Factors Influencing Estimated Time En Route

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Abstract

We investigated the influence of several factors that might be expected to influence a flight's estimated time en route (ETE): origin airport, destination airport, month of year, day of week, hour of day, aircraft type, and carrier. Our main interest was to see whether the ETEs in filed flight plans differed within and among carriers. We found much variation in ETEs. Sustained and significant trends in ETE have occurred for a number of origin-destination pairs. Route, month, hour of day and carrier are all statistically significant influences on ETE. Some routes have ETE distributions that are well modeled by a mixture of lognormals; in simple cases, this pattern can be regarded as a mixture of regular and irregular operations. Some origin-destination pairs show large differences in the number of different routes specified in flight plans, but variations in flight plan distances are too small to account for all the variation in ETEs. Overall, the simple question of how long it should take to fly from point A to point B turns out to have an intriguing number of revealing answers.

Keywords: Flight planning, Airline operations

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1. Introduction

Every airline flight is conducted on the basis of a flight plan filed with the Federal Aviation Administration (FAA). One component of flight plans is an estimate of the time between takeoff and landing, known as the estimated time en route (ETE). As part of a study of deviations from flight plans, we became interested in the phenomenon of variation in ETEs. Like so many other elements of the air transport system, ETEs are dynamic. In a system as stressed as the US National Airspace System (NAS), even single digit percentage changes in ETEs can carry operational significance. We investigated variations in ETEs in hopes of better understanding the behavior of airlines and the influence of external factors on that behavior.

There is a good deal of variability in ETEs. For instance, the average ETE for flights from Memphis (MEM) to Cincinnati (CVG) was 76 minutes in winter 1998-99 but dropped to 73 minutes, 66 minutes, and 58 minutes in the three succeeding winters. Over the same period, the average winter ETE for commercial flights between Baltimore (BWI) and New York (LGA) rose steadily from 36 to 52 minutes.

Variability is also present at a more microscopic level. Consider the case of one route served by two major carriers during the first five months of 2002. For one of the carriers, there was a 6% difference in ETEs for its own flights just two hours apart. And for flights departing at one particular hour, there was an 8% difference between the ETEs of the two carriers.

Several factors could account for such differences. Some can be thought of as background influences: the origin and destination airports, the anticipated weather along the route, and the type of aircraft. Of particular relevance to the FAA would be links between anticipation of airspace congestion and carriers' flight planning.

Of interest to the study of airline behavior would be evidence of differences in carriers' flight planning styles. We have heard anecdotal evidence of substantial differences in carriers' flying styles, such as the efforts taken to insure a ride free of turbulence and sudden maneuvers; similar differences might appear in flight planning. We know that airlines devote attention to flight planning issues: in summer 2002, United Airlines was advertising for an operations researcher to lead its SkyPath project for development of new flight planning software.

To better understand the influences on the ETE component of flight planning, we undertook two statistical analyses. The first, broader study looked at trends in average ETEs for flights among 31 major airports. The second, more detailed study examined data on individual flights during early 2002. Section 2 reports trends in ETEs for flights among 31 major airports. Section 3 reports the results of bivariate statistical analyses, which relate ETEs to factors such as airline, day of week, etc. Section 4 reports the results of a multivariate analysis. Finally, Section 5 summarizes our findings and relates them to issues in air traffic management and airline operations.

2. Trends in ETEs

As part of an ongoing study on sources of delay, we obtained information on average ETEs for 31 major airports in the US. These data were retrieved from the FAA's ASPM database (see http://www.apo.data.faa.gov). The data include operations of several major airlines, which we cannot name. The 31 airports were ATL, BOS, BWI, CLE,

CLT, CVG, DCA, DEN, DFW, DTW, EWR, IAD, IAH, JFK, LAS, LGA, LAX, MCO, MEM, MIA, MSP, ORD, PHL, PHX, PIT, SAN, SEA, SFO, SLC, STL, and TPA.

To minimize the effect of convective weather on flight times, we narrowed our analysis to include only operations during the winters of 1998-1999, 1999-2000, 2000-2001 and 2001-2002. We defined winter as December, January, and February.

Out of over 900 routes, this analysis identified 33 for which the average ETE decreased across all four winters at an average rate of at least 2% per year. We also found six routes for which the average ETE increased for four consecutive winters at an average rate of at least 2% per year. Table 1 lists the routes and rates of change in ETEs. Memphis (MEM) figured prominently in OD pairs with decreasing ETEs. Most of the increases were on routes in the northeast. Table 1 establishes that, even averaging over many flights from many carriers, ETEs on some routes can be quite dynamic.

3. Influences on ETEs of Individual Flights

3.1 Data

To better understand the influences on ETEs, we used ASPM data for nearly 60,000 individual flights during the first five months of 2002. Five relevant variables were available to us:

- Carrier: A primary interest was to see if there were any systematic differences that could be attributed to differences in flight planning styles among major airlines. We studied flights from six major carriers, which we will refer to as AAA, BBB, CCC, DDD, EEE, FFF.
- Route: The distance between the origin and destination airports is a major influence on ETE. Beyond that obvious fact, however, we could study whether the relative

behavior of competing airlines was different along different routes. We selected origin-destination (OD) pairs from among the 31 major airports listed in the previous section. Because we wanted to subdivide the data by hour of the day and to study routes served by more than one major carrier, we were forced to discard most of the possible OD pairs. In the end, we identified 14 OD pairs, accounting for 28 routes (i.e., we treated the two directions between the airports as different routes). Generally, we combined results from the two directions to give one overall set of results for any given OD pair. For a given OD pair, we excluded carriers with very few flights. Figure 1 shows these routes on a map. Table 2 shows the count of flights in our study broken down by OD pair and carrier. Overall, we analyzed about 60,000 individual flights. (We excluded flights made by carriers having very little traffic for a given OD pair.)

- Month: Seasonal weather patterns can have an effect on ETEs. We could expect the effect to be different for routes at different latitudes. We used ASPM data from January – May 2002.
- Hour: Both winds aloft and airspace congestion can be expected to vary by time of day. To insure stable estimates, we excluded from analysis any hour on any route that did not have at least four flights per month. In general, there were sufficient flights to study operations from 0600 to 2200 local time. To the extent that there are large hourly variations in ETEs along the same routes, we could suspect that congestion would play some part. (To investigate this association, we would need to compile data on hourly airport arrival and departure rates relative to airport capacity. This

would expose any correlation between congestion in terminal airspace and longer ETEs. We have not done this analysis.)

