable.

We can re-write the results of Table 2-8 and Table 2-9 in terms of odds ratios for a unit change in the explanatory variables. For some of the explanatory variables, we present the odds ratio for a smaller than unit change in the variable, since a unit change would not represent changes that appear in our dataset. For example, a 1 unit change in load factor is the entire range for that variable. The odds ratios are presented below in Table 2-10.

The odds ratios presented in Table 2-10 can give us a better sense for the magnitude of the impact each explanatory variable has on the relative likelihood of cancelling a flight. For example, the odds of cancelling a given flight is only 76% that of cancelling a flight with 100 fewer seats. Conversely, the odds of cancelling a flight are 1.32 times greater than those for an otherwise identical flight with 100 more seats (1/0.76 = 1.32). The magnitudes of the odds ratios for flight characteristics are much smaller than the magnitude for the odds ratios for weather effects. Consider the odds ratio for the GDP variable, which indicates that the odds of cancelling a flight with a GDP at the destination airport are 1.43 times greater than the odds of cancelling an identical flight without a GDP at the destination airport. Clearly the weather effects are very strong, especially considering many of them could happen at the same times. For example, consider a flight with a GDP at the destination, and snow at both the origin and destination. The combined odds ratio for these three conditions is the product of the three individual odds ratios, or 9.69. The odds of cancelling such a flight are nearly 10 times those of cancelling a similar flight without the presence of snow and a GDP.

Table 2-10: Logit Model Odds Ratios

Variable	Unit Increase	Odds Ratio	Variable	Unit Increase	Odds Ratio
Fare($100)	$100	1.07	IMCDest	1	0.97
DepTime(9:00-15:00)	1	0.94	TempDest	10 deg	0.99
DepTime(15:00-21:00)	1	1.04	VisDest	1 mi	0.93
DepTime(21:00-3:00)	1	1.17	WindDest	10 mph	1.21
Miles<500	1	1.00	IMCOrigin	1	1.03
Miles750-1000	1	0.87	TempOrigin	10 deg	0.97
Miles1000-1500	1	0.90	VisOrigin	1 mi	0.91
Miles1500more	1	0.74	WindOrigin	10 mph	1.35
Num.Seats (100)	100	0.76	Hub Origin	1	0.78
LoadFactor	10%	0.81	Hub Dest	1	0.92
FlightFrequency(flight/day)	1	1.04	GDP	1	1.43
Departure Delay (min)	10 min	1.35	Dest_Rain	1	1.05
Arrival Delay (min)	10 min	1.38	Dest_Snow	1	2.54
Dep. Delay Squared (min^2)	100 min^2	0.986	Dest_TStorm	1	2.38
Arr. Delay Squared (min^2)	100 min^2	0.986	Origin_Rain	1	1.22
Sunday	1	0.86	Origin_Snow	1	2.67
Monday	1	0.80	Origin_TStorm	1	2.58
Tuesday	1	0.93	Regional_Carrier	1	1.13
Thursday	1	0.92			
Friday	1	0.86			
Saturday	1	0.77			

2.7 Cancellation Predictions

We can use the results from the above cancellation model to predict cancellations. We use the cancellation probability formula shown in Equation 2-2, and using our estimates from Table 2-8 and Table 2-9, we can generate a cancellation probability for each flight based on the observable characteristics. Flights that have characteristics favorable for increased chances of cancellations, such as low load factor, small aircraft, short flights in bad weather, will have higher cancellation probabilities than flights with characteristics not favorable for cancellation, such as large aircraft, high load factor, hub-to-hub flights in the morning hours on a good weather day. The coefficients above give us a qualitative sense for which characteristics will lead to a higher or lower cancellation probability, but do not give us a sense for the magnitude of those cancellation probabilities. To illustrate the magnitude that we are talking about here, we calculated the

predicted cancellation probability for the sample of flights used in our analysis and plotted the cumulative probability distribution of the predicted cancellation probabilities. This plot is shown below in Figure 2-2.
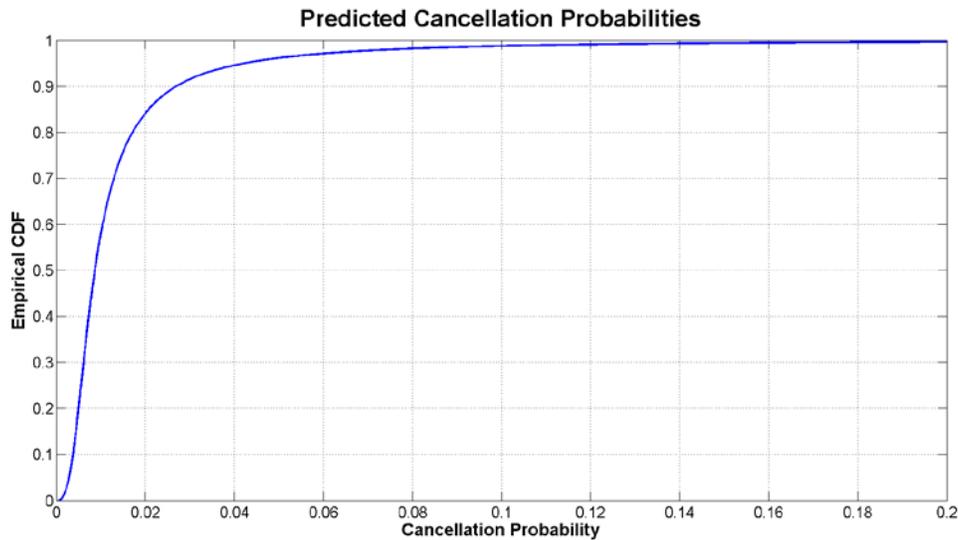
Figure 2-2: CDF of Cancellation Probability for One Month Sample

The curve in Figure 2-2 represents the cumulative probability of the cancellation probability defined by the x-axis. For example, the median cancellation probability for our sample is just below 1%. The mean cancellation probability for our sample is 1.5%. The 90th percentile of cancellation probabilities is less than 3%. The flights with a predicted cancellation probability higher than this typically have a combination of favorable flight characteristics for cancellations and poor weather. We almost never see cancellation probabilities above 20%, even when considering all of these effects.

2.8 Model Fit

So far we have interpreted the cancellation model coefficients in terms of their effect on a predicted cancellation probability, but we have not addressed how well the predicted cancellation probabilities match the cancellations that actually happened. We investigate this by predicting the cancellation probabilities for all flights in our sample and aggregating them over a single day for a single airport. This will give us a total number of predicted flight cancellations for a particular airport and a particular day. We can then compare this number to the total actual number of cancellations on that same day.

Across all airports and all days, our model should give the exact number of cancelled flights. This is a result of us including an alternative-specific constant in the model specification. Doing this forces the actual percentage of flight cancellations to equal the predicted percentage of flight cancellations. This does not have to be true across any subset of our sample, however, so we can use the comparison described above to determine how robust the model is for cancellation predictions at a smaller level.

The method of sample enumeration is used for predicting flights for a single day. The following formula illustrates the technique:

$$\hat{C}_{i,t} = \sum_{j=1}^{n} p_j * d_{t,i} \qquad \text{(Equation 2-6)}$$

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The predicted number of cancellations on day t at airport i is given by $\hat{C}_{i,t}$. The total number of flights in our sample is given by n, each flight j of which has a predicted cancellation probability by p_j. $d_{t,i}$ is a dummy variable, equal to 1 when the flight is on day t with destination airport i, and 0 otherwise.

We will compare the predicted number of cancellations based on our model, $\hat{C}_{i,t}$, with the actual number of cancellations, $C_{i,t}$. For a destination airport i, each day t will be represented by two numbers, $(\hat{C}_{i,t}, C_{i,t})$. We can plot these points to compare the predicted number to the actual number. If the model perfectly predicts the number of cancellations for a given destination airport-day, then all points will lie on the 45 degree line.

Consider an example of flights into ATL from our sample, shown below in Figure 2-3. On the x-axis we have the predicted number of cancellations and the y-axis the actual number of cancellations. The line shown is the 45 degree line, where the actual number of cancellations equals the predicted number of cancellation. Each point shown is a day from our sample. Points above this line represent cases of under-prediction, where the actual number of cancellations was more than the predicted number of cancellations. Points below the line represent over-prediction. We have investigated departure flights and see the same trend as arrival flights, so we will use only arrival flights for the following analysis.
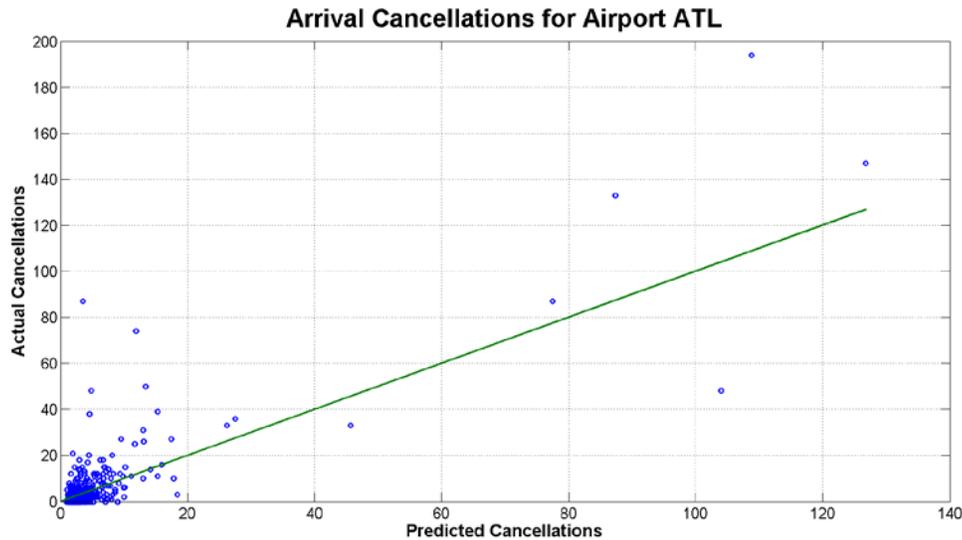
Figure 2-3: Actual vs Predicted Daily Cancellations at ATL

We see a spread of points on both sides of the line. Our model tends to under-predict cancellations on some days while over-predicting on others. We do see many points not far from the 45 degree line, however. Consider the same plot with a different scale, shown below in Figure 2-4.
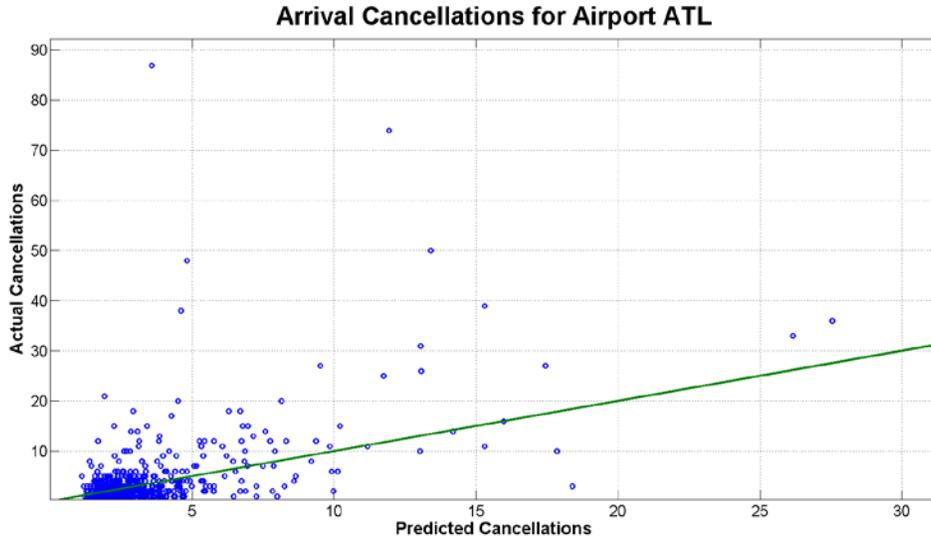
Figure 2-4: Actual vs Predicted Daily Cancellations at ATL (Zoomed in)

Now we can clearly see a large cluster of days when both the actual and predicted number of cancellations is less than 5. Beyond this some spread exists in both directions. These prediction results are similar at other airports. Consider the sample plot for BOS, shown in Figure 2-5, below.
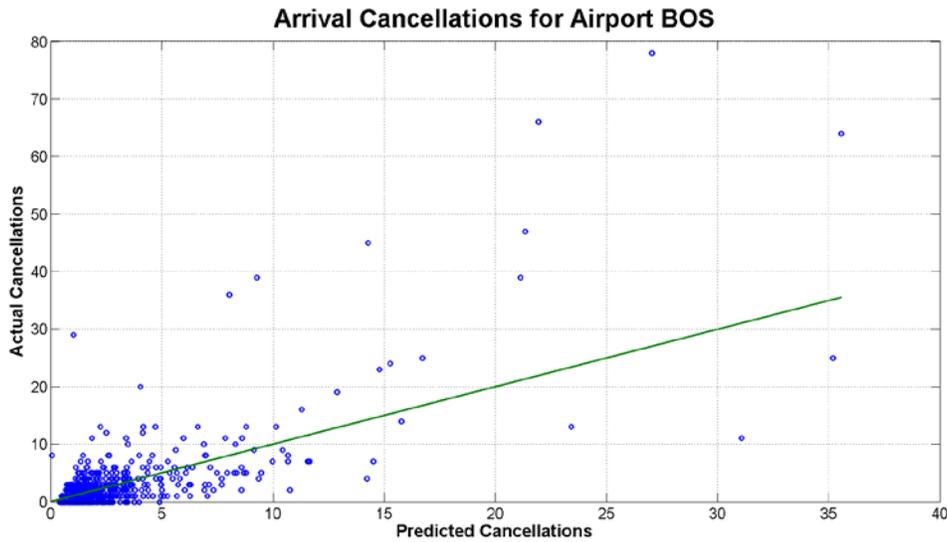
Figure 2-5: Actual vs Predicted Daily Cancellations at BOS

Again, we can see there is a large cluster of flights around less than 5 cancellations, with some spread in both directions from the 45-degree line. From inspection it is hard to distinguish these results from those at ATL, however. At first glance, it might appear that our model is not doing a good job predicting cancellations, since not many of the points lie exactly along the 45 degree line. Some discrepancy is to be expected, however, since cancellations are low probability events. Thus we need a more formal way of evaluating the model fit than the naked eye.

As another form of model fit, we can compare the number of predicted cancellations aggregated across all days for a given airport with the total number of actual cancellations aggregated across all days for the same airport. A plot with many days of over-prediction and many days of under-prediction can cancel out and result in a total number of predicted cancellations similar to that which was observed. The number of predicted and actual cancellations for each airport is shown below in Table 2-11.