- Aircraft type: To the extent that different aircraft operate at different speeds and different carriers use different types of aircraft, this variable can confound any comparison of airline flight planning. Likewise, it can contribute to ETE variability within a single carrier. However, because it is quite common for aircraft to cruise at speeds much lower than their maximum speeds, this problem may not be acute.

3.2 Analysis of ETEs by OD pair and carrier

We begin the data analysis with a series of tables showing ETE statistics broken down by OD pair and carrier. Table 2 shows counts of flights. The Range/Average column divides the difference between the largest and smallest values in a row by the average of the values in the row. It is a measure of the relative variability across carriers serving a given OD pair. For Table 2, it provides a measure of dominance for each OD pair: a large value indicates very unequal market shares across carriers. By this measure, the most balanced market was DTW:CLT, where DDD and FFF had nearly equal numbers of flights; the least balanced was SFO:LAX, which EEE dominated. (Note that in all our OD pairs, only two or three of our six carriers had appreciable market share, so the overall level of competition was not as high as it might have been.)

Table 3 shows average ETEs. While the averages are very route-specific, the Range/Average column standardizes for this to show which OD pairs saw the greatest variability across carriers. The variability ranged from 1% to 10%, the latter being for the SFO:LAX route. These levels of variability in average ETEs suggest that the differences among carriers' flight plans are large enough to be interesting and merit further investigation.

Table 4 shows the standard deviations in ETEs. The value in each cell represents the extent to which flight plans vary from flight to flight for the same carrier flying the same route. The absolute levels of the standard deviations tend to increase with the mean ETEs shown in Table 3; it is not surprising that longer flights would have more room for variety in flight plans. The Range/Average column can be interpreted here as a measure of differences in internal variability of carriers' flight planning processes: a high value indicates that there are substantial differences across airlines in the extent to which they vary the ETE from flight to flight on the same route. Note that a big difference in standard deviation between two carriers does not imply which flight planning process is better. The fact that one carrier has a lower standard deviation might mean that the carrier strives for predictability, which is good, but it could also mean that the carrier is not very particular about its planning and rarely changes ETEs to account for weather or other variable factors, which is bad.

Table 5 shows the coefficients of variation of the ETEs in each cell. The coefficient of variation is the standard deviation divided by the mean. Using the coefficient of variation places the raw differences in standard deviations in context by standardizing them for the mean ETE on a route. This makes it easier to read down a column and develop an overall impression of the variability of flight planning for a particular carrier. For some OD pairs and carriers, there is a fairly high relative variability in ETEs, up to 17% for BBB flying between EWR and ORD. Because the data have already been standardized to take account of differences in route distance, the rightmost column shows the simple range. The OD pair that provoked the greatest difference in carrier behavior was ATL:CLT, where the internal variability in ETEs was 15% for FFF but only 10% for CCC. In fact, CCC had the most consistent ETE (i.e., lowest coefficient of variation) in all six of the markets in which we compared it to other carriers.

We analyzed the data in Table 5 using unbalanced two-way analysis of variance (ANOVA). Both the carrier and route effects were highly significant. Multiple comparisons using Tukey's method with $\alpha = 0.05$ showed that the differences between CCC and all five other carriers were statistically significant. In the same way, many pairs of routes had differences in coefficients of variation that were statistically significant. Thus it is clear that the level of consistency in ETEs varies by route and by carrier.

3.3 Bivariate analyses of standardized ETEs

Earlier, we listed five variables that we expected to influence ETE. In this section, we show how each in turn changed the distribution of ETE.

The response variable in these analyses is the standardized ETE, not the raw ETE studied in the previous section. To allow us to combine results across routes, we removed the obvious effect of route distance by dividing every ETE for flights between a given OD pair by the average ETE for all such flights. For example, every ETE for flights involving EWR and ORD was divided by the ETE of all flights between EWR and ORD. We present the bivariate results in a series of side-by-side boxplots supplemented by tables of summary statistics.

Figure 2 shows how standardized ETE varied by carrier. CCC had the lowest average standardized ETE, indicating the most aggressive flight planning; it also had the lowest interquartile range (IQR), indicating the greatest internal consistency in its flight planning. At the other extreme, AAA had the highest mean and IQR. The difference between the means was about 5%, which is a significant difference operationally. We still need to understand whether other factors, such as fleet mix, might explain this difference. The most visually striking feature of Figure 2 is the presence of many outliers, especially on the high side. Considering the large counts summarized in each boxplot (roughly 6,000 to 12,000), the outliers represent only a small fraction of all flights. However, they indicate that ETEs can vary by a factor of two, and such exceptions are important because they represent major disruptions in scheduled operations.

Figure 3 shows how standardized ETE varied by month. Of the first five months in 2002, March had the highest mean, median and IQR, while February had the lowest. While these differences are highly significant statistically (since they are based on over 10,000 flights each), they are of little operational significance compared to the differences across carriers. (Similar analyses not shown here also established that standardized ETE showed little difference by day of the week.)

Figure 4 shows how standardized ETE varied by hour of the day. The mean and median were relatively constant throughout the day. However, the IQR showed some larger changes, ranging from a low of 0.0684 at 22 hours to a high of 0.0951 at 13 hours. This suggests that the variability in the flight planning process can change substantially throughout the day. Together with differences in when the largest outliers occur, it also hints that anticipated congestion in the NAS could play an important role in airlines' flight planning.

Figure 5 shows how standardized ETE varied by equipment, i.e., type of aircraft. The mean result for Boeing 747s was quite high, but the small sample size of 20 flights means we should disregard these results. The 1,736 flights for which the aircraft type was not recorded had the highest mean of all, leading us to speculate that these were regional jets. Among the known types with many flights, the Fokker aircraft had standardized ETEs about 5% above average and the Airbus aircraft about 2% low. Not surprisingly, then, aircraft type had a large influence on standardized ETE relative to other variables, roughly comparable to carrier. However, we suspect that variations in ETE associated with aircraft type are primarily reflections of differences in aircraft mix across carriers (and differences in routes across carriers).

Since both carrier and type seemed to be important influences on standardized ETE, we wondered whether these variables might be confounded in the dataset, i.e., whether different carriers had very different mixes of aircraft. If so, it would be more difficult to untangle the separate effects of each factor. Unfortunately, the answer was yes. Figure 6 shows the mix of aircraft types by carrier. Consider the Boeing 737: this type made up the majority of the fleets of BBB, EEE and FFF, whereas there were few or none in the fleets of AAA, CCC and DDD. In general, the carriers varied widely in the variety present in their fleets. Variety in nominal data, like fleet mix data, is measured by entropy, more entropy indicating greater variety. CCC had the highest equipment entropy and FFF the lowest. Flying a randomly chosen flight on FFF, one was very likely to get a 737; flying CCC, one was quite uncertain what type of aircraft would be used.

Finally, Figure 7 shows standardized ETE by OD pair. By definition, the mean value of this variable was 1.0, but the other statistics were revealing. Even after standardizing, there were substantial variations across OD pairs. This conclusion is the same as that reached in the discussion of Table 5. The transcontinental SFO:PHL routes showed the least relative variation, while the relatively short SFO:LAX routes had the most extreme outliers in both directions. The two largest IQRs were for EWR:ORD and ORD:PHL. We note, however, that some of the variation might be traced to asymmetry in the times required to fly the two routes between any given pair of airports, since this analysis combines information for flights in both directions.

3.3 Modeling ETE distributions

The preceding analyses have studied the issue of variations in ETEs from a macroscopic perspective, examining trends and summary statistics such as the coefficient of variation. In this section, we look in detail at selected routes, develop probability models for ETE distributions, and use the model parameters to characterize the flight planning of carriers competing on those routes.

For any given route and carrier, the distribution of ETE values can have a very complex shape. In many cases, this is because there is a large number of possible flight paths between the origin and destination, making for a multimodal distribution of estimated times en route. Even when there is only one flight path filed, there can be many different planned ETEs along the same path, corresponding to differences in planned airspeed, which can in turn depend on differences in planned altitude and other factors.

Examining many ETE distributions, we noticed that some were well described as a combination of a main, unimodal distribution of typical ETE values with an attached tail of unusually high values. Accordingly, our approach to modeling the distribution of ETEs was to use mixture models. Now, when a distribution has many modes, mixture models become very difficult to estimate numerically. However, for some routes whose

ETE distributions have only a few modes, it is both feasible and instructive to use mixture models.

In certain cases, we can think of the ETE distribution as a combination of two components. One component, accounting for most of the data, represents regular operations. The second component, accounting for the tail of high values, represents irregular operations. We found that both components could be represented with a discretized version of the lognormal distribution. The discretization is necessary because flight plans are filed in units of one minute. The use of a lognormal rather than normal distribution not only fits the data better but guarantees that ETE will be nonnegative. This decomposition of the distribution allows us to characterize each airline's flight plans using five parameters: the mean and standard deviation of ETE for regular operations, the mean and standard deviation of ETE for irregular operations, and the proportion of operations that are regular.

Mathematically, the mixture model can be described as follows. Let

X = filed ETE, regarded as a discrete, positive-valued random variable

$g(X \mid \mu_r, \sigma_r)$ = conditional probability mass function of X for regular operations

$h(X \mid \mu_i, \sigma_i)$ = conditional probability mass function of X for irregular operations

$f(X) = \pi g(X) + (1-\pi)h(X)$ = probability mass function of X

where

π = proportion of filed flight plans representing regular operations

μ_r = location parameter of g()

σ_r = scale parameter of g()

μ_i = location parameter of h()

σ_i = scale parameter of h().

For a lognormal distribution, the location parameter μ and scale parameter σ combine to form the mean and standard deviation as follows:

$E[X] = \exp\{\mu + \sigma^2/2\}$

$S[X] = E[X]\sqrt{(\exp\{\sigma^2\} - 1)}$.

We estimated the five parameters by the method of maximum likelihood, programmed as a constrained optimization. The constraints were that π, σ_r, and σ_i had to be positive and μ_i had to exceed μ_r.

We fitted lognormal mixtures to ETEs for three origin-destination pairs: ATL:CLT, SFO:LAX, and EWR:LAX. The first two had the largest range in coefficient of variation (see Table 5) and appeared to fit the paradigm of a combination of regular and irregular operations. The EWR:LAX route represented the more complex, multimoCCC situation one would expect for transcontinental flights.

3.3.1 Flights from ATL to CLT

Figure 8 shows the distributions of ETE for flights by FFF and CCC from ATL to CLT. Both distributions had a main body and a long tail to the right.

We applied the mixture model to these ETE distributions. Figures 9 and 10 show the observed and fitted distributions, the estimated parameter values, and the ETE means and standard deviations corresponding to the parameter values. The mixture models fitted the data for both carriers well. Comparing the two carriers' results, we see an interesting difference. Both carriers filed normal plans 98% of the time, and both carriers' normal plans called for an ETE of 35 minutes with a standard deviation of 1 or 2 minutes. However, irregular operations at FFF had an average ETE of 49 minutes, compared to 39 minutes for CCC, and a standard deviation of 5 minutes, compared to 2 minutes for CCC. Thus, irregular operations at CCC were much "tighter" than those at FFF for flights from ATL to CLT.

3.3.2 Flights from CLT to ATL

The distributions of ETEs for flights from CLT to ATL are shown in Figure 11. These westbound ETEs were longer and more clearly skewed to the right than the corresponding eastbound ETEs in Figure 8.

Figures 12 and 13 show the fits from the two-component mixture model. There was a clear difference between the two airlines. CCC had regular operations slightly more often than FFF (88% versus 85%), and both its regular and irregular operations had more desirable, i.e., smaller, values for the mean and standard deviation.

3.3.3 Flights from SFO to LAX

Figure 14 shows the ETEs for flights from SFO to LAX by AAA, CCC and EEE. The distributions of the three carriers were clearly different.