Table 2-11: Total Predicted and Actual Cancellations by Airport

Airport	Total Actual	Total Pred.	% Diff.	Airport	Total Actual	Total Pred.	% Diff.
ATL	3032	2885	-4.8%	LAS	585	711	+21.6%
BOS	1988	1931	-2.9%	LAX	1293	1411	+9.2%
BWI	851	677	-20.5%	LGA	2464	2507	+1.8%
CLT	1167	983	-15.8%	MCO	574	637	+10.9%
DCA	1487	1211	-18.5%	MDW	522	565	+8.2%
DEN	878	1082	+23.3%	MIA	727	583	-19.9%
DFW	2092	2098	+0.3%	MSP	807	1022	+26.6%
DTW	1063	1206	+13.5%	ORD	3629	3943	+8.7%
EWR	1930	1445	-25.1%	PHL	1123	1086	-3.3%
IAD	830	608	-26.7%	PHX	1076	885	-17.8%
IAH	645	859	+33.2%	SAN	511	627	+22.7%
JFK	1581	1183	-25.2%	SFO	1128	1039	-7.9%

The airports with the closest number of predicted and actual cancellations are ATL, BOS, DFW, LGA, PHL, and SFO, each with less than 8% difference between the predicted and actual. The airports with the largest discrepancy between the actual number of cancellations and the predicted number were EWR, IAD, IAH, JFK, and MSP, each with over 25% difference in the number of cancellations. Also since cancellations are rare events, one would expect the standard errors of the predicted numbers to be approximately the square root of the predicted number – generally between 25 and 50 for the airports listed. Clearly, in many cases, the difference between predicted and observed cancellation numbers is well outside the $\pm 2\sigma$ 95% confidence bounds derived from these standard errors.

To further investigate the distribution of daily cancellations, we can think of the number of cancellations predicted by our model as the expected number for a given day. Even if the predicted number, on average, matches the actual number, any number of realizations will show a discrepancy between the two numbers. In particular, think about the days of very high under-prediction shown in the plot for ATL in Figure 2-4. Considering that we are looking at 730 airport-days, we might expect one or two of them to be very far away from the mean values predicted by our model, simply due to statistical fluctuations. We need to do something more than just inspect the plots of actual versus predicted cancellations in order to tell how well the model predicts cancellations for single airport-days.

We can use a statistical test to determine how well the predicted distribution of cancellations matches the actual distribution of cancellations. We will assume that the number of cancellations

for a given day follows a Poisson distribution with a mean value equal to the number predicted by our model. Therefore, for a single day, we can define the probability of observing a specific number of cancellations by the following equation:

$$P\big(C_{i,t} = k\big) = \frac{e^{-\lambda_{i,t}}\lambda_{i,t}^{\,k}}{k!} \qquad\qquad \text{(Equation 2-7)}$$

where: $C_{i,t}$ = number of cancellations at airport i on day t

   λ_i = predicted number of cancellations at airport i on day t, equal to $\hat{C}_{i,t}$

Similarly, the probability of observing less than or equal to some specific number of cancellations is shown in the following formula:

$$P\big(C_{i,t} \le K\big) = \sum_{k=0}^{K} \frac{e^{-\lambda_{i,t}}\lambda_{i,t}^{\,k}}{k!} \qquad\qquad \text{(Equation 2-8)}$$

We will calculate, for each airport-day, the probability of observing less than or equal to the number of cancellations actually observed for that airport-day, using the formula above. If the model correctly predicts the distribution of the number of cancellations, then we would expect the calculated probability to be approximately equal to the empirical probability based on the number of days in the data set. For example, we expect roughly 50% of the days to have a probability of less than or equal to 50% based on equation 2-8. We can compare these two distributions for a given airport by plotting the empirical CDF of the cumulative probabilities calculated using equation 2-8, for all days in our sample. The result for Atlanta is shown below in Figure 2-6.
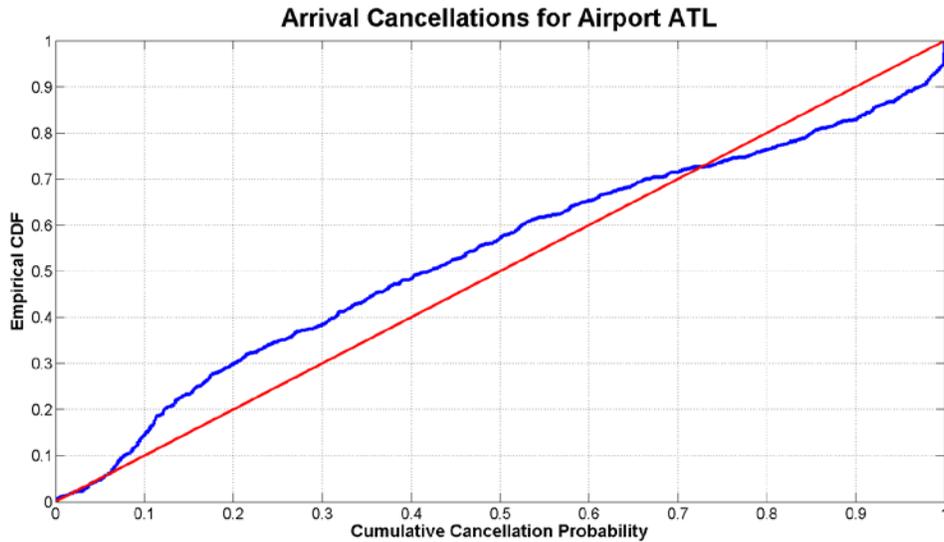
Figure 2-6: Empirical CDF of Cumulative Cancellation Probabilities at ATL

The empirical CDF of cumulative probability calculated from equation 2-8 is shown in the blue curve. The red line is the 45-degree line represents the empirical CDF of the observed

cancellations for each day. The probability calculated for each airport-day using equation 2-8 is sorted in ascending order, then each day is assigned a cumulative probability defined as follows:

$$P_n = \sum_{n=1}^{N} \frac{n}{N} \qquad \text{(Equation 2-9)}$$

Where n is the number of the day in the ordered sample, and N is the total number of days. Thus, the empirical CDF of these probabilities is simply the 45 degree line. We use this as a basis of comparison for the empirical CDF of the cumulative probabilities calculated using equation 2-8. The closer the blue curve is to the red line, the better the model does in predicting the distribution of the number of cancellations for individual airport-days.

As seen in Figure 2-6, the blue curve oscillates around the red line, being above the line for probabilities below 0.7 and below the line for higher probabilities. In the former case, a larger fraction of days has a calculated probability below a certain value than the model predicts. For example, about 60% of days have a probability below 50%. Put another way, for 60% of days, the number of cancellations, based on the cancellation model and equation 2-8, is on the low side of what might be expected. In contrast, only about 84% of days have a calculated probability below 90%. Conversely 16% of days have numbers of cancellations that, according to the model, should be exceeded 10% of the time. Similarly, on roughly 4% of days, the number of cancellations is almost impossibly large according to the model, since the calculated cumulative probability is well above 99%. These days are represented by the nearly vertical part of the curve on the right of Figure 2-6.

Another way to interpret Figure 2-6 is to compare the slopes of the blue and red curves. When the slope of the blue curve over some region of the CDF is steeper than 45 degrees, there are more observed days in this region than the probability model would suggest, and vice versa. It is evident that there are more days with cancellations in the 0-0.2 range of the predicted distribution than expected, fewer days in the 0.4-0.9 range, and then many more days on the far right tail of the distribution.
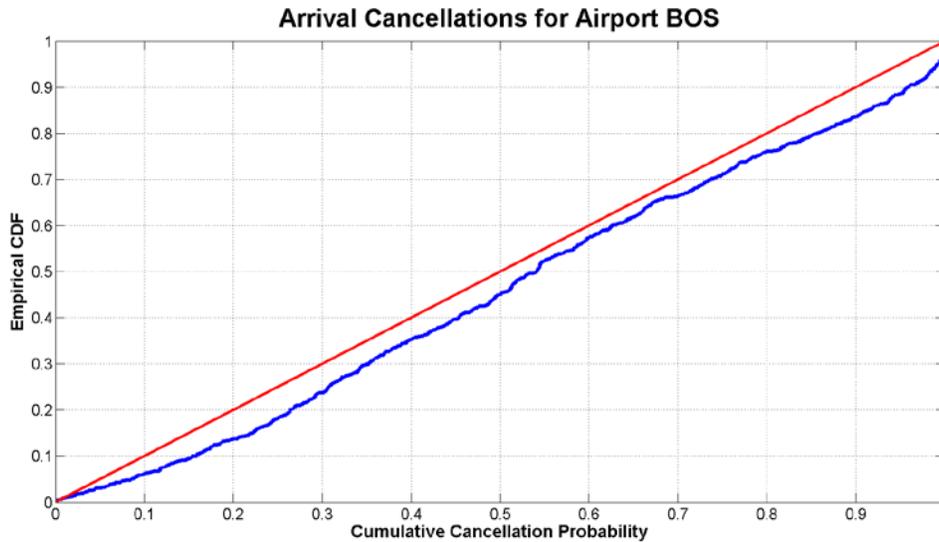
Figure 2-7: Empirical CDF of Cumulative Cancellation Probabilities at BOS

A similar plot for BOS appears in Figure 2-7. The blue curve tracks the red curve more closely in this case, although even here we see a vertical segment of the blue curve on the right, indicating days in which cancellations on the far right tail of the distribution are overrepresented. Consider IAD, shown below in Figure 2-8. In this case there are many fewer days with realized cancellations on the left tails of the distributions – up to a cumulative probability of about 0.4, more days in the range between 0.4 and 0.6, and again more days on the right tail starting at about 0.85.
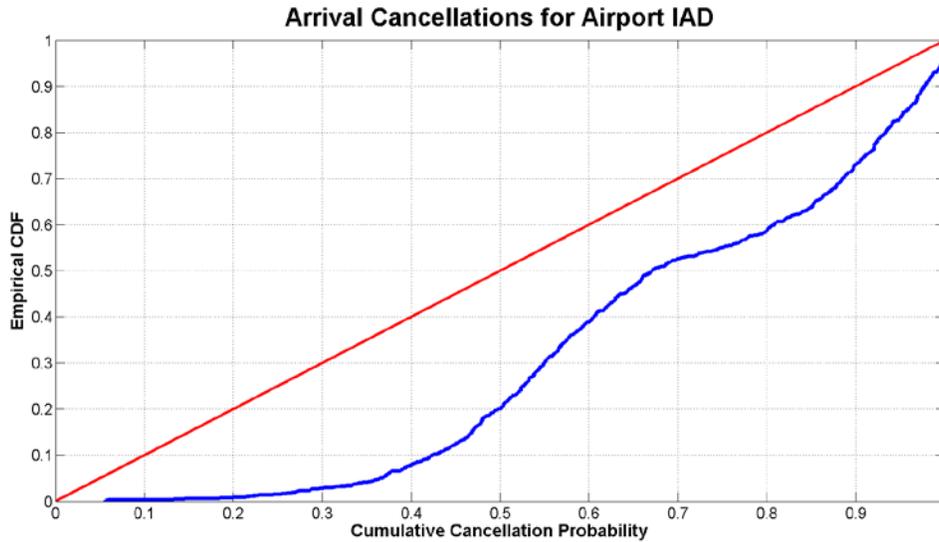
Figure 2-8: Empirical CDF of Cumulative Cancellation Probabilities at IAD

Some variation between the modeled and observed distributions for the number of cancellations will result from random fluctuations. It is therefore of interest to formally test the statistical significance of the observed differences. The Kolmogorov-Smirnov Test (KS Test) is a well-known statistical test that is used for comparing whether two datasets come from the same distribution. In our case, we are comparing the blue curve and the red curve. Conceptually, the KS Test is very easy to perform. The test statistic is simply the largest vertical difference between the two curves at a single x value. The test statistic is then used to calculate a p-value, by which the null hypothesis (the two datasets come from the same distribution) is either reject or not rejected. Mathematically, the test statistic is calculated as follows:

$$D_n = \sup_x |F_n(x) - F(x)| \qquad\qquad \text{(Equation 2-10)}$$

Where $\sup_x$ is the supremum set of distances between the two curves, $F_n(x)$ is the empirical cumulative distribution function of the data, and $F(x)$ is the cumulative distribution function of the red curve, which follows a uniform distribution between 0 and 1. The test statistic, D_n follows the Kolmogorov distribution and from this we can calculate a p-value, which represents the probability of observing the distributions we saw given the assumption that they both come from the same underlying distribution. Thus, for the statistical test to suggest that the distributions are the same, it would yield a high p-value, indicating that we cannot reject the null hypothesis that the two distributions are the same. The p-values calculated for the largest airports in our sample

are shown below in Table 2-12. Along with the p-values, we report the test statistic calculated using Equation 2-10.

Table 2-12: KS Test P-Values for Logit Model

Airport	P-val for KS Test	D_n	Airport	P-val for KS Test	D_n
ATL	0.00053	0.10	LAS	0.0000	0.25
BOS	0.0405*	0.07	LAX	0.0114*	0.08
BWI	0.0000	0.26	LGA	0.0015	0.10
CLT	0.0000	0.19	MCO	0.0000	0.28
DCA	0.0000	0.17	MDW	0.0000	0.32
DEN	0.0000	0.12	MIA	0.0000	0.34
DFW	0.0000	0.13	MSP	0.0000	0.17
DTW	0.00053	0.10	ORD	0.0000	0.12
EWR	0.0000	0.14	PHL	0.0000	0.23
IAD	0.0000	0.33	PHX	0.0000	0.25
IAH	0.0000	0.16	SAN	0.0000	0.28
JFK	0.0000	0.13	SFO	0.0000	0.20

* Not significant at 1% significance level

We can see from Table 2-12 that the p-values are very small for the most part. The p-values that were written with no significant digits (as 0.0000) were so small that we can consider them to be zero. There are only two airports where we cannot reject the null hypothesis at a 1% level of significance: BOS and LAX. The rest of the airports result in a distribution that is different enough from what we would expect that we can reject the null hypothesis with a very high level of confidence. BOS and LAX both had a small percentage difference between the total predicted and total actual cancellations (see Table 2-11), but were not the two best airports for this metric.

These results are not surprising, since they are testing a hypothesis that is very strong: that cancellations are independent events whose probabilities can be predicted by a model that applies to all airports in all situations. Our results show that this is clearly not the case. Beyond this, we can consider the test statistic shown as D_n in Table 2-12, above, to be a metric for how well the model does at predicting cancellations for each airport. Although we may not be able to statistically validate the predictions for most of these airports, we can distinguish between them in terms of model fit. For instance, ATL and LGA have a much better fit than PHL, although the total number of cancellations predicted at the three airports is roughly the same compared to the actual number, from Table 2-11. These results help identify the airports where the hypothesis is closer to and further from the truth, although it is not completely valid for any airport.

In a similar vein, hypothesis tests aside, our cancellation model is fairly good at predicting the number of cancellations by airport. In many cases it predicts the distribution of cancellations by airport-day reasonably well. The airport-day results show that the hypothesis that cancellations are independent events whose probabilities can be calculated from the estimated model must be rejected, but they also show that the model predictions, for most airports and most days, are reasonably accurate.

3 Cancellation Model Extensions

3.1 Airline-Specific Models

Given the large amount of heterogeneity observed due to the airline fixed effects in the binary logit model presented in Table 2-9, we estimated separate models for each airline. In addition to the cancellation rates being different across airlines, we suspect that the coefficients for the flight characteristics differ across airlines as well. We used the same sample as before and estimated a binary logit model with the same specification as that for the aggregate model. The results are presented below in Table 3-1 and Table 3-2. Due to the quantity of estimation results, we only present the estimate values themselves, and note statistical significance at the 5% level with a bolded estimate. Table 3-3 and Table 3-4 present the airline-specific results in the form of odds ratios for each coefficient.

Some variables have large differences across airlines, such as the hub fixed effects. United has positive coefficients for both origin and destination, while the rest of the airlines either have either negative coefficients for both or a mixture of not significant and negative coefficients. Fare, departure time, and day of week are also quite varied across airlines.

There is generally more consistency across coefficients for the legacy carriers than the low cost carriers. For instance, the distance effects are roughly constant for all legacy carriers – longer flights are less likely to be cancelled. We see that for the low cost carriers, there are few airlines with a clear trend at all for the distance effects. Load factor is negative and significant for all legacy carriers, but positive for JetBlue and not significant for Frontier. The most consistent variables across all airlines are queueing delay, snow, visibility, and winds.

The regional carrier dummy variable is positive and significant for United, Continental, and American, not significant for US Airways and Alaska, and negative and significant for Delta and AirTran. Although we saw this variable enter as positive and significant in the aggregate model, we see different effects for each airline individually. We would suspect the regional carrier effect to be positive and significant, so it is interesting that we find a negative and significant estimate for Delta and AirTran. Based on the odds ratios, Delta is almost twice as likely to cancel a mainline flight as a regional carrier flight, while American and Continental are almost twice and three times as likely, respectively, to cancel a regional carrier flight.

Overall we see some consistent effects across all airlines, but in general there is significant heterogeneity with respect to many of the explanatory variables.

Table 3-1: Airline-Specific Logit Estimates 1

	Delta	United	US Airways	American	Continental	Southwest	JetBlue	Frontier	AirTran	Alaska
ASC(Cancel)	**-0.553**	**-2.288**	**-2.088**	-3.162	-2.884	-0.122	-0.821	**-4.930**	1.116	0.584
Fare($100)	-0.038	0.042	**0.231**	**0.340**	-0.010	**0.188**	**-0.296**	0.040	**0.193**	**0.142**
DepTime(9:00-15:00)	**-0.169**	-0.061	0.059	**-0.120**	-0.082	**0.154**	0.130	-0.195	0.001	-0.051
DepTime(15:00-21:00)	**-0.069**	0.047	-0.002	0.014	**0.219**	**0.183**	**0.306**	0.342	0.106	-0.118
DepTime(21:00-3:00)	-0.083	0.057	**0.268**	-0.016	0.135	-0.001	**0.829**	**1.434**	**0.453**	-0.097
Miles<500	0.005	-0.017	0.036	-0.051	0.018	-0.046	**-0.233**	-0.186	-0.026	**-0.914**
Miles750-1000	**0.100**	**-0.492**	-0.089	**-0.425**	0.011	0.112	0.088	**-0.397**	-0.068	-0.413
Miles1000-1500	**-0.230**	**-0.666**	**-0.310**	**-0.364**	0.031	**0.266**	**0.307**	**-0.728**	-0.315	-0.369
Miles1500more	**-0.327**	**-0.728**	**-0.457**	**-0.835**	**-0.252**	**0.351**	0.122	**-0.763**	**-0.615**	-0.283
Num.Seats(100s)	**-0.572**	**-0.352**	-0.176	**0.387**	**0.352**	0.027	**-0.870**	1.051	**-2.160**	-0.881
LoadFactor	**-2.110**	**-0.998**	**-1.432**	**-1.434**	**-1.829**	**-4.478**	**0.905**	0.437	**-2.886**	**-3.292**
FlightFrequency(flight/day)	**0.008**	**0.020**	**0.029**	**-0.011**	**-0.061**	**0.132**	0.029	-0.045	**0.031**	**0.086**
Dep. Delay (10s min)	**0.351**	**0.249**	**0.256**	**0.365**	**0.473**	**0.337**	**0.272**	-0.034	**0.292**	0.226
Arr. Delay (10s min)	**0.330**	**0.318**	**0.239**	**0.313**	**0.450**	**0.324**	**0.279**	**0.517**	**0.427**	0.319
Dep. Delay Squared (100s min^2)	**-0.020**	**-0.010**	**-0.009**	**-0.021**	**-0.026**	**-0.021**	**-0.012**	0.010	-0.005	-0.005
Arr. Delay Squared (100s min^2)	**-0.014**	**-0.015**	**-0.011**	**-0.016**	**-0.017**	**-0.021**	-0.005	**-0.041**	**-0.021**	-0.004
Sunday	-0.031	**-0.189**	**0.309**	**-0.159**	0.094	**-0.635**	-0.167	-0.451	**-0.338**	-0.011
Monday	**-0.118**	**-0.245**	0.093	**-0.157**	**-0.257**	**-0.478**	**-0.495**	-0.338	-0.207	-0.091
Tuesday	**-0.079**	-0.084	0.061	**-0.168**	-0.067	**0.114**	**-0.316**	-0.046	-0.179	-0.035
Thursday	0.009	-0.015	**0.138**	-0.046	0.123	**-0.350**	**-0.356**	0.165	**-0.228**	**-0.664**
Friday	**-0.101**	-0.082	**0.208**	-0.031	**0.351**	**-0.799**	**-1.159**	-0.157	-0.155	-0.109
Saturday	0.025	**-0.389**	**0.187**	**-0.290**	**0.247**	**-0.956**	**-0.868**	**-0.668**	-0.033	0.058

Table 3-2: Airline-Specific Logit Estimates 2

	Delta	United	US Airways	American	Continental	Southwest	JetBlue	Frontier	AirTran	Alaska
IMCDest	0.047	-0.024	**-0.219**	-0.019	0.079	-0.037	-0.149	-0.110	0.012	0.078
TempDest (10s deg F)	-0.009	-0.007	**0.029**	**0.029**	**-0.039**	**-0.080**	**-0.122**	-0.011	**-0.078**	-0.109
VisDest (mi.)	**-0.064**	**-0.070**	**-0.101**	**-0.071**	**-0.032**	**-0.081**	**-0.104**	**-0.153**	**-0.072**	**-0.072**
WindDest (mph)	**0.019**	**0.017**	**0.013**	**0.015**	**0.016**	**0.024**	**0.030**	0.016	**0.033**	-0.002
IMCOrigin	**0.078**	**0.149**	-0.084	0.062	0.031	**-0.090**	0.018	**0.484**	0.169	-0.133
TempOrigin (10s deg F)	**-0.017**	-0.003	**-0.061**	0.012	**-0.108**	**-0.101**	**-0.145**	0.089	-0.007	-0.035
VisOrigin (mi.)	**-0.083**	**-0.090**	**-0.107**	**-0.089**	**-0.088**	**-0.123**	**-0.116**	**-0.113**	**-0.124**	**-0.149**
WindOrigin (mph)	**0.029**	**0.022**	**0.024**	**0.023**	**0.037**	**0.034**	**0.035**	**0.028**	**0.043**	**0.037**
Hub Origin	**-0.502**	**0.152**	**-0.519**	0.056	**-0.306**	-0.030	**-0.406**	-0.495	**-0.408**	**-0.782**
Hub Dest	**-0.284**	**0.349**	-0.089	0.041	0.010	**-0.090**	-0.114	**-0.660**	**-0.175**	-0.073
GDP	**0.475**	**0.186**	**0.429**	**0.508**	**0.171**	**0.536**	-0.088	-0.015	**0.382**	-0.631
Dest_Rain	**-0.125**	-0.061	0.107	**0.125**	**0.195**	**0.140**	**0.251**	-0.762	0.130	-0.345
Dest_Snow	**0.744**	**0.760**	**1.214**	**0.986**	**0.811**	**1.033**	**1.720**	**0.690**	**0.576**	0.761
Dest_TStorm	**0.699**	**0.963**	**0.822**	**1.091**	**0.487**	**0.546**	**0.906**	0.965	**0.726**	-0.330
Origin_Rain	**0.178**	**0.132**	0.079	**0.170**	**0.508**	**0.220**	**0.438**	-0.429	0.193	-0.157
Origin_Snow	**1.017**	**0.708**	**1.169**	**1.017**	**0.673**	**0.964**	**1.352**	**0.998**	**0.905**	**1.189**
Origin_TStorm	**0.722**	**0.948**	**0.721**	**1.304**	**0.766**	**0.362**	**0.680**	**1.848**	**0.956**	1.003
Regional_Carrier	**-0.407**	**0.416**	-0.071	**0.673**	**1.184**	--	--	--	**-1.269**	-0.287

Bolded parameters: Significant at 5% level

Table 3-3: Airline-Specific Odds Ratios 1

	Change	Delta	United	US Airways	American	Continental	Southwest	JetBlue	Frontier	AirTran	Alaska
Fare($100)	$100	0.96	1.04	1.26	1.40	0.99	1.21	0.74	1.04	1.21	1.15
DepTime(9:00-15:00)	1	0.84	0.94	1.06	0.89	0.92	1.17	1.14	0.82	1.00	0.95
DepTime(15:00-21:00)	1	0.93	1.05	1.00	1.01	1.24	1.20	1.36	1.41	1.11	0.89
DepTime(21:00-3:00)	1	0.92	1.06	1.31	0.98	1.14	1.00	2.29	4.19	1.57	0.91
Miles<500	1	1.01	0.98	1.04	0.95	1.02	0.95	0.79	0.83	0.97	0.40
Miles750-1000	1	1.10	0.61	0.91	0.65	1.01	1.12	1.09	0.67	0.93	0.66
Miles1000-1500	1	0.79	0.51	0.73	0.69	1.03	1.30	1.36	0.48	0.73	0.69
Miles1500more	1	0.72	0.48	0.63	0.43	0.78	1.42	1.13	0.47	0.54	0.75
Num.Seats(100s)	100	0.56	0.70	0.84	1.47	1.42	1.03	0.42	2.86	0.12	0.41
LoadFactor	10%	0.81	0.91	0.87	0.87	0.83	0.64	1.09	1.04	0.75	0.72
FlightFrequency(flight/day)	1	1.01	1.02	1.03	0.99	0.94	1.14	1.03	0.96	1.03	1.09
Dep. Delay (10s min)	10 min	1.42	1.28	1.29	1.44	1.60	1.40	1.31	0.97	1.34	1.25
Arr. Delay (10s min)	10 min	1.39	1.37	1.27	1.37	1.57	1.38	1.32	1.68	1.53	1.38
Dep. Delay Squared (100s min^2)	100 min^2	0.98	0.99	0.99	0.98	0.97	0.98	0.99	1.01	1.00	1.00
Arr. Delay Squared (100s min^2)	100 min^2	0.99	0.99	0.99	0.98	0.98	0.98	0.99	0.96	0.98	1.00
Sunday	1	0.97	0.83	1.36	0.85	1.10	0.53	0.85	0.64	0.71	0.99
Monday	1	0.89	0.78	1.10	0.85	0.77	0.62	0.61	0.71	0.81	0.91
Tuesday	1	0.92	0.92	1.06	0.85	0.93	1.12	0.73	0.96	0.84	0.97
Thursday	1	1.01	0.99	1.15	0.95	1.13	0.70	0.70	1.18	0.80	0.51
Friday	1	0.90	0.92	1.23	0.97	1.42	0.45	0.31	0.85	0.86	0.90
Saturday	1	1.02	0.68	1.21	0.75	1.28	0.38	0.42	0.51	0.97	1.06

Table 3-4: Airline-Specific Odds Ratios 2

	Change	Delta	United	US Airways	American	Continental	Southwest	JetBlue	Frontier	AirTran	Alaska
IMCDest	1	1.05	0.98	0.80	0.98	1.08	0.96	0.86	0.90	1.01	1.08
TempDest	10 deg	0.99	0.99	1.03	1.03	0.96	0.92	0.89	0.99	0.93	0.90
VisDest	1 mi	0.94	0.93	0.90	0.93	0.97	0.92	0.90	0.86	0.93	0.93
WindDest	10 mph	1.21	1.18	1.14	1.16	1.17	1.27	1.35	1.17	1.39	0.98
IMCOrigin	1	1.08	1.16	0.92	1.06	1.03	0.91	1.02	1.62	1.18	0.88
TempOrigin	10 deg	0.98	1.00	0.94	1.01	0.90	0.90	0.87	1.09	0.99	0.97
VisOrigin	1 mi	0.92	0.91	0.90	0.92	0.92	0.88	0.89	0.89	0.88	0.86
WindOrigin	10 mph	1.33	1.25	1.27	1.26	1.45	1.41	1.42	1.33	1.54	1.45
Hub Origin	1	0.61	1.16	0.59	1.06	0.74	0.97	0.67	0.61	0.67	0.46
Hub Dest	1	0.75	1.42	0.91	1.04	1.01	0.91	0.89	0.52	0.84	0.93
GDP	1	1.61	1.20	1.54	1.66	1.19	1.71	0.92	0.99	1.46	0.53
Dest_Rain	1	0.88	0.94	1.11	1.13	1.21	1.15	1.28	0.47	1.14	0.71
Dest_Snow	1	2.10	2.14	3.37	2.68	2.25	2.81	5.59	1.99	1.78	2.14
Dest_TStorm	1	2.01	2.62	2.28	2.98	1.63	1.73	2.47	2.62	2.07	0.72
Origin_Rain	1	1.19	1.14	1.08	1.19	1.66	1.25	1.55	0.65	1.21	0.85
Origin_Snow	1	2.76	2.03	3.22	2.77	1.96	2.62	3.87	2.71	2.47	3.28
Origin_TStorm	1	2.06	2.58	2.06	3.68	2.15	1.44	1.97	6.35	2.60	2.73
Regional_Carrier	1	0.67	1.52	0.93	1.96	3.27	--	--	--	0.28	0.75

3.2 Random Effects Model

Next, we used a model specification that allows us to capture unobserved random effects that are correlated across time. We attempt to capture the heterogeneity in unobserved variables through the use of a random effects term in a mixed logit model. The random effect enters the utility as follows:

$$U_{n,cancel} = \beta' x_{n,cancel} + \mu_n' z_{n,cancel} + \varepsilon_{n,cancel} \qquad \text{(Equation 3-1)}$$

where β is a vector of fixed coefficients, $x_{n,cancel}$ and $z_{n,cancel}$ are vectors of observed variables, μ_n is a vector of random terms with zero mean, and $\varepsilon_{n,cancel}$ is iid extreme value. For our random effects model, the terms $z_{n,cancel}$ are a constant value of 1, similar to an alternative-constant. Together with $\varepsilon_{n,cancel}$, the term μ_n' defines the stochastic portion of utility:

$$\eta_{n,cancel} = \mu_n' z_{n,cancel} + \varepsilon_{n,cancel} \qquad \text{(Equation 3-2)}$$

The subscript n, in the above equation represents a set of flights that are treated as sharing similar unobserved characteristics. This is analogous to the case of repeated choices for an individual. This technique is commonly used in panel data, where individuals make repeated choices over time (Revelt and Train, 1998 and Johannesson and Lundin, 2000). The sequential choices made by the same person are correlated due to the unobserved tastes, attitudes, or preferences specific to each individual that is constant for them across all choices they make:

$$Cov\left(\eta^1{}_{n,cancel}, \eta^2{}_{n,cancel}\right) = E\left(\mu_n' z_{n,cancel}{}^1 + \varepsilon_{n,cancel}{}^1, \mu_n' z_{n,cancel}{}^2 + \varepsilon_{n,cancel}{}^2\right)$$
$$Cov\left(\eta^1{}_{n,cancel}, \eta^2{}_{n,cancel}\right) = z'_{n,cancel}{}^1 W z_{n,cancel}{}^2 \qquad \text{(Equation 3-3)}$$

Where W is the covariance of μ_n. We can think of flight cancellations as having a similar behavior as panel data. While we do not have the structure of individuals making repeated decisions, we do have specific agents (i.e. airlines) making sequential choices across a given day (flight cancellations). In a way, the airlines can act as individuals with a constant set of attitudes and preferences, since flight dispatchers work in shifts from day-to-day. The morning flights might have physically the same person making the cancellation decisions from day-to-day.

Moreover, the set of unobservable effects that we will be capturing are likely constant for all flights within some time interval that is much shorter than a complete day. In other words, some time intervals are more "cancellation prone" than others, even accounting for all the various factors included in the previous models. The problem we have is how to define the sets of flights for which this random time interval effect is assumed constant. We estimated several models and the one that provided the best fit was one that grouped the flights for a given airline into four-hour intervals based on scheduled departure time (GMT).