We also fitted mixture models to these flights. Every flight plan filed by these three carriers during the time period studied called for exactly the same route, so any differences in ETE must be attributed to expected differences in airspeeds. Figures 15, 16, and 17 show the observed and fitted distributions, the estimated parameter values, and the ETE means and standard deviations corresponding to the parameter values. Again the mixture models fitted the data for the carriers well. The EEE flights from SFO to LAX followed the same pattern as the CCC and FFF flights from ATL to CLT. That is, there was a main body of data that was lognormal and accounted for 98% of the flights, and there was a high tail also described by a lognormal distribution. Regular operations had a mean ETE of 52 minutes and a standard deviation of 2 minutes. Irregular operations had a mean of 65 minutes and a standard deviation of 3 minutes. There were a relatively small number of CCC flights, so it is not surprising that there were no irregular operations observed for CCC. Regular operations for CCC had a mean ETE of 53 minutes and a standard deviation of 1 minute, similar to EEE. Finally, flights by AAA defined a new pattern. As shown in Figure 17, the ETE distribution for AAA flights was left-skewed, not right-skewed like the others. It appears that there was a minor mode at lower rather than higher ETE levels, and it also appears that irregular operations were nearly as common as regular operations. Regular operations occurred in only 52% of the flights, averaging 59 minutes with a standard deviation of 2 minutes. Irregular operations averaged 58 minutes with a standard deviation of 4 minutes. In this case, the distinction between regular and irregular operations seems to break down, though the mixture model does a good job of fitting the data.

3.3.4 Flights from LAX to SFO

Figure 18 shows the ETE distributions for flights from LAX to SFO. These were shifted to the right but otherwise similar to the distributions for flights in the opposite direction (see Figure 14) in their reflection of differences among AAA, CCC and EEE.

Figures 19, 20, and 21 show the fits to the ETE distributions. Only the CCC distribution fit the pattern of a dominant mode for regular operations at lower ETEs and a minor mode for irregular operations at higher ETEs. For both AAA and EEE, the lower mode had the smaller probability. The ETE distribution for CCC was the best overall, combining low average ETE with greater consistency.

3.3.5 Flights from EWR to LAX

We have demonstrated that one can model the ETE distribution for some routes as a mixture of two lognormal distributions, one corresponding to regular operations and the other to irregular operations. Doing so provides a new way to characterize the flight planning behavior of various carriers flying the same route. Unfortunately, many routes have a much more complex, multimodal distribution of ETEs.

Figure 22 shows the ETE distributions for westbound flights from EWR to LAX by AAA, BBB, and EEE. These distributions had complex shapes: they were multimodal and very widely dispersed.

Figures 23, 24, and 25 show the mixture models fit to these distributions. If one wanted to fit every bump in these distributions, the resulting model would probably be too complex to be useful. However, we found that we could do an excellent job of approximating the ETE distribution using a mixture of either two (AAA) or four (EEE and BBB) lognormals. These mixture models provide excellent approximations to the cumulative distribution function (CDF), which shows the probability that the ETE will be less than or equal to any given number of minutes.

3.3.6 Flights from LAX to EWR

Figure 26 shows the ETE distributions for eastbound flights from EWR to LAX by AAA, BBB, and EEE. In contrast to the distributions for westbound flights shown in Figure 22, the modes in these distributions were much less dominant and the averages were, thanks to the jet stream, shifted lower.

Figure 27, 28, and 29 show the mixture model fits to these ETEs using either three or four components. All three carrier’s distributions had one dominant mode, with a mean of 266 minutes for BBB and EEE and 272 minutes for AAA.

3.4 Variations in filed flight paths

As noted above, some of the variation in ETEs can be traced to differences in filed flight paths. To study this phenomenon, we used the POET data mining software (Metron Aviation 2002) to identify and plot filed flight paths for flights flown during early Fall 2002 .We found some large differences among carriers in their choice of filed routes. For instance, Figure 30 contrasts the flight paths filed by BBB and FFF for the CLT to IAH route. Whereas BBB filed only one path for all its flights during the period studied, FFF filed a multitude of alternatives. The paths actually flown always showed more variety than those filed, but large differences in planned routes translated into correspondingly large differences in flown routes. Figure 31 illustrates this for the routes actually flown from CLT to IAH.

POET provides information on the distances of the filed flight paths. We used this information to compare the distances in carriers' flight plans using ANOVA. The data for this analysis were from the period 10 September to 7 November 2002. On average, each combination of route and carrier involved about 400 flights.

Table 6 shows the mean flight plan distance by route and carrier. Although the differences across carriers were statistically significantly different on many routes, the sizes of the differences were negligible.

Table 7 shows the standard deviation of flight plan distance by route and carrier. There are a variety of interesting comparisons in Table 7:

- For most of the routes, all carriers had the same or nearly the same standard deviation (see, e.g., PHL to ORD and LAX to EWR).

- For some routes, there were substantial differences across carriers, e.g., ATL to IAH, DTW to CLT, DTW to DEN, and ORD to EWR.
- For other routes, there was a large difference between the consistency of flight plan distances in the two directions of flight. For example, flight plan distances from LAX to SFO had a standard deviation of about 17 miles, while plans in the opposite direction had a standard deviation of only 0.1 mile.
- As one might expect, the standard deviation of flight plan distance increased with the mean (correlation = +0.85, $p < 0.001$).

To standardize for the effect of mean distance on the standard deviation, one can shift focus to the coefficient of variation (CV), which is the standard deviation divided by the mean. Table 8 shows the CV of flight plan distance by route and carrier. The typical CV was about 2.7%, but some routes were noticeably higher: ATL to IAH at 4.8%, DTW to ATL at 4.3%, EWR to DTW at 4.6%, and LAX to SFO at 5.6%. The most important result in Table 8 comes from comparison with Table 5, which showed the CVs for estimated times rather than distances. The CVs were much greater for times, which establishes that the variation in flight plan distances does not by itself explain the variation in ETEs.

4. Multivariate Analysis of Standardized ETEs

Bivariate analyses of the type reported in the previous section are usually helpful but can be misleading. This is because our data arose from an observational study, which is a relatively weak way to establish links between input factors (e.g., carrier) and a response (e.g., ETE). In contrast, a designed experiment creates a complete and balanced dataset, so one can aggregate over all other variables to get a meaningful view of how changes in any one factor affect changes in a response variable. Still, even with designed experiments, this simplicity requires that there be no significant interactions among variables.