We choose μ_n to have a normal distribution with zero mean: $\mu_n \sim N(0, \sigma^2)$. Our estimation procedure estimates the value of σ, which is the standard deviation of the distribution.

The model specification for the other variables was identical to that of the first binary logit model estimated earlier. For the estimation we use a maximum simulated likelihood routine coded in Matlab. We select a small sample of 100,000 flights from the larger sample, using simple random sampling, to perform this estimation. The reason for the smaller sample is the long simulation time associated with mixed logit estimation. The final model estimation results are shown below in Table 3-5 and Table 3-6

Table 3-5: Random Effects Model Estimates 1

Variable	Estimate		Std. Err.	Variable	Estimate		Std. Err.
ASC(Cancel)	-2.049	**	0.032	IMCDest	-0.088	**	0.034
Fare($100)	0.012		0.032	TempDest	0.001		0.024
DepTime(9:00-15:00)	-0.098	**	0.022	VisDest	-0.089	**	0.010
DepTime(15:00-21:00)	0.071	*	0.036	WindDest	0.016	**	0.005
DepTime(21:00-3:00)	0.004		0.033	IMCOrigin	0.001		0.025
Miles<500	-0.046		0.026	TempOrigin	-0.051	**	0.019
Miles750-1000	-0.305	**	0.019	VisOrigin	-0.102	**	0.010
Miles1000-1500	-0.169	**	0.034	WindOrigin	0.036	**	0.005
Miles1500more	-0.348	**	0.027	Hub Origin	-0.380	**	0.031
Num.Seats(100)	-0.361	**	0.063	Hub Dest	-0.158	**	0.022
LoadFactor	-2.018	**	0.082	GDP	0.446	**	0.075
FlightFrequency(flight/day)	0.036	**	0.008	Dest_Rain	0.002		0.077
Dep. Delay (10s min)	0.261	**	0.059	Dest_Snow	0.977	**	0.029
Arr. Delay (10s min)	0.289	**	0.046	Dest_TStorm	0.730	**	0.071
Dep. Delay Squared (100s min^2)	-0.011	*	0.007	Origin_Rain	0.152	**	0.026
Arr. Delay Squared (100s min^2)	-0.010		0.005	Origin_Snow	1.115	**	0.061
Sunday	-0.134	**	0.023	Origin_TStorm	0.980	**	0.039
Monday	-0.209	**	0.042	Regional_Carrier	-0.037		0.027
Tuesday	-0.046		0.039	** Significant at 1% level			
Thursday	-0.007		0.036	* Significant at 5% level			
Friday	-0.183	**	0.037				
Saturday	-0.306	**	0.059				
Random Effect: σ	0.057	**	0.018				

Table 3-6: Random Effects Model Estimates 2

Variable	Estimate		Std. Err.
DL (Delta)	1.231	**	0.047
UA (United)	1.380	**	0.020
US (US Airways)	0.952	**	0.080
AA (American)	1.689	**	0.049
CO (Continental)	1.029	**	0.047
WN (Southwest)	0.859	**	0.029
B6 (JetBlue)	1.390	**	0.021
F9 (Frontier)	0.292	**	0.047
FL (Air Tran)	0.671	**	0.029
AS (Alaska)	0.134	**	0.032

** Significant at 1% level

We can see in Table 3-5 that the random effect coefficient has a positive and significant sign, indicating that some level of unobserved heterogeneity does exist during the time windows we defined. The magnitude of this effect appears to be small, however. This is an indication that some amount of unobserved heterogeneity does exist across groups of flights for the same airline within four hour buckets.

The random effects model was estimated on a reduced dataset, so we estimated the same MNL specification as before on the same reduced dataset. We can thus compare the coefficient estimates between the two models directly. These estimates are shown below in Table 3-7 and Table 3-8. The estimates from the random effects model are mostly consistent with the MNL estimates, both in magnitudes and significance. We would expect the random effects coefficients to have a larger magnitude than the MNL coefficients, as has been documented extensively in the literature (Revelt and Train, 1998), but we do not see a clear trend here.

We would expect the random effects term to have a large magnitude if there exist similar characteristics between flights within a given time window that substantially affect the cancellation utility that are otherwise not explicitly captured in our specification. We have a fairly comprehensive list of variables, however, which include flight characteristics, queueing delay, and weather effects. For the time windows we have considered, these effects appear to capture most of the similarity between flights that are affecting the cancellation utility.

Table 3-7: MNL Estimates for Reduced Dataset 1

Variable	Estimate		Std. Err.	Variable	Estimate		Std. Err.
ASC(Cancel)	-2.153	**	0.046	IMCDest	-0.042		0.027
Fare($100)	0.045	*	0.020	TempDest	-0.025		0.018
DepTime(9:00-15:00)	-0.147	**	0.043	VisDest	-0.086	**	0.010
DepTime(15:00-21:00)	0.006		0.030	WindDest	0.014	**	0.005
DepTime(21:00-3:00)	0.003		0.059	IMCOrigin	0.084		0.047
Miles<500	0.047	*	0.023	TempOrigin	-0.035	*	0.016
Miles750-1000	-0.079	*	0.035	VisOrigin	-0.096	**	0.011
Miles1000-1500	-0.037		0.041	WindOrigin	0.033	**	0.005
Miles1500more	-0.359	**	0.072	Hub Origin	-0.319	**	0.079
Num.Seats(100)	-0.425	**	0.049	Hub Dest	-0.071		0.041
LoadFactor	-2.074	**	0.028	GDP	0.556	**	0.097
FlightFrequency(flight/day)	0.036	**	0.007	Dest_Rain	0.126	**	0.049
Dep. Delay (10s min)	0.314	**	0.043	Dest_Snow	0.762	**	0.030
Arr. Delay (10s min)	0.250	**	0.019	Dest_TStorm	0.973	**	0.043
Dep. Delay Squared (100s min^2)	-0.017	**	0.006	Origin_Rain	0.251	**	0.034
Arr. Delay Squared (100s min^2)	-0.008	**	0.003	Origin_Snow	1.152	**	0.051
Sunday	0.065	*	0.027	Origin_TStorm	0.894	**	0.072
Monday	-0.160	**	0.058	Regional_Carrier	0.027		0.029
Tuesday	-0.004		0.040	** Significant at 1% level			
Thursday	0.053	**	0.020	* Significant at 5% level			
Friday	-0.073	*	0.034				
Saturday	-0.197	**	0.033				

Table 3-8: MNL Estimates for Reduced Dataset 2

Variable	Estimate		Std. Err.
DL (Delta)	1.224	**	0.035
UA (United)	1.293	**	0.028
US (US Airways)	0.934	**	0.054
AA (American)	1.592	**	0.040
CO (Continental)	0.851	**	0.068
WN (Southwest)	0.814	**	0.030
B6 (JetBlue)	1.336	**	0.063
F9 (Frontier)	0.133	**	0.033
FL (Air Tran)	0.886	**	0.052
AS (Alaska)	-0.232	**	0.060

** Significant at 1% level

3.3   Model Fit

The different estimates we found in each airline-specific suggest different cancellation behavior for different airlines. Thus, we would think that splitting up the analysis into airline-specific models would prove to be beneficial. One way we can evaluate this is to perform the same aggregation techniques used in section 2.8 to assess the ability of the models to predict cancellations at specific airport-days. We used the results from each airline-specific choice model and calculated the predicted cancellation probability for each flight in our sample. As a first measure of comparison with the aggregate model, consider the total number of cancellations predicted at each airport, shown below in Table 3-9.

Table 3-9: Total Predicted and Actual Cancellations by Airport: Airline-Specific Models

Airport	Total Actual	Total Pred.	% Diff.	Airport	Total Actual	Total Pred.	% Diff.
ATL	3032	2798	-7.7%	LAS	585	368	-37.0%
BOS	1988	1807	-9.1%	LAX	1293	1145	-11.4%
BWI	851	402	-52.7%	LGA	2464	2390	-3.0%
CLT	1167	1142	-2.1%	MCO	574	520	-9.4%
DCA	1487	1162	-21.8%	MDW	522	135	-74.2%
DEN	878	845	-3.8%	MIA	727	692	-4.8%
DFW	2092	2104	+0.6%	MSP	807	953	+18.1%
DTW	1063	1121	+5.5%	ORD	3629	3834	+5.6%
EWR	1930	1507	-21.9%	PHL	1123	998	-11.1%
IAD	830	636	-23.4%	PHX	1076	626	-41.8%
IAH	645	760	+17.8%	SAN	511	377	-26.1%
JFK	1581	1247	-21.1%	SFO	1127	953	-15.4%

The airports with the closest number of total of predicted and actual cancellations are CLT, DEN, DEN, LGA, and MIA, each with less than a 5% difference. The worst airports are BWI, LAS, and PHX, each with over a 35% difference between actual and predicted cancellations. There is not strong evidence from these numbers that airline-specific models improve the cancellation prediction at an airport level.

In the same way as before, we performed sample enumeration to aggregate the cancellations across airport-days. We then calculated the empirical CDF of the Poisson probabilities using equation 2-5. For each airport we calculated the KS test statistic and the associated p-value. The p-values and the max vertical difference between the two curves (Equation 2-7) for each airport are shown below in Table 3-10.

We find that none of the airports have a p-value less than 0.01, compared to two airports for the aggregate model. The best airports in terms of p-value are ATL, LGA, ORD, and BOS. It appears that although the airline-specific choice models do a better job of capturing the heterogeneity in cancellation decisions across airlines, they do not perform better than the aggregate model for predicting cancellations for specific airport-days.

Table 3-10: KS Test P-Values for Airline-Specific Models

Airport	P-val for KS Test	D_n	Airport	P-val for KS Test	D_n
ATL	0.0057	0.09	LAS	0.0000	0.49
BOS	0.00014	0.11	LAX	0.0000	0.19
BWI	0.0000	0.49	LGA	0.00053	0.11
CLT	0.0000	0.14	MCO	0.0000	0.35
DCA	0.0000	0.19	MDW	0.0000	0.75
DEN	0.0000	0.23	MIA	0.0000	0.27
DFW	0.0000	0.13	MSP	0.0000	0.19
DTW	0.0000	0.15	ORD	0.00035	0.11
EWR	0.0000	0.16	PHL	0.0000	0.29
IAD	0.0000	0.32	PHX	0.0000	0.39
IAH	0.0000	0.20	SAN	0.0000	0.47
JFK	0.0000	0.12	SFO	0.0000	0.22

* Not significant at 1% significance level

The random effects term for the mixed logit model is very small in magnitude compared to the other coefficient estimates, suggesting that only a small amount of heterogeneity exists across 4-hour time intervals. Moreover, at the daily level for a given airport it is very unlikely that the net effect of the random effects term, draws from which are independent for different airlines and from one four-hour period to the next for a given airline, result in a major difference in total cancellations. Thus, we do not perform the KS test for model fit using the mixed logit results.

4 Delay Analysis

4.1 Delay Simulation

We will now implement the results from our cancellation models into delay simulation models to evaluate the impact of flight cancellations on delay forecasts. First we will dicsuss our queueing algorithm and how it compares with the algorithm used by the FAA, ACASAT. Then we will incorporate flight cancellations into the queueing model in two ways. First, we will use Monte Carlo simulation to evaluate the probabilistic flight cancellations as draws of realized flight demand from a distribution, and then we will illustrate a first-order approximation to Monte Carlo simulation that does not require any simulation. Finally we conduct a simulation experiment on a large number of airport-days to illustrate the effect flight cancellations have on delay forecasts as well as the differences between the first-order approximation and Monte Carlo simulation.

We will estimate the impacts of flight cancellations on delay using a deterministic queueing model. The model is a simplified representation of the queueing model that is used in ACASAT that we use to calculate delays given a flight schedule and capacities. The framework for our cancellation models and the delay forecasts is shown below in Figure 4-1.
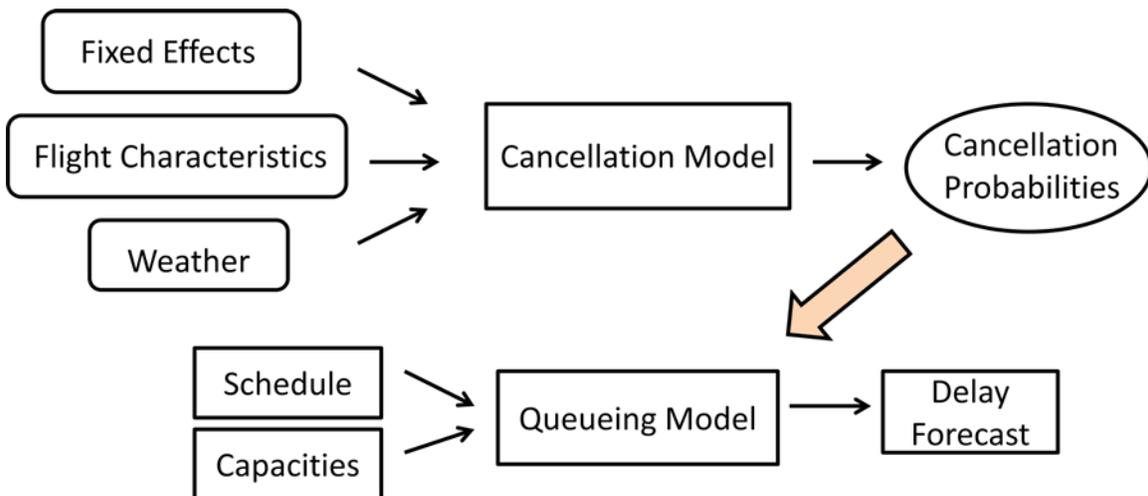

Figure 4-1: Cancellation and Delay Model Framework

As seen in the top of Figure 4-1, the cancellation model inputs consist of various explanatory variables, such as fixed effects for airlines, flight characteristics, and weather. The outputs from our cancellation model, as described previously, are in the form of cancellation probabilities for each flight. The queueing delay framework used by the FAA is shown in the bottom of Figure 4-1. Schedules and capacities for specific airports are the inputs to a queueing model which outputs delay forecasts. We will incorporate flight cancellations into the queueing models in the form of cancellation probabilities, and evaluate the effect of cancellations on the delay forecast output from the queueing models.

4.2 Queueing Model

4.2.1 Queueing Algorithm

We will use an iterative queueing model that models the demand, capacity, throughput, and queue length on a minute-by-minute basis. We choose one minute as our time step since that is the

smallest level of granularity of flight schedules. The capacities, which are constant hourly rates for each 15 minute interval, are converted to minute-by-minute capacities in a continuous fashion. That is, we take the capacity for a given 15 minute interval, measured in flights/hour and divide it by 60 to get a flight/min capacity.

Consider an example of a set of departures, shown below in Table 4-1.

Table 4-1: Sample Set of Departures

Departure Time
8:00
8:00
8:00
8:01
8:03
8:03
8:05

We can then create the demand and capacity for each minute in the time interval, shown in Table 4-2. We assume a constant capacity of 75 departures / hr, which translates to 75/60 = 1.25 departures / min.

Table 4-2: Sample Demand and Capacity

Time Period, i	Demand, D_i	Capacity, C_i
8:00	3	1.25
8:01	1	1.25
8:02	0	1.25
8:03	2	1.25
8:04	0	1.25
8:05	1	1.25

The demand column is generated by aggregating the flight schedule for each minute from 8:00 to 8:05. The queueing algorithm begins with the first time step:

$$Q_1 = min(D_1, C_1) \qquad \text{(Equation 4-1)}$$

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Where Q_1, D_1, and C_1 are the throughput, demand, and capacity at time period 1, respectively. The throughput is equal to the demand unless the demand exceeds capacity. In that case, the throughput is equal to the capacity. The excess demand beyond the throughput is the amount in queue at time period 1, N_1:

$$N_1 = D_1 - Q_1 \qquad \text{(Equation 4-2)}$$

In the second time step, the throughput is now the minimum of the demand in the current time period plus the queue from the previous time period and the capacity in the current period:

$$Q_2 = min(D_2 + N_1, C_2) \qquad \text{(Equation 4-3)}$$

The amount in queue is the difference between the demand at this time period (plus the queue from the previous time period) and the throughput at this time period:

$$N_2 = D_2 + N_1 - Q_2 \qquad \text{(Equation 4-4)}$$

The following time steps follow the same form as that of time period 2. The general formula can be written as follows:

$$Q_i = \begin{cases} min(D_i, C_i) & i = 1 \\ min(D_i + N_{i-1}, C_i) & i > 1 \end{cases} \qquad \text{(Equation 4-5)}$$

$$N_i = \begin{cases} D_i - Q_i & i = 1 \\ D_i + N_{i-1} - Q_i & i > 1 \end{cases} \qquad \text{(Equation 4-6)}$$

This process is continued until the end of the queue has cleared or the end of the flight schedule, whichever is later. The total delay is the sum of the amount in queue at each time period times the time step used. In our case, we use one minute time steps, so the delay is simply the sum of the queues at each time period:

$$Delay = \sum_i N_i \, \Delta t = \sum_i N_i \qquad \text{(Equation 4-7)}$$

This procedure was applied to the schedule presented earlier and the resulting throughput and queue for each time step are shown below in Table 4-3.

Table 4-3: Sample Demand, Capacity, and Throughput

Time Period	Demand, D_i	Capacity, C_i	Throughput, Q_i	Queue, N_i
8:00	3	1.25	1.25	1.75
8:01	1	1.25	1.25	1.5
8:02	0	1.25	1.25	0.25
8:03	2	1.25	1.25	1
8:04	0	1.25	1	0
8:05	1	1.25	1	0

The total delay for this example is simply the sum of the queue for each time period. In this case we have 4.5 aircraft-minutes of delay.

4.2.2 ACASAT Algorithm

We are going to use our queueing model to illustrate the effects of flight cancellations on flight delays. Due to the techniques we will be using and the large number of flight schedules we will be performing calculations on, we have chosen to use our own queueing algorithm (shown above) implemented in Matlab. We need to verify that the queueing model is producing delay calculates similar to those from ACASAT. We verify this so that we can infer that the results shown here would be reproducible if the cancellations were properly incorporated into ACASAT.

The ACASAT queueing algorithm is different from the one described above. The algorithm is continuous in time and discrete in the number of services. Instead of iterating through a discrete time step as our algorithm does, the ACASAT algorithm iterates through discrete flight operations. Take for example the same flight schedule from before, shown here in Table 4-4.

Table 4-4: Sample Set of Departures

Departure Time
8:00
8:00
8:00
8:01
8:03
8:03
8:05

The capacity of 75 / hr that we used earlier is now incorporated into the algorithm as a minimum inter-departure time. In this case we have 60 / 75 = 0.8 minutes. The algorithm starts with the first scheduled departure time, 8:00 and services one flight. The next departure time is the previous departure time plus the maximum of two quantities, the minimum inter-departure time (determined by capacity) and the time until the next scheduled departure.

$$T_2 = max(H, S_2 - S_1) + T_1 \qquad \text{(Equation 4-8)}$$

Where T_i is the actual departure time of flight i, S_i is the scheduled departure time of flight i and H is the current minimum inter-departure time, determined by the capacity. The only time this algorithm deviates from this formula is when the time between actual departures crosses a boundary of the 15 minute intervals within which capacity is constant. For example, assume capacity is constant between 8:00 and 8:15, and there is an actual departure at 8:14. If the next departure is supposed to occur at 8:16 based on the above formula, the algorithm is interrupted.

In this case, the minimum inter-departure time (and thus the capacity) is no longer constant between two flight operations. The algorithm uses linear interpolation to determine the time of the next operation. The original inter-departure time is treated as the time required to service

another flight. The time between the capacity change and the previous flight relative to this time is effectively the proportion of the time that has elapsed until the flight is allowed to be served.

This is illustrated below in Figure 4-2. Consider two periods of constant capacity, divided at time T. For flights served prior to time T, the inter-departure time is Δt_1 and for flights served after time T, the inter-departure time is Δt_2. The last flight served in the first period is served x time units prior to T, where x < Δt_1. That is, the time until the capacity change is less than the inter-departure time for the current capacity. After T, the time until the next flight is served will be denoted by y. The sum of x and y, which we will denote $\Delta \tilde{t}$, is the inter-departure time between the last flight served prior to T and the first flight served after T.
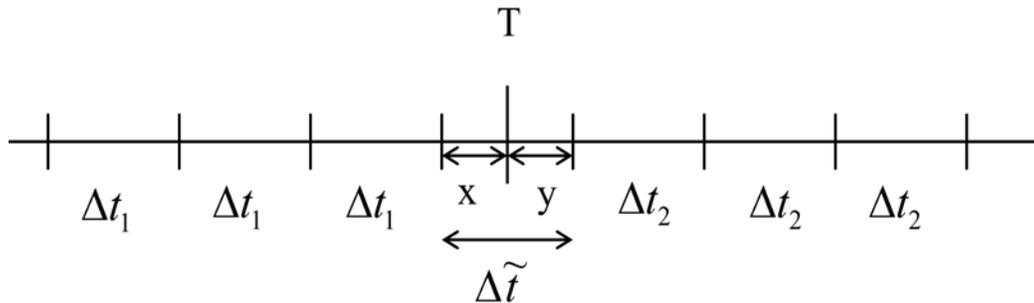
Figure 4-2: Linear Interpolation for ACASAT Queueing Model

The ACASAT algorithm uses linear interpolation to calculate the inter-departure time that spans across periods of two different capacities. We can think of the percentage of a flight serve that was completed prior to T as being the ratio between x and Δt_1. The remaining portion of the flight that must be served must be equal to the portion of the flight served after time T:

$$1 - \frac{x}{\Delta t_1} = \frac{y}{\Delta t_2}$$ (Equation 4-9)

Consider a numerical example, where we assume the following values for inter-departure times, in minutes:

$$\Delta t_1 = 2$$
$$\Delta t_2 = 3$$

Also, assume that the last flight prior to T is served 1 minute before T. The portion of the current flight service that is served before T is the ratio between x and Δt_1, or ½ = 0.5. Thus, the remaining portion of the flight to be served is 0.5. We can calculate y using the following equation:

$$y = \Delta t_2 \left(1 - \frac{x}{\Delta t_1}\right)$$ (Equation 4-9)

In this case, y = 1.5 minutes. Thus, the inter-departure time that spans across the capacity change is given by the sum of x and y, 2.5 minutes.

Once the actual service times are established using the queueing algorithm, the total delay is simply the sum of the difference between the scheduled service time and the actual service time for each flight:

$$Delay = \sum_i T_i - S_i$$ (Equation 4-9)

4.2.3 Queueing Algorithm Comparison

We implemented both algorithms in Matlab and used a large sample of airport-days to evaluate the difference between our minute-by-minute queueing algorithm and the true ACASAT algorithm. We use a sample of eight airports that represent some of the most congested airports in the country, and thus are of great interest to the FAA regarding the prediction of flight delays. The airports are SFO, ORD, ATL, JFK, EWR, LGA, PHL, and BOS. The days are sampled from months ranging from October 2010 to December 2011.

The two algorithms above were used to calculate the total departure delay due to queueing for each day at the various airports. A total of 65 airport-days were used in our analysis. The percentage difference between the total daily delay from each queueing method was calculated. The difference between the delay from the ACASAT algorithm and the delay using our approximation is shown below in Figure 4-3.
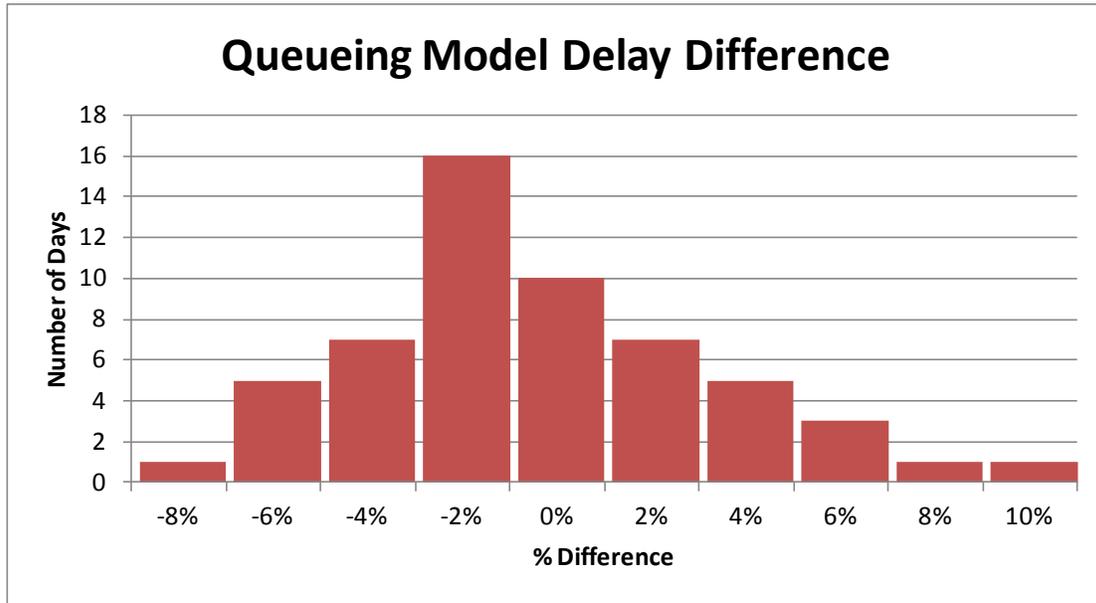

Figure 4-4: ACASAT vs Queueing Model Comparison

The mean difference between our queueing model and the ACASAT queueing model is -1.91%. That is, the ACASAT model on average produced estimates 1.91% below our model estimates. However, since our model did not produce results strictly above or below the estimates from the ACASAT model, we can take the absolute value of each percentage difference and use these as another basis for comparison. The distribution of the absolute value percentage differences between the two algorithms is presented below in Figure 4-4.
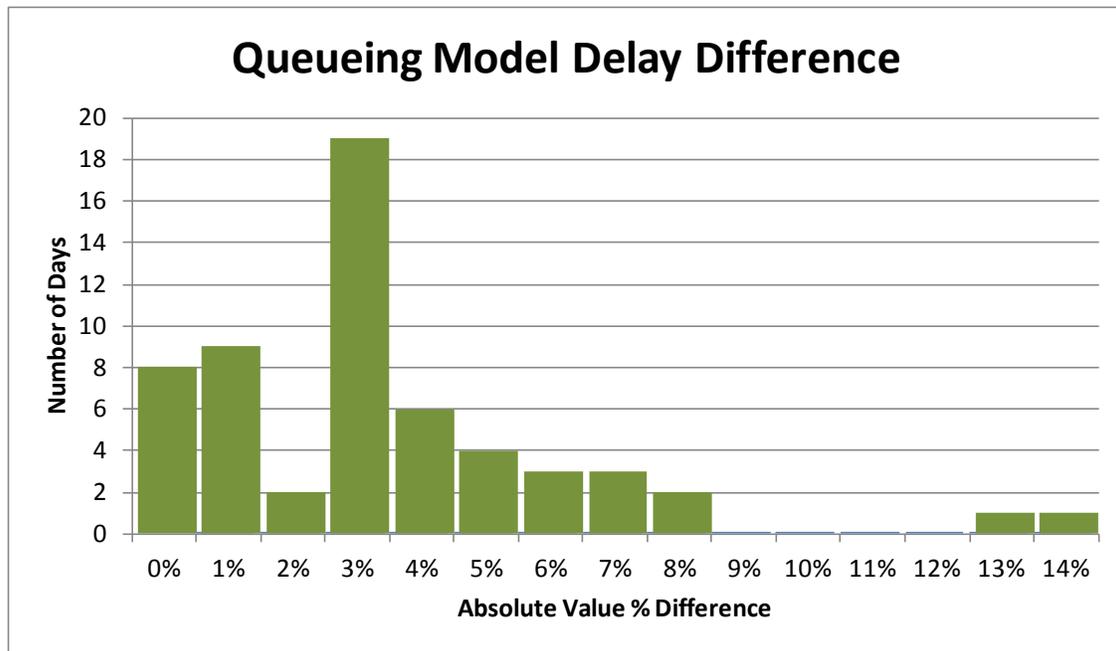
Figure 4-4: ACASAT vs Queueing Model Comparison (Absolute Value)

The average absolute value of percentage difference between the two models is 3.6%, with a couple high outliers. For the most part, the absolute value of delays differences are below 5%. We consider this an acceptable difference to consider an analysis using our queueing model to be equivalent to the results that would be found with the queueing model from ACASAT.

4.3   Incorporating Flight Cancellations into Queueing Models

A probabilistic approach is required to introduce cancellations into a queueing model. As shown previously, the outputs of the cancellations models are cancellation probabilities for each flight. Therefore, we can no longer assume a deterministic demand for each minute according to the flight schedules. We can think of demand, and thus delay, as being a random variable, taking on different values for different sets of flight cancellations, the likelihood of which depend on the cancellation probabilities. The exact relationship between delay and cancellation probabilities would be very difficult to derive analytically. Thus, we will estimate the expected value of delay using Monte Carlo simulation. We will compare the results from the Monte Carlo simulation, which we can assume (for a sufficient number of simulation runs) yields a consistent and very accurate estimate for the true expected delay, with a first-order approximation of the effect of flight cancellations.

4.3.1   Monte Carlo Simulation

We use Monte Carlo simulation to estimate the expected queueing delay when there are probabilistic flight cancellations. From a given binary logit model, each flight is assigned a cancellation probability. For each run in the Monte Carlo simulation, a realized set of flight cancellations is drawn from the set of cancellation probabilities for each flight.

We can think of demand as being the sum of the number of flights in the schedule for each minute, where each flight is a Bernoulli random variable with cancellation probability p_j. We will denote X_j as an indicator for the status of each flight, where 1 means the flight is not cancelled.

Therefore, we use a Bernoulli random variable with 1 minus the cancellation probability as the probability of a successful draw:

$$D_i = \sum_{j \in i} X_j \qquad \text{(Equation 4-10)}$$

$$X_j \sim Bernoulli(1 - p_j) \qquad \text{(Equation 4-11)}$$

Where D_i is the demand in time period i, and X_j is an indicator variable for the demand for flight j within time period i. X_j is equal to 1 with probability $1 - p_j$ and 0 with probability p_j. We assume that the cancellations are made independently and thus we use successive draws for each flight in our dataset. We then use the deterministic queueing model described above to calculate the queueing delay for that particular realization of flight cancellations. Successive draws are made, the delays are calculated for each set, and then finally averaged together to get an unbiased estimated of the delay due to flight cancellations. The sampling error is calculated as the standard deviation of the set of realized delay calculations divided by the square root of the number of simulation draws made. We use 1000 draws and thus get a fairly small sample error.