In our observational study, the factors were by no means balanced. Thus, if two carriers show substantial differences in their ETEs for a given route, we cannot safely conclude that the difference is due to different flight planning philosophies or practices. Such a difference might instead be caused by differences in when the planes fly during the day or in the types of aircraft used. Ultimately, lacking experimental data, we must resort to some form of multivariate statistical model to try to identify, quantify and control for the separate influences of all the factors.

Another basic issue in data analysis is choice of level of aggregation. It is attractive to pool all the data from all the routes. However, this form of data combination creates a danger of confounding from multiple factors. For instance, more southern routes (e.g., IAH:ATL) might not be as strongly influenced by a change in weather from January to May as would more northern routes (e.g., EWR:ORD). While we pooled both directions on each routes in the analyses in Section 3, here we consider flights between the same cities but in different directions to be separate routes.

As a first attempt at a multivariate analysis, we conducted an analysis of variance (ANOVA) on standardized ETE as a response to several factors: equipment, airline, month, day of week, hour of day, city pair, and eastward direction of flight. We included two-way interactions in the analysis but excluded higher-way interactions because of the difficulty of interpreting what they might mean.

Formal statistical inference of the ANOVA was not possible because the residuals did not satisfy the assumptions for inference (they were neither normal nor homoscedastic). Nevertheless, we include them because the mean square values are useful descriptive statistics to show the relative importance of the factors. Table 9 shows the ANOVA results, sorted by size of mean square (equivalently, sorted by F ratio). The main conclusions from Table 9 were:

- All seven factors combined explained only about one quarter of the variation in standardized ETE. We conjecture that weather and anticipated congestion accounted for more of the variation.
- The factor with the largest main effect mean square was equipment (type of aircraft).
- Airline had the next greatest main effect. This confirms our previous findings that there are significant differences in ETE across carriers.
- All other main effects and interactions were much smaller (by about a factor of 10 or more).

5. Summary and Conclusions

We investigated the influence of several factors that might be expected to influence a flight's estimated time en route (ETE): origin airport, destination airport, month of year, day of week, hour of day, aircraft type, and carrier. Our main interest was to see whether the ETEs in filed flight plans differed within and among carriers. We found much variation in ETEs.

- Sustained and significant trends in ETE have occurred for certain origin-destination pairs.

- Route, month, hour of day and carrier are all statistically significant influences on ETE.
- Some routes have ETE distributions that are well modeled by a mixture of two or more lognormal distributions. In simple cases, these mixture models can be regarded as characterizing regular and irregular operations.
- Some routes show large differences in the number of different flight paths filed. The average distances of the filed plans did not vary much across airlines, though the standard deviations of the filed distances varied more. The standard deviations increased with the mean distances. The coefficients of variation of filed distances were smaller than the coefficients of variation for ETEs, so the variations in ETEs cannot be explained simply by differences in the distances of filed flight plans.

Overall, the simple question of how long it should take to fly from point A to point B turns out to have an intriguing number of revealing answers.

The next phase of this research should investigate two questions. First, is there any relationship between ETE and deviation from ETE? If there is a link, then it may not be appropriate to study deviations from planned flight times to diagnose problems in the NAS, because such a relationship would suggest that carriers are gaming their flight plans. Second, is it possible to isolate the effect of winds on ETEs. The multivariate analysis in section 4 explained only about one quarter of the variation in ETEs. One assumes that the bulk of the variability can be attributed to daily changes in winds aloft, but it would be good to confirm this assumption.

Table 1: Origin-destination pairs with notable trends in average estimated times en route (ETEs) during winter seasons

	City Pair		Average Estimated Air Times				Annual % Change			
OD Pair #	Origin	Destination	'98-'99	'99-'00	'00-'01	'01-02	1st	2nd	3rd	Average
1	MEM	CVG	76	73	66	58	-4.1%	-9.5%	-11.4%	-8.3%
2	MEM	IAH	98	85	78	77	-13.5%	-7.8%	-1.1%	-7.5%
3	IAD	CLE	72	68	60	57	-5.8%	-12.6%	-3.8%	-7.4%
4	CVG	CLT	73	65	63	58	-10.4%	-3.6%	-7.7%	-7.3%
5	CLE	IAD	62	58	51	50	-7.3%	-12.0%	-1.2%	-6.8%
6	CLT	CVG	80	76	73	66	-4.2%	-4.6%	-9.2%	-6.0%
7	CVG	MEM	80	79	76	68	-2.1%	-2.6%	-11.2%	-5.3%
8	JFK	BOS	43	41	39	37	-4.4%	-5.2%	-4.3%	-4.6%
9	MEM	STL	57	53	51	50	-6.4%	-4.0%	-2.7%	-4.4%
10	PHL	DCA	33	32	30	29	-2.4%	-7.5%	-2.9%	-4.3%
11	DCA	EWR	43	42	40	37	-1.4%	-6.1%	-5.3%	-4.3%
12	BWI	JFK	48	48	44	43	-1.1%	-7.4%	-3.9%	-4.1%
13	DCA	PHL	29	28	27	26	-4.5%	-3.2%	-3.4%	-3.7%
14	IAD	JFK	53	51	49	47	-3.3%	-4.3%	-3.2%	-3.6%
15	IAH	JFK	176	175	173	158	-0.1%	-1.5%	-8.3%	-3.3%
16	BOS	PHL	70	65	64	63	-7.3%	-1.1%	-1.2%	-3.2%
17	CVG	DTW	43	43	41	40	-1.2%	-2.9%	-4.3%	-2.8%
18	DFW	MEM	65	64	60	60	-1.5%	-6.3%	-0.2%	-2.7%
19	EWR	SLC	297	280	278	274	-5.7%	-0.8%	-1.5%	-2.7%
20	DTW	CLE	26	25	25	24	-3.9%	-0.4%	-3.5%	-2.6%
21	ATL	CLT	38	38	37	35	-0.9%	-2.8%	-4.1%	-2.6%
22	EWR	DCA	44	43	41	41	-4.2%	-3.1%	-0.4%	-2.6%
23	MCO	BWI	107	107	104	99	-0.4%	-2.3%	-4.8%	-2.5%
24	PHL	BOS	52	49	49	48	-5.9%	-1.2%	-0.1%	-2.4%
25	TPA	BWI	112	112	112	104	-0.1%	-0.2%	-6.9%	-2.4%
26	PHX	LAX	65	65	64	61	-0.4%	-0.8%	-5.7%	-2.3%
27	CLE	LAX	297	281	279	277	-5.3%	-0.9%	-0.7%	-2.3%
28	MEM	DFW	78	76	73	73	-2.8%	-3.0%	-1.0%	-2.3%
29	PHX	PIT	208	202	198	195	-2.8%	-1.8%	-1.8%	-2.1%
30	PHX	CLT	201	194	191	189	-3.6%	-1.5%	-1.2%	-2.1%
31	DTW	CVG	46	45	44	43	-2.2%	-1.9%	-2.0%	-2.0%
32	PHL	MSP	157	154	151	147	-1.9%	-1.4%	-2.8%	-2.0%
33	DEN	BWI	174	172	166	163	-1.1%	-3.1%	-1.9%	-2.0%
34	PHL	IAD	41	43	43	44	5.6%	0.1%	3.1%	2.9%
35	PHL	EWR	23	24	25	26	4.5%	2.1%	4.0%	3.5%
36	DFW	IAH	41	42	42	46	3.1%	0.4%	7.8%	3.7%
37	CLE	PIT	31	31	34	39	2.6%	9.0%	15.4%	9.0%
38	PIT	CLE	34	34	40	45	1.9%	16.6%	13.4%	10.6%
39	BWI	LGA	36	42	51	52	14.8%	22.4%	2.3%	13.2%

Table 2: Counts of flights by city pair and carrier

		Carrier						
OD pair		AAA	BBB	CCC	DDD	EEE	FFF	Range/Average
ATL	ORD	1666		3129		1995		65%
EWR	LAX	897	1566			896		60%
EWR	ORD	2578	1876			3240		53%
SFO	LAX	2253		242		6268		206%
ORD	PHL	2520				3096	1638	60%
DTW	CLT				1121		1191	6%
MIA	ATL	1398		2557				59%
IAH	ATL		1964	1767				11%
DTW	EWR		1030		2122			69%
DTW	ATL			2060	2241			8%
ATL	CLT			2030			1733	16%
DTW	DEN				936	606		43%
SFO	PHL					599	949	45%
IAH	CLT		257				1032	120%

Note: Cells with small counts were excluded from the analysis.

Table 3: Average ETE (minutes) by OD pair and carrier

		Carrier						
OD pair		AAA	BBB	CCC	DDD	EEE	FFF	Range/Average
ATL	ORD	93		87		88		7%
EWR	LAX	302	300			297		2%
EWR	ORD	106	104			102		4%
SFO	LAX	58		53		55		10%
ORD	PHL	102				98	99	4%
DTW	CLT				77		75	3%
MIA	ATL	87		83				4%
IAH	ATL		96	94				2%
DTW	EWR		75		75			1%
DTW	ATL			85	89			4%
ATL	CLT			37			39	5%
DTW	DEN				146	148		1%
SFO	PHL					310	306	1%
IAH	CLT		127				123	3%

Table 4: Standard deviation of ETE (minutes) by OD pair and carrier

OD pair		Carrier						
		AAA	BBB	CCC	DDD	EEE	FFF	Range/Average
ATL	ORD	8		7		9		27%
EWR	LAX	33	37			33		13%
EWR	ORD	16	18			16		9%
SFO	LAX	4		2		5		70%
ORD	PHL	14				14	15	10%
DTW	CLT				4		5	17%
MIA	ATL	6		4				39%
IAH	ATL		10	9				9%
DTW	EWR		12		12			1%
DTW	ATL			5	7			22%
ATL	CLT			4			6	48%
DTW	DEN				18	15		20%
SFO	PHL					36	35	2%
IAH	CLT		16				16	4%

Table 5: Coefficient of variation of ETE by OD pair and carrier

OD pair		Carrier						
		AAA	BBB	CCC	DDD	EEE	FFF	Range
ATL	ORD	9%		8%		10%		2%
EWR	LAX	11%	12%			11%		2%
EWR	ORD	15%	17%			16%		2%
SFO	LAX	7%		4%		8%		4%
ORD	PHL	14%				14%	15%	2%
DTW	CLT				6%		7%	1%
MIA	ATL	7%		5%				2%
IAH	ATL		10%	9%				1%
DTW	EWR		16%		16%			0%
DTW	ATL			6%	8%			1%
ATL	CLT			10%			15%	5%
DTW	DEN				12%	10%		2%
SFO	PHL					12%	11%	0%
IAH	CLT		12%				13%	1%

Note: Results rounded to nearest whole percentage

Table 6: Average planned distance by route and carrier

OD pair		Carrier AAA	BBB	CCC	DDD	EEE	FFF	All
ATL	CLT			205.0			205.3	205.2
ATL	DTW			542.7	542.5			542.6
ATL	IAH		610.2	604.6				607.4
ATL	MIA	548.8		543.8				546.3
ATL	ORD	563.7		562.0		563.6		563.1
CLT	ATL			212.3			205.0	208.7
CLT	DTW				452.3		452.4	452.4
CLT	IAH						823.3	823.3
DEN	DTW				1018.0	1029.8		1023.9
DTW	ATL			548.0	539.8			543.9
DTW	CLT				467.7		464.0	465.8
DTW	DEN				1002.8	1004.2		1003.5
DTW	EWR		427.0		427.0			427.0
EWR	DTW		465.9		466.0			466.0
EWR	LAX	2167.1	2169.4			2178.8		2171.8
EWR	ORD	653.2	653.0			652.9		653.0
IAH	ATL		627.7	623.2				625.4
IAH	CLT						838.2	838.2
LAX	EWR	2170.0	2167.5			2168.7		2168.7
LAX	SFO	314.1				314.0		314.1
MIA	ATL	535.0		532.4				533.7
ORD	ATL	551.7		550.8		546.2		549.6
ORD	EWR	625.6	626.8			625.4		625.9
ORD	PHL	605.1				604.2	601.0	603.4
PHL	ORD	613.4				613.5	613.8	613.6
PHL	SFO					2232.7	2231.1	2231.9
SFO	LAX	311.0				311.0		311.0
SFO	PHL					2208.1	2211.5	2209.8
	All	804.9	968.4	492.5	657.7	1070.5	864.6	827.4