The Monte Carlo simulation steps are performed as follows:

1. For each flight, X_j, take a draw from a Uniform distribution between 0 and 1.
2. If the draw is less than the cancellation probability for that flight, set $X_j = 0$, otherwise, set $X_j = 1$.
3. For each minute in the schedule, add up the total flights, $D_i = \sum_j X_j$
4. Use the queueing algorithm to calculate the queueing delay, $Delay$, for this draw.
5. Perform steps 1 to 4 R times, obtaining $Delay^r$ for $r = 1, \dots R$.
6. Average the R draws, $\overline{Delay} = \frac{\sum_r Delay^r}{R}$.

To illustrate the Monte Carlo simulation technique, consider the flight schedule from earlier, but now assume that each flight has an associated cancellation probability, shown below in Table 4-5.

Table 4-5: Sample Set of Departures with Cancellation Probabilities

Departure Time	Cancellation Prob.
8:00	0.02
8:00	0.01
8:00	0.03
8:01	0.03
8:03	0.04
8:03	0.03
8:05	0.02

We can translate the flight schedule and cancellation probabilities into many different realized demands. Each demand set is created by taking random draws for each flight from a Bernoulli distribution with mean equal to the cancellation probability for a particular flight. An example of a few possible realized demand sets are shown below in Table 4-6. For illustration purposes, the demand sets feature more cancellations than would normally be expected given the cancellation probabilities in Table 4-5.

Table 4-6: Realized Demand Sets for Monte Carlo Simulation

Time Period, i	D_i^1	D_i^2	D_i^3	D_i^4
8:00	3	2	3	3
8:01	1	1	0	1
8:02	0	0	0	0
8:03	2	1	2	2
8:04	0	0	0	0
8:05	1	1	1	0

The highlighted cells represent time periods where a cancellation took place. The expected value of delay is calculated by averaging together the delay calculations for each realized demand set. As the number of realized demand sets grows large, the average delay value converges on the true value of expected delay due to cancellations.

4.3.2   First-Order Approximation

Monte Carlo simulation, while accurate, is quite computationally cumbersome due to the large number of simulation runs required to get an accurate estimate. Thus, we developed an approximation that we can perform once, rather than having to take successive draws from a distribution for each flight and performing the queueing algorithm calculations repeatedly. The queueing delay from our model is deterministic, conditional on a realized set of flights. This was seen in each draw of the Monte Carlo simulation shown above in Table 4-6. If we can develop an approximation that reduces the realized set of demand to a single set, we can speed up the algorithm tremendously.

With the introduction of the cancellation probabilities, the delay for each time period is now a random variable, which is itself a function of another random variable, demand. We'll call the demand vector random variable $\bar{D}$ and the function of demand that represents delay, $f(\bar{D})$. We express demand as a vector to account for each time period separately. We can write an approximation for the expected value of delay using a Taylor series expansion:

$$E[f(\bar{D})] = f(E[\bar{D}]) + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} COV[D_i, D_j]\frac{d^2 f(\bar{D})}{\partial D_i \partial D_j}\bigg|_{E[\bar{D}]} + \cdots \qquad \text{(Equation 4-12)}$$

In the above equation $\overline{D}$ is the demand vector, containing n demand variables, one for each time period, and $f(\overline{D})$ is the relationship between the demand and the flight delay. The equation shows the first two terms in the Taylor Series for $E[f(\overline{D})]$, the expected value of delay for each time period. The higher order terms beyond the second are not shown here. The first term is simply the delay function evaluated at the expected value of demand, $E[\overline{D}]$. The second term includes the covariance of the demand for each pair of demand variables, $COV[D_i, D_j]$, and the second derivative of the delay function with respect to each pair of the demand variables, $\frac{d^2 f(\overline{D})}{\partial D_i \partial D_j}$. The derivative of delay with respect to the demand would be difficult to calculate analytically, given the nature of our queueing process. The algorithm we use is iterative over each minute and uses the minimum function in the calculation of delay. The minimum function is not continuously differentiable over all points, so an approximation to the minimum function would need to be used. This is not trivial, so we first consider the Taylor Approximation with only one term. This is the first-order approximation that we use to estimate the expected delay as a function of demand which is a random variable:

$$E[f(\overline{D})] \cong f(E[\overline{D}]) \qquad \text{(Equation 4-13)}$$

We can now consider the demand at each time period independently. The demand random variable for each time period, as shown earlier, is simply the sum of independent Bernoulli random variables for each flight within a given time period. The demand within a single time period is given by Equation 4-13, above. Now we can take the expected value of the demand random variable. Since demand for a given time period is the sum of independent Bernoulli random variables, we can take the expected value operation inside the summation:

$$E[D_i] = E\left[\sum_{j \in i} X_j\right] = \sum_{j \in i} E[X_j] \qquad \text{(Equation 4-14)}$$

The expected value for each Bernoulli random variable is simply the probability that the Bernoulli random variable is equal to 1, which is our case is 1 minus the cancellation probability, p_j. Thus, we can write the expected value of demand, for a given time period, as the following:

$$E[D_i] = \sum_{j \in i}(1 - p_j) \qquad \text{(Equation 4-15)}$$

We can illustrate this approximation through the same sample schedule we used earlier, shown below in Table 4-7.

Table 4-7: Sample Set of Departures with Cancellation Probabilities

Departure Time	Cancellation Prob.
8:00	0.02
8:00	0.01
8:00	0.03
8:01	0.03
8:03	0.04
8:03	0.03
8:05	0.02

For each minute, we calculate the expected value of demand using Equation 4-15. The results are shown in Table 4-8, below.

Table 4-8: First-Order Approximation of Demand

Time Period, i	$E[D_i]$
8:00	2.94
8:01	0.97
8:02	0
8:03	1.93
8:04	0
8:05	0.98

We then calculate the queueing delay once, using $E[D_i]$ as an estimate for the expected demand. We will compare the results from this first-order approximation to the results from Monte Carlo simulation to determine the amount of bias caused by such an approximation.

4.3.3   Simulation Experiment

To compare the results of Monte Carlo simulation and a first-order approximation, we will estimate the queueing delay for 150 airport-days of flight departures. The following airports are used: ORD, JFK, LGA, EWR, ATL, PHL, BOS, and SFO. The days are chosen uniformly from months ranging from October 1st, 2010 to December 31st, 2011.

We'd also like to compare the effects of different cancellation model specifications on the final delay estimates. Thus, we will be calculating the cancellation probabilities for each flight from six different cancellation specifications. The specifications are shown below in Table 4-9, where an X indicates the variable is included in the specification shown.

Table 4-9: Cancellation Model Specifications for Delay Analysis

Explanatory Variables	Cancellation Model Specifications					
	Model 0	Model 1	Model 2	Model 3	Model 4	Model 5
Constant	X	X	X	X	X	X
Time		X	X	X	X	X
Distance		X	X	X	X	X
Hub Origin			X	X	X	X
Hub Destination			X	X	X	X
Airline Fixed Effects			X	X	X	X
Number of Seats				X	X	X
Load Factor				X	X	X
Fare				X	X	X
Frequency				X	X	X
Queueing Delay					X	X
GDP					X	X
Weather						X

Each model is successively built from the previous one by adding more and more variables. The first model is a very naïve specification, using a constant cancellation probability for each flight. The first model uses variables for time of departure and flight distance, with no flight-specific variables used. Model 5 is used as a proxy for our "perfect information" case, since it includes some variables can only be known after the day-of-operation. Thus, this model represents the theoretical limit for being able to predict cancellations using a binary logit model specification given the information we are observing. In practice, however, predictions cannot be made on these variables, so a model specification such as Model 2 or Model 3 would more likely be used. Model 4 could also be used for prediction assuming specific capacity scenarios that can be inferred from historical data and the presence (or lack) of GDPs.

We use the average delay for each flight across all 150 airport-days, without considering cancellations, as a baseline. The delays due to cancellations were calculated using the probabilities predicted from each cancellation model and then were compared with the baseline delay in terms of how much the baseline delay was reduced. The reduction in delay reflects the

changes to the flight schedule that are captured in our cancellation model and predicted in the form of cancellation probabilities.

The baseline delay was 9 minutes / flight for our entire sample. The delay reduction estimates range from 0.74 minutes / flight, or 8.02% of the baseline delay, to 2.11 minutes / flight, or 22.87% of baseline delay. A summary of the results for both queuing algorithms and all cancellation model specifications is shown below in Table 4-10.

Table 4-10: Delay Reduction Comparison

Delay reduction due to cancellations	First-Order Approx.		Monte Carlo	
Baseline delay (no cancellations) = 9 min. / flight	Min / Flight	% of delay	Min / Flight	% of delay
Model 0: Constant	-0.77	8.33%	-0.74	8.02%
Model 1: Time, Distance, Day of week	-0.77	8.30%	-0.74	8.00%
Model 2: Model 1 + Hubs, Airlines	-0.86	9.30%	-0.83	8.94%
Model 3: Model 2 + Fare, LF, Seats, Freq.	-0.95	10.25%	-0.91	9.85%
Model 4: Model 3 + Queueing Delay, GDP	-1.72	18.59%	-1.67	18.03%
Model 5: Model 4 + Weather	-2.11	22.87%	-2.05	22.24%

As the cancellation model specification becomes more sophisticated, the percentage reduction in delay increases. The largest jump between model specifications is between Model 3 and Model 4, with an increase in delay reduction of over 8 percentage points. The only variables different between these two models are the queueing delay at the origin and destination and the presence of a GDP. Model 5, which incorporates weather at the origin and destination, only increases the delay reduction by approximately 4 percentage points. Although the weather variables have large and significant coefficient estimates in the logit model, their incremental impact on predicting delay reductions due to cancellations is not very large compared to the marginal impact of the queueing delays and GDPs. Queueing delays are a function of the realized capacity, which itself a function of the weather and operating conditions. These results indicate that using queueing delay in our model captures a very large portion of these effects.

The most naïve models can capture around a third of the delay reduction that is found from using the most sophisticated models. The marginal impacts of adding fixed effects for time, distance, day of week, airline, and hubs as well as flight characteristics are relatively small compared to the incremental effect of accounting for operating conditions through the use of queueing delay variables.

The other important result from this analysis is the small difference between the estimates for each model using the first-order approximation, as compared to the Monte Carlo simulation estimates. In general, the predicted delay reduction resulting from cancellations estimated from the first-order approximation is about 3% more than the prediction based on the Monte Carlo

simulation. The magnitudes of delay reduction that come from improving the model specification are much larger than the magnitude of the differences between the two techniques. Thus, we conclude that the first-order approximation is sufficient for predicting the delay results using the estimates from our cancellation models.

As a further exploration of the differences between the model results, consider the empirical CDF of the daily percentage delay reduction, across all 150 airport days, for each model specification using the first-order approximation method, shown below in Figure 4-5.
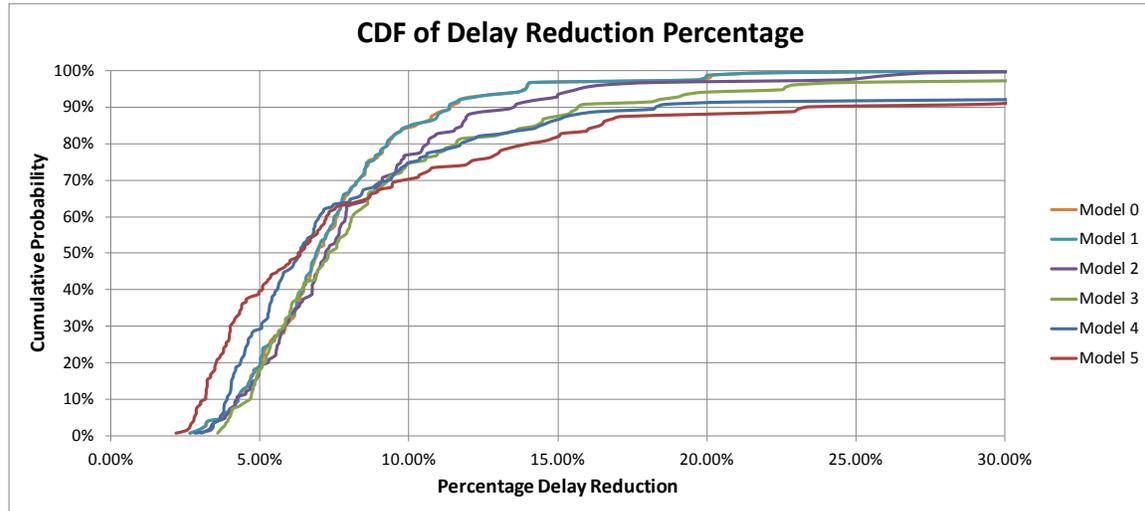
Figure 4-5: Empirical CDF of Delay Reduction

There are a few interesting results from this plot. First, the difference in the percentiles for each model is relatively small up until the 80th percentile. The primary difference between the mean values reported in Table 4-10 result from the difference in the size of the upper tails for each model. To illustrate the difference between the curves, consider the difference in delay reduction for the 70th percentile for each model. From the figure, this looks to be no more than 2 or 3 percentage points. Looking at the 90th percentile, however, we can see a much larger difference of at least 20 percentage points between Model 1 and Model 5. Thus, the primary driver behind the differences in the mean delay reduction in Table 4-10 are the small number of days with a large number of cancellations, and thus, a large delay reduction. From investigating the specific data points in this range, the days in the upper tails are typically associated with winter and airports in the northeast, such as LGA, EWR, JFK, PHL, and BOS.

Second, the weather, queueing delay, and GDP variables capture days with very large delay reductions that are not found in the other models. For purposes of predicting cancellations and their effect on delay reduction for days that do not have many cancellations, the difference between the model specifications becomes less important. Using a completely naïve model will produce a similar delay reduction for most days as the most sophisticated model.

Third, notice the variation in skewness between the different models. The most naïve models are the most symmetric, and the most sophisticated models are more skewed to the right. For airport-days below the median, Model 5 has a much smaller delay reduction than the other models. This makes sense because the more sophisticated models are better able to distinguish between days with many cancellations and days with fewer cancellations since we are using more information about the operating conditions on any particular day Model 5 (and 4 as well) better captures the tendency of airlines to cancel flights in circumstances where the delay impact of the cancellations is large.

The median value of delay reduction is stable across all model specifications, having a value of approximately 6-7%. The median value is much lower than the mean value for each model, as a result of the long tails of the delay reduction distributions.

5 SFO Case Study

Finally, we present an application of our cancellation models in a case study at the San Francisco International Airport (SFO). We will use the cancellation models presented in this report to predict flight cancellations that are cause by changes in operating conditions at the airport. Specifically, we will be looking at changes in capacity, which are manifested in terms of changes in queueing delay. The results presented here are applicable for any changes in capacity or demand and can be used to predict the change in cancellations under a number of different operating conditions.

SFO operates four runways, arranged in two crossing sets of two parallel runways (see Figure 5-1 below). Under typical operating conditions, one pair of runways (1L & 1R) is used for departures and the other pair (28L and 28R) is used for arrivals. When weather conditions are favorable, simultaneous departures and arrivals are permitted. However, when weather conditions deteriorate, simultaneous departures and arrivals are not allowed, effectively cutting the airport's operational capacity in half.
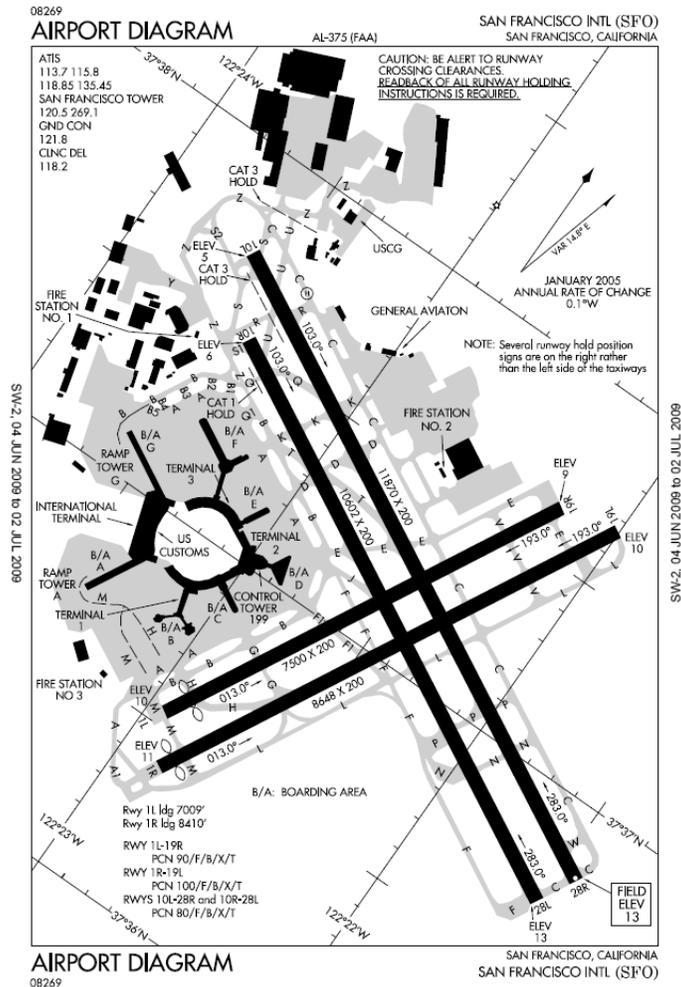
Figure 5-1: SFO Runway Layout

Two of the runways, 1L/19R and 1R/19L, are scheduled to be closed during the summer of 2014 due to construction. All operations will take place on the remaining two runways for the duration of the construction period. Thus, delays are expected to rise during this time. To add to the congestion that is likely to result from the reduction in capacity, airlines have planned flight schedules for next summer with 5% more demand on average than previous summers. The new flight schedule added to an already capacity-constrained airport will result in even higher delays. We expect, however, that the high delays will result in an increase in flight cancellations. The goal of this analysis is to determine to what extent the increase in demand will affect the number of flight cancellations during the construction time period next summer at SFO.

5.1 FAA Simulation

The FAA has modeled the effect of capacity restrictions and schedule increases through use of their airport delay simulation tool, ACASAT. Two demand scenarios were used for the analysis, one flight schedule from summer 2013 and a proposed schedule for summer 2014 with approximately 5% more flights. These demand scenarios were used along with 90 days of capacity profiles, where the capacities were modified to account for the runway closures. The resulting delay profiles were averaged together to get a representative profile for each demand scenario for summer 2014. The delay profiles for arrivals and departures, generated by the FAA, are shown below in Figure 5-2 and Figure 5-3.
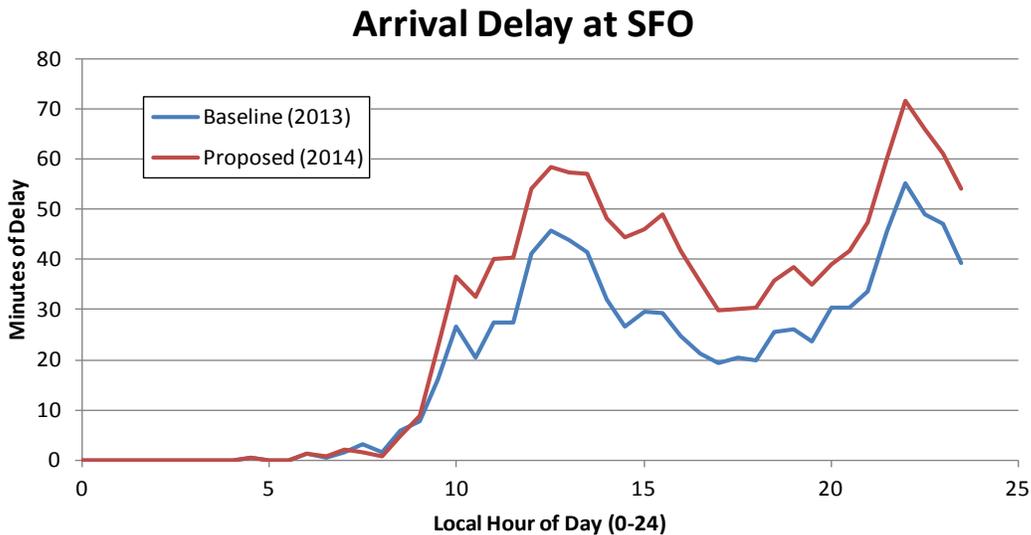

Figure 5-2: SFO Arrival Queueing Delay
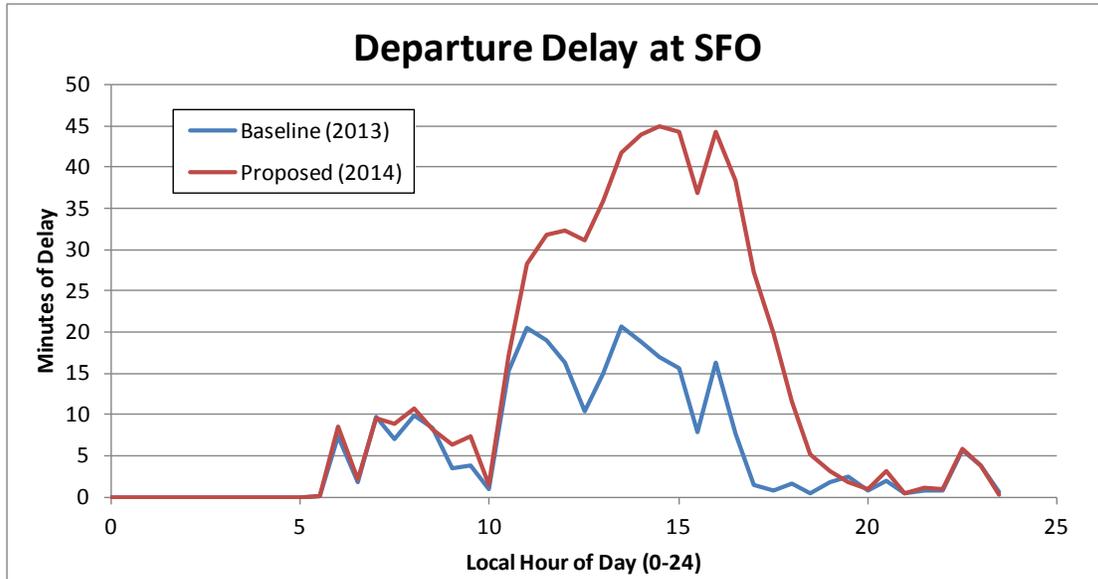
Figure 5-3: SFO Departure Queueing Delay

As seen in Figure 5-2, the average arrival delays increase to 60 minutes by early afternoon and reach just above 70 minutes in the late evening. The difference between the two curves represents the increase in queueing delay caused by the increase in demand. Arrival delay increases by around 15 minutes for most of the day due to the proposed schedule.

Shown in Figure 5-3, departure delays are not as high as the arrival delays, with a peak of 45 minutes in the late afternoon. The increase in departure delays due to the new schedule is quite significant, however. The baseline schedule only generates between 15 and 20 minutes of delay in the busy period between 1000 and 1700, but the new schedule will increase these numbers by 20-25 minutes.

These delays seem to very high and are not representative of the numbers typically seen for actual flight delays at SFO. These high delays are unlikely to be realized on the day-of-operation, because airlines will likely respond to high delays with flight cancellations, which effectively reduce the demand, and thus the queueing delay. To what extent the delays will be reduced due to flight cancellations is not obvious, however. We will develop a model that will allow us to predict flight cancellations as a function of queueing delay and apply the results to the two demand scenarios shown above. We can predict how airlines will respond given the increase in schedule, and thus, delay.

5.2   Cancellation Model

We will use a similar cancellation model specification as before, from section 2. Since we will be predicting cancellations for flights that have yet to be operated, we do not have access to information such as weather and GDPs. Therefore, we will remove those variables from our model, as shown in Figure 5-4, below. In our aggregate MNL model, we used queueing delay that was a function of scheduled demand and realized capacity. In this case, we will use queueing delay forecasts that were created using historical capacity profiles by the FAA to predict the delay for the time period of interest. The arrival and departure queueing delay forecasts shown in Figure 5-2 and Figure 5-3 will provide the values of queueing delay that we will use in our cancellation model.

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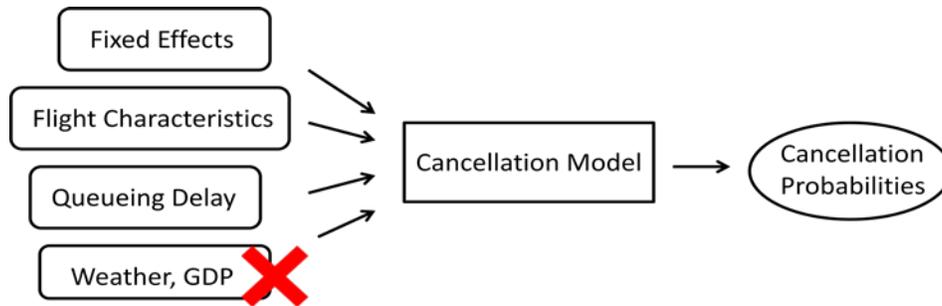
Figure 5-4: Cancellation Model Structure

We are primarily interested in SFO and our queueing delay forecast is limited to that airport, so we will develop a cancellation model only for flights into or out of SFO during our two-year time span (2010-2011). We estimate two different models, one for departures and one for arrivals. The model will be the same structure as the model used in section 2, with the exception of the weather and GDP variables. The estimation results for the arrival model are shown below in Table 5-1.

Table 5-1: SFO Arrival Cancellation Model Results

Variable	Estimate		Std. Err.		Estimate		Std. Err.
ASC(Cancel)	-2.614	**	0.593	DL (Delta)	0.085		0.528
Fare($100)	0.001		0.033	UA (United)	0.321		0.526
ArrTime(9:00-15:00)	-0.115	**	0.054	US (US Airways)	-0.408		0.541
ArrTime(15:00-21:00)	-0.016		0.053	AA (American)	0.958		0.526
ArrTime(21:00-3:00)	-0.321	**	0.082	CO (Continental)	-0.334		0.539
Miles<500	0.453	**	0.072	WN (Southwest)	0.458		0.530
Miles750-1000	-0.128		0.117	B6 (JetBlue)	-0.230		0.543
Miles1000-1500	-0.009		0.113	F9 (Frontier)	-0.245		0.609
Miles1500more	0.387	**	0.089	FL (Air Tran)	-1.987	**	0.785
Num.Seats(100)	-0.244	**	0.093	AS (Alaska)	-0.945		0.566
LoadFactor	-2.902	**	0.232	Regional_Carrier	0.616	**	0.101
FlightFrequency	0.049	**	0.007	Arrival Delay (min)	0.033	**	0.002
Sunday	-0.467	**	0.068	Arrival Delay Squared (100s min^2)	-0.012	**	0.002
Monday	-0.204	**	0.061	** Significant at 1% level			
Tuesday	0.091		0.058	* Significant at 5% level			
Thursday	-0.191	**	0.062				
Friday	-0.203	**	0.062				
Saturday	-0.518	**	0.074				

Some of the coefficients are similar to those found in the model from section 2, such as number of seats, load factor, frequency, and day-of-week. Others show different trends, such as the positive coefficient and significant for the longest flight distance category and the negative sign for the latest arrival time category. The airline fixed effects are not very large and generally not significant, in contrast to the strong inter-airline differences found in the aggregate model. The regional carrier coefficient is positive and significant, indicating a higher propensity of cancellations for regional carrier flights.

The queueing delay variable estimates are both significant. The negative sign of the quadratic term indicates a decreasing effect on the cancellation utility as the queueing delay increases. This is consistent with what we have seen in the aggregate model from section 2. To illustrate this effect, consider a graph of cancellation utility versus arrival queueing delay, shown below in Figure 5-5. In this graph we show two curves, one corresponding to the results from the linear + quadratic model specification above and another corresponding to just a linear specification. The graph illustrates the way the models are capturing any non-linear effects of queueing delay.
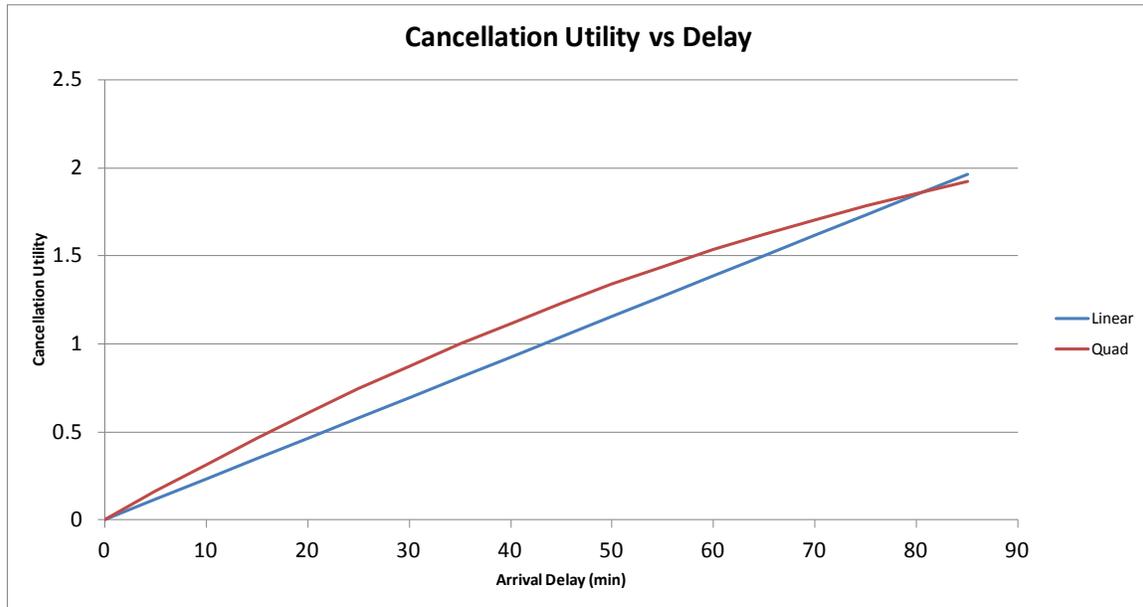

Figure 5-5: Cancellation Utility vs Arrival Delay

The two curves tend to track each other very closely, indicating that the effect of the quadratic term does not largely change the behavior of cancellation utility with respect to queueing delay. The model fit, however, is greatly improved by using a linear + quadratic specification, so we will use this as our final choice.

The estimation results for the departure model are shown below in Table 5-2.