Table 7: Standard deviation of planned distance by route and carrier

OD pair		Carrier						
		AAA	BBB	CCC	DDD	EEE	FFF	All
ATL	CLT			0.0			5.1	2.6
ATL	DTW			6.6	5.8			6.2
ATL	IAH		37.4	20.9				29.2
ATL	MIA	9.3		5.9				7.6
ATL	ORD	14.8		0.0		14.3		9.7
CLT	ATL			16.7			0.0	8.4
CLT	DTW				4.5		5.0	4.8
CLT	IAH						41.2	41.2
DEN	DTW				31.9	32.1		32.0
DTW	ATL			23.4	23.2			23.3
DTW	CLT				7.7		0.0	3.9
DTW	DEN				15.8	22.7		19.3
DTW	EWR		0.0		0.0			0.0
EWR	DTW		21.6		21.6			21.6
EWR	LAX	46.6	46.6			46.7		46.6
EWR	ORD	25.6	25.6			25.6		25.6
IAH	ATL		25.1	25.0				25.1
IAH	CLT						24.0	24.0
LAX	EWR	46.6	46.6			46.6		46.6
LAX	SFO	17.7				17.7		17.7
MIA	ATL	23.1		23.1				23.1
ORD	ATL	15.4		12.3		15.9		14.5
ORD	EWR	10.9	19.0			9.6		13.2
ORD	PHL	24.6				24.6	24.5	24.6
PHL	ORD	24.8				24.8	24.8	24.8
PHL	SFO					47.3	47.2	47.3
SFO	LAX	0.2				0.0		0.1
SFO	PHL					47.0	47.0	47.0
	All	21.6	27.7	13.4	14.0	26.5	21.9	21.2

Table 8: Coefficient of variation of planned distance by route and carrier

OD pair		Carrier AAA	BBB	CCC	DDD	EEE	FFF	All
ATL	CLT			0.0%			2.5%	1.2%
ATL	DTW			1.2%	1.1%			1.1%
ATL	IAH		6.1%	3.5%				4.8%
ATL	MIA	1.7%		1.1%				1.4%
ATL	ORD	2.6%		0.0%		2.5%		1.7%
CLT	ATL			7.9%			0.0%	3.9%
CLT	DTW				1.0%		1.1%	1.1%
CLT	IAH						5.0%	5.0%
DEN	DTW				3.1%	3.1%		3.1%
DTW	ATL			4.3%	4.3%			4.3%
DTW	CLT				1.6%		0.0%	0.8%
DTW	DEN				1.6%	2.3%		1.9%
DTW	EWR		0.0%		0.0%			0.0%
EWR	DTW		4.6%		4.6%			4.6%
EWR	LAX	2.2%	2.1%			2.1%		2.1%
EWR	ORD	3.9%	3.9%			3.9%		3.9%
IAH	ATL		4.0%	4.0%				4.0%
IAH	CLT						2.9%	2.9%
LAX	EWR	2.1%	2.1%			2.1%		2.1%
LAX	SFO	5.6%				5.6%		5.6%
MIA	ATL	4.3%		4.3%				4.3%
ORD	ATL	2.8%		2.2%		2.9%		2.6%
ORD	EWR	1.7%	3.0%			1.5%		2.1%
ORD	PHL	4.1%				4.1%	4.1%	4.1%
PHL	ORD	4.0%				4.0%	4.0%	4.0%
PHL	SFO					2.1%	2.1%	2.1%
SFO	LAX	0.1%				0.0%		0.0%
SFO	PHL					2.1%	2.1%	2.1%
	All	2.9%	3.3%	2.8%	2.1%	2.7%	2.4%	2.7%

Table 9: Analysis of variance of standardized ETE

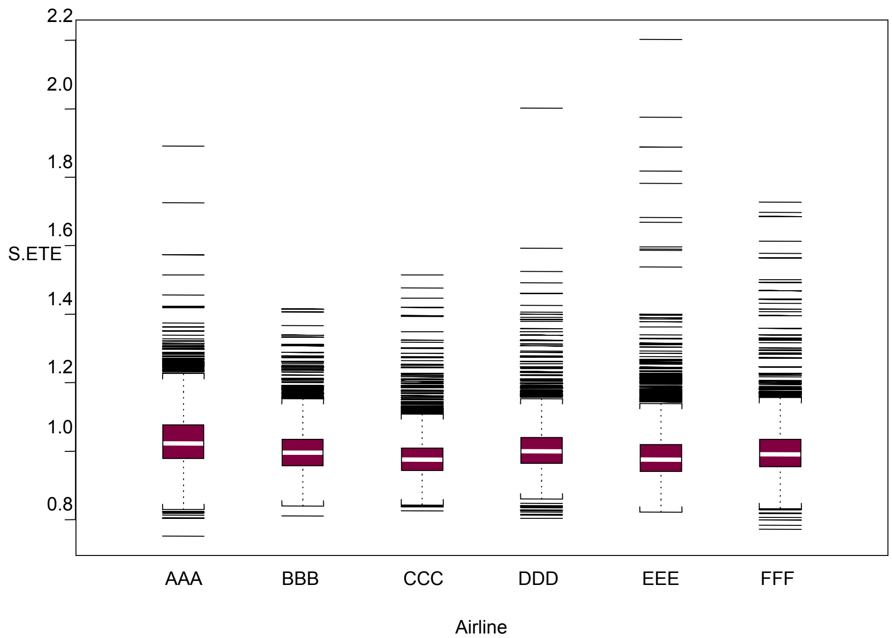
R-Square	Coeff of Var	Root MSE	S_ETE Mean
0.25	6.68	0.067	1.00