Table 5-2: SFO Departure Cancellation Model Results

Variable	Estimate		Std. Err.		Estimat		Std. Err.
ASC(Cancel)	-1.759	**	0.601	DL (Delta)	-0.241		0.536
Fare($100)	0.093	**	0.029	UA (United)	-0.225		0.535
DepTime(9:00-15:00)	0.136	**	0.044	US (US Airways)	-0.200		0.543
DepTime(15:00-21:00)	0.114	*	0.049	AA (American)	0.724		0.534
DepTime(21:00-3:00)	-0.179	*	0.080	CO (Continental)	-0.665		0.546
Miles<500	0.501	**	0.074	WN (Southwest)	0.211		0.538
Miles750-1000	-0.277	*	0.134	B6 (JetBlue)	-0.161		0.547
Miles1000-1500	0.144		0.116	F9 (Frontier)	-0.112		0.610
Miles1500more	0.733	**	0.090	FL (Air Tran)	-0.294		0.592
Num.Seats(100)	-0.544	**	0.099	AS (Alaska)	-0.485		0.557
LoadFactor	-3.606	**	0.227	Regional_Carrier	0.671	**	0.109
FlightFrequency	0.072	**	0.007	Dep. Delay (min)	0.056	**	0.006
Sunday	-0.433	**	0.070	Dep. Delay Squared (100s min^2)	-0.033	**	0.009
Monday	-0.084		0.062	** Significant at 1% level			
Tuesday	0.217	**	0.059	* Significant at 5% level			
Thursday	-0.195	**	0.064				
Friday	-0.167	**	0.064				
Saturday	-0.529	**	0.075				

We see similar coefficient estimates as the arrival model, such as a negative sign for the latest departure time category and a large positive sign for the longest distance category. The flight characteristic coefficients, day-of-week, airlines, and regional carrier are all similar to those found in the arrival model. The delay coefficients are slightly larger in magnitude than those in the arrival model, but follow the same trend with a positive linear term and a negative quadratic term.

Similarly, a graphical depiction of the differences between the linear + quadratic specification and a linear specification is shown below, in Figure 5-6. We plot the utility vs departure queueing delay for each of the two specifications. The graph illustrates the way the models are capturing any non-linear effects of queueing delay.
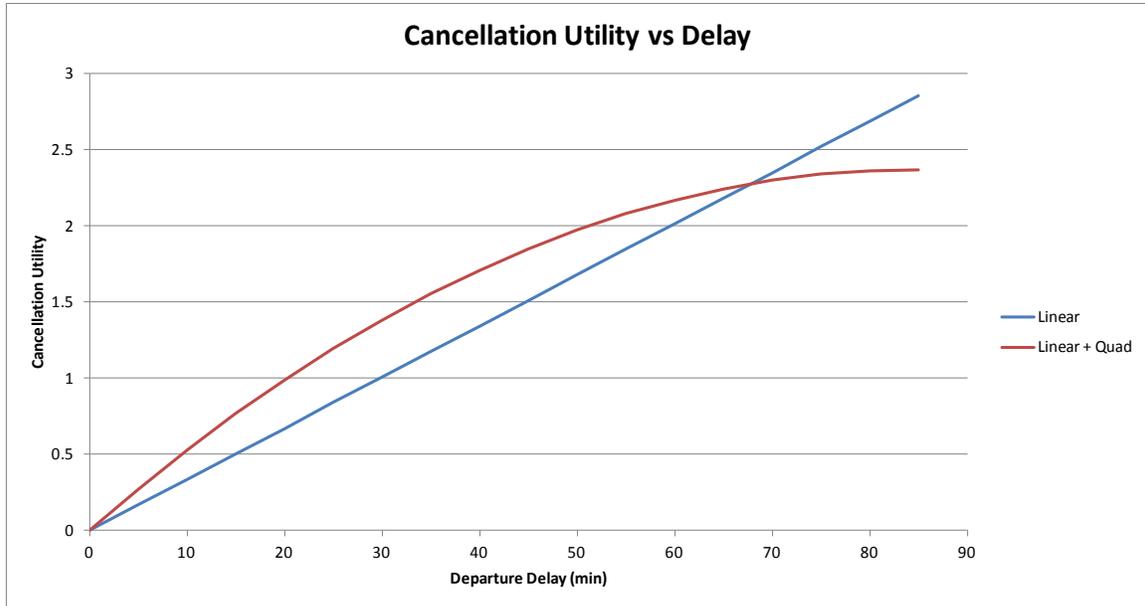
Figure 5-6: Cancellation Utility vs Departure Delay

The linear + quadratic model does not track the linear model as closely as the figure corresponding to the arrival delays. We see that the quadratic effect is much stronger in this case, which eventually causes the cancellation utility for the linear + quadratic model to be lower than that for a linear model once the queueing delay is higher than approximately 70 minutes. However, we do not see queueing delays for our prediction time period higher than 50 minutes (see Figure 5-3). The linear + quadratic specification has a much better model fit, so we will use this as our final specification.

5.3   Prediction Results

We now use the estimation results to calculate cancellation probabilities for new flight schedules and delay estimates for summer 2014. Airlines are proposing a 5% increase in the number of flights next summer, when the capacity of the airport will be greatly reduced due to runway closures. The FAA has shown that delays will be higher for the new schedule compared to a baseline schedule from summer 2013. We will evaluate the impact the new delay values will have on flight cancellations.

Since we are predicting cancellation probabilities on future flights, we do not know the values of some of our explanatory variables, such as load factor or average fare. We are limited to a flight schedule that contains the airline, origin, destination, and aircraft type. We used our historical data to estimate these values based on similar flights in our dataset. We calculated the average values for each explanatory variable over the summer months of June, July, and August for the year 2011. The averages were calculated across airline, origin, destination, and aircraft type. We then matched these values to the new flight schedule for summer 2014 to be used in our cancellation prediction.

We use the two flight schedules and their respective delay profiles to predict cancellation probabilities for each flight across the day. The cancellation probabilities are aggregated over 30 minute windows and plotted against the queueing delay for each flight operation, shown below in Figure 5-7 and Figure 5-8.
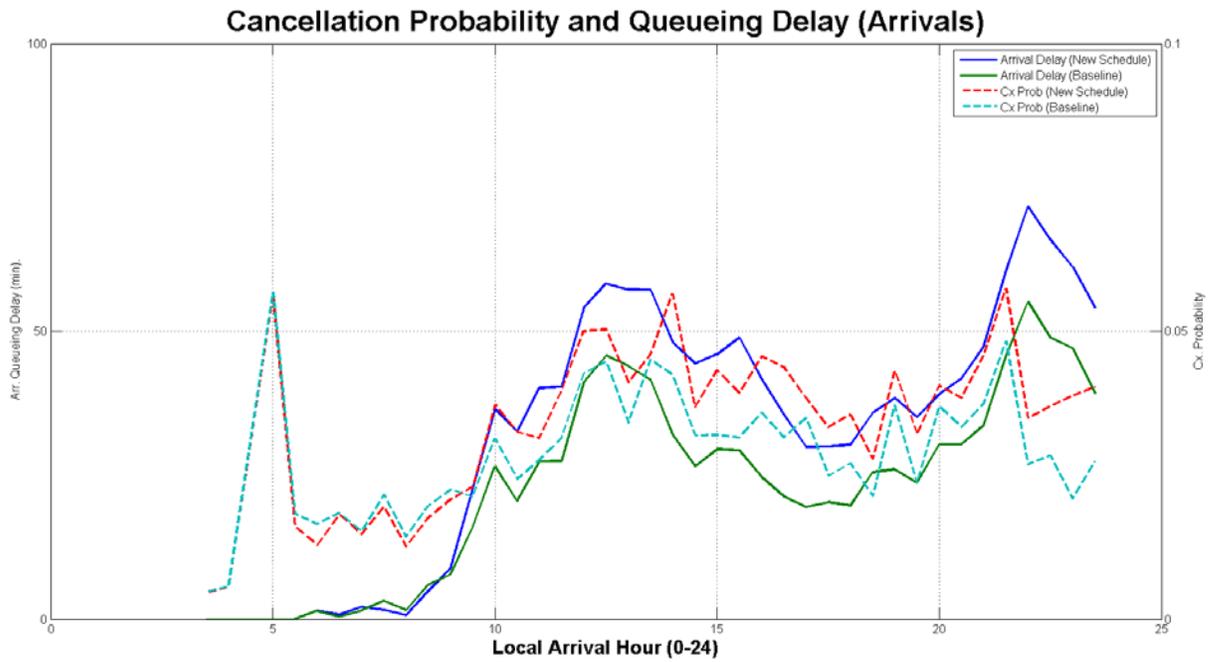
Figure 5-7: Arrival Queueing Delay and Cancellation Probability
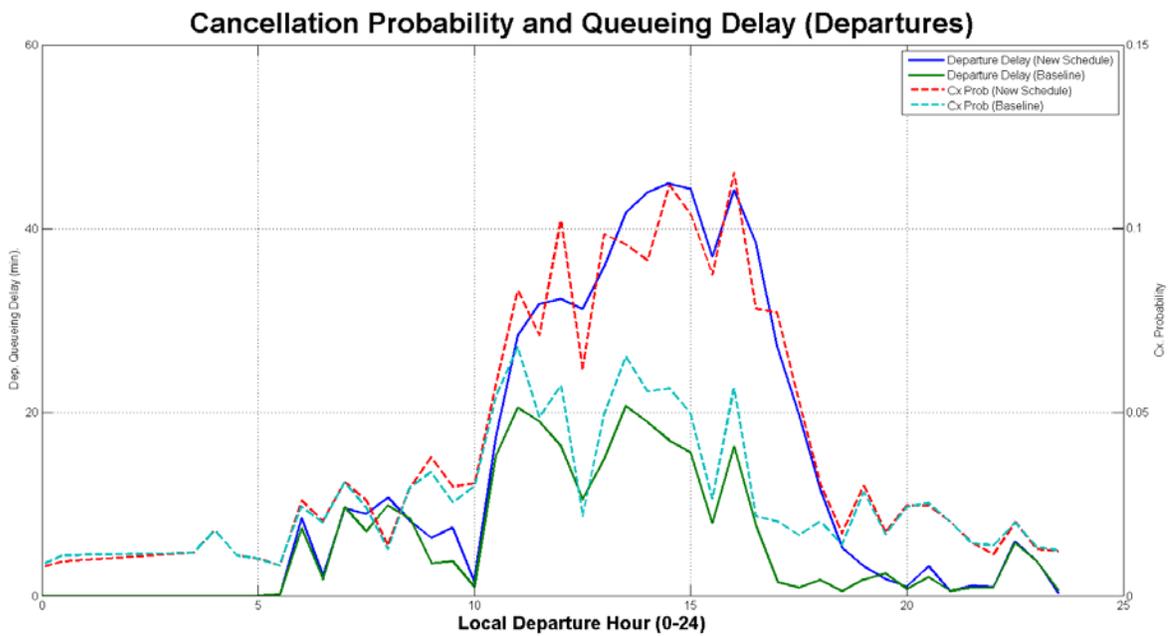
Figure 5-8: Departure Queueing Delay and Cancellation Probability

As seen in Figure 5-7, the arrival cancellation probability profile for the new schedule is similar to that of the baseline schedule. The delay increase due to the new schedule is not very large, and thus the increase in cancellation probabilities is not larger either. An increase in about 1% of flight cancellations during the busy period can be expected.

The results when considering departures, however, are quite different. From Figure 5-8, the difference in delay profiles is quite large for the two schedules, and thus the cancellation probability profiles are very different as well. The cancellation probability during the busy period in the middle of the day roughly doubles, from 5% to 10%. The percentage point increase in cancellation probability is roughly the same magnitude as the increase in demand for the same time period.

A summary of the changes in demand and cancellation probabilities for arrivals and departures is shown below in Table 5-3. An increase in flight cancellations of 0.8% can be expected for arrivals during the busy period, while the increase for departures is 4%.

The new schedule has about 4% more flights during the peak period (1100-1800) for both arrivals and departures. However, the increase in arrival cancellation probability is much larger for departures (+4%) than arrivals (+0.8%). Therefore, we estimate that the increase in scheduled flights, particularly for departures, will not be fully realized. Due to the large increase in delay, flight cancellations will increase due to the increase in queueing delay that will result from the reduction in capacity. The departure delay will increase to the point where the number of cancelled departures increases by roughly the same amount as the departure demand. It seems obvious that increasing a planned flight schedule in a manner such that increased flight cancellation returns flight volume to its original level benefits neither airlines nor passengers.

Table 5-3: SFO Cancellation Prediction Summary

		Demand			Average Cancellation Prob.		
		Baseline	New Sched.	Diff.	Baseline	New Sched.	Diff.
Arrivals	All Day (0000-2300)	636	675	+6.1%	2.4%	2.8%	+0.5%
	Peak Period (1100-1800)	283	295	+4.2%	3.3%	4.1%	+0.8%
Departures	All Day (0000-2300)	635	663	+4.4%	2.5%	3.9%	+1.4%
	Peak Period (1100-1800)	281	293	+4.3%	4.0%	8.0%	+4.0%

6 Conclusions

In this research we have investigated the factors influencing flight cancellations, including operational conditions, flight characteristics, weather, and airline-specific effects. We used discrete choice models applied to a set of historical domestic flights over two years to infer the preferences and behaviors of airlines with respect to flight cancellation decisions.

Our cancellation behavioral analysis results indicate that, as expected, adverse weather is a major contributor to flight cancellations. In addition, we discovered that flight characteristics are still

important factors for airlines. Particularly, average load factor plays a very large role in driving flight cancellations. We found large differences in the cancellation behavior across airlines, as measured through fixed effects and airline-specific cancellation models. The trends in behavior are somewhat consistent for the legacy carriers, but less so for the low cost carriers. We also developed a random effects model to capture correlation between unobserved variables across multiple flight cancellation decisions. We grouped flight decisions into time windows of four hours and treated the sequential decisions in the same vein as panel data for repeated choices for an individual. We estimated a significant, albeit small, random effect term which indicates that the proclivity to cancel fluctuates from one four-hour period to the next.

We evaluated the model fit for all of our cancellation models by using sample enumeration to aggregate the predicted cancellations across airport-days. Evidence of under-prediction and over-prediction were both seen, and the results vary across airports. We used the assumption that the number of flight cancellations for a particular airport-day are independent events and follow the Poisson distribution to develop a formal statistical test for the model fit of our models. Through these statistical tests we did not find evidence that the more detailed model specifications, i.e. airline-specific models and the random effects model, resulted in a better model fit than the aggregate cancellation model. However, these models are useful for distinguishing cancellation behavior across airlines and across time.

We then developed a queueing algorithm to evaluate the effect of cancellations on delay forecasts. We compared our model to the queueing algorithm used by the FAA in their ACASAT simulation and showed a small difference in the overall delay estimates. We used Monte Carlo simulation and first-order approximation for modeling the demand as a random variable, as derived from the cancellation probabilities for each flight. We performed a simulation experiment to compare the effects of different cancellation model specifications and the two queueing algorithm techniques. Our results indicate that a naïve cancellation model can accurately predict a third of the overall delay reduction found from a sophisticated cancellation model (8% vs 23%, respectively). Beyond a naïve cancellation model, the largest increase in delay prediction comes from adding the queueing variables to the cancellation model sophistication (8 percentage point increase in delay reduction). We also found that the difference between the Monte Carlo technique and the first-order approximation were very small compared to the differences in delay prediction caused by changes in the cancellation model specification.

Finally, we illustrated a case study at SFO. In the summer of 2014, runway construction will result in reduced capacity at SFO and airlines are proposing more aggressive schedules with approximately 5% more flights than in 2013. We used a cancellation model with queueing delay as an explanatory variable to predict the increase in cancellation that will be caused by the increase in flight schedules. We see evidence that the departures will be affected more than arrivals, with an increase in departure cancellations almost one-to-one with the increase in demand. The benefits of an increased schedule are likely to not be realized by airlines or air travelers, given the large number of cancellations that we anticipate based on our model. Thus, these results suggest that airlines should reconsider their schedules at SFO in summer 2014.

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