Source	DF	Type I SS	Mean Square	F Value	Pr > F[a]
Eqpt	9	35.11	3.90	873.40	<.0001
Airline	5	10.32	2.06	462.11	<.0001
Citypair	13	3.81	0.29	65.57	<.0001
Hour	16	3.39	0.21	47.50	<.0001
Eqpt*Airline	21	3.71	0.18	39.54	<.0001
Month	4	0.64	0.16	35.79	<.0001
Eqpt*Citypair	58	5.22	0.09	20.17	<.0001
Day	6	0.43	0.07	15.97	<.0001
Eqpt*Hour	129	6.02	0.05	10.45	<.0001
Eastward(Citypair)	14	0.59	0.04	9.47	<.0001
Airline*Hour	78	3.27	0.04	9.39	<.0001
Airline*Month	20	0.79	0.04	8.86	<.0001
Month*Day	24	0.85	0.04	7.94	<.0001
Hour*Citypair	173	5.82	0.03	7.53	<.0001
Eqpt*Month	35	1.15	0.03	7.34	<.0001
Airline*Citypair	16	0.40	0.02	5.57	<.0001
Month*Hour	64	1.47	0.02	5.13	<.0001
Month*Citypair	52	0.79	0.02	3.40	<.0001
Eqpt*Day	53	0.70	0.01	2.96	<.0001
Day*Citypair	78	0.85	0.01	2.45	<.0001
Airline*Day	30	0.24	0.01	1.78	0.0053
Day*Hour	96	0.66	0.01	1.53	0.0006

[a] p-values not trustworthy because residuals do not satisfy assumptions for inference

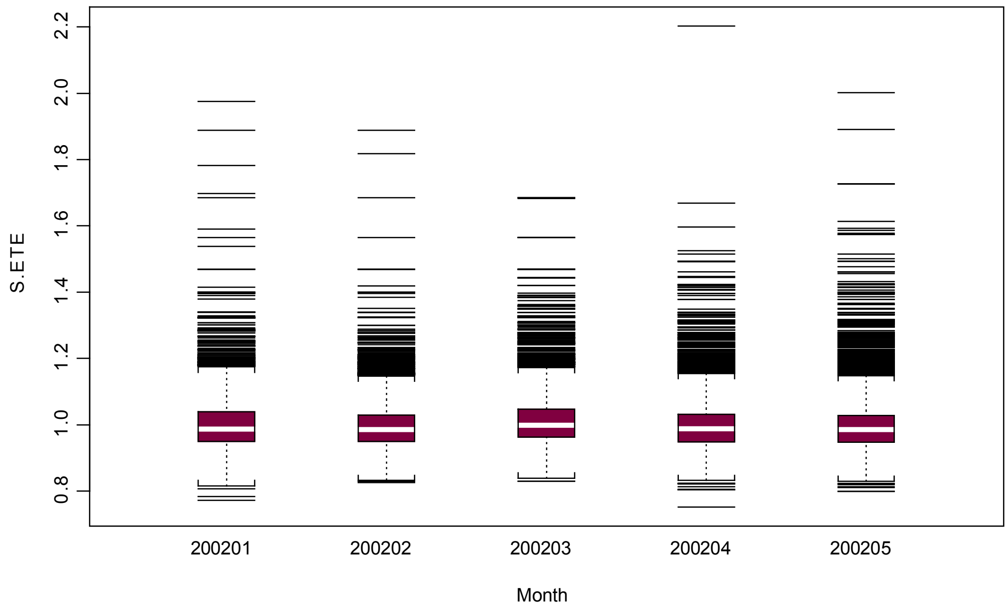
Figure 1: Routes used in this study

Figure 2: Standardized ETE by carrier



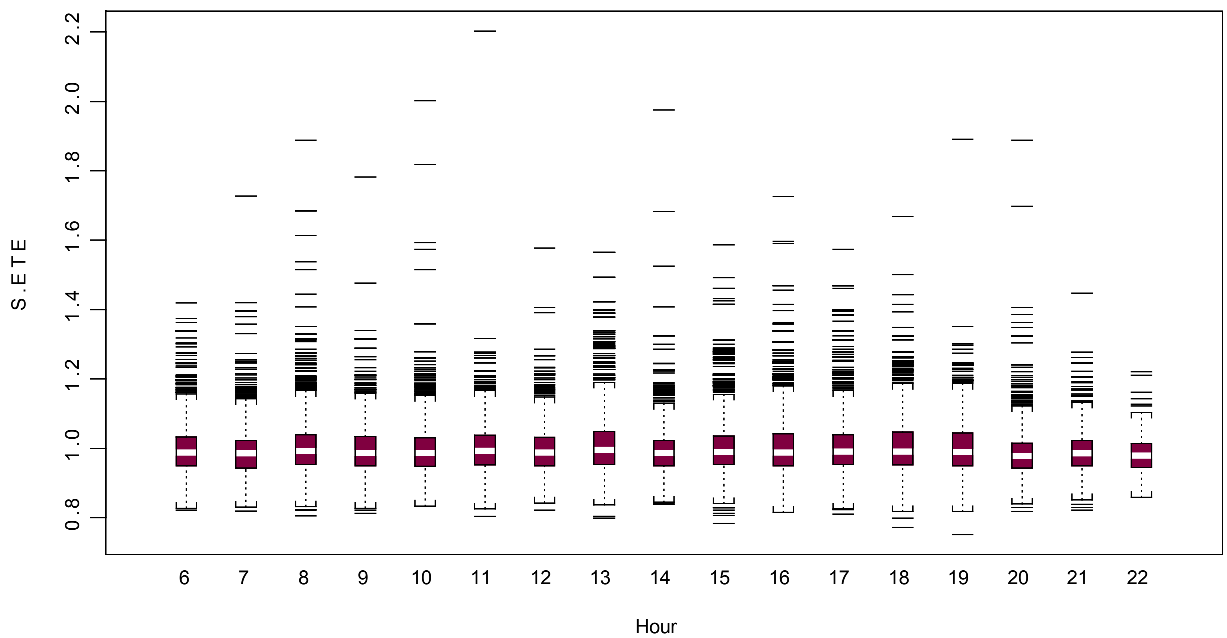
Airline	AAA	BBB	CCC	DDD	EEE	FFF
Mean	1.0302	1.0027	0.9804	1.0098	0.9869	1.0042
SE of Mean	0.0008	0.0009	0.0005	0.0010	0.0006	0.0011
75%Quantile	1.0775	1.0349	1.0094	1.0405	1.0202	1.0352
50%Quantile	1.0227	0.9958	0.9754	0.9996	0.9752	0.9906
25%Quantile	0.9777	0.9555	0.9417	0.9639	0.9394	0.9537
IQR	0.0998	0.0795	0.0677	0.0765	0.0808	0.0814
n	11313	6693	11785	6432	16700	6543

Figure 3: Standardized ETE by month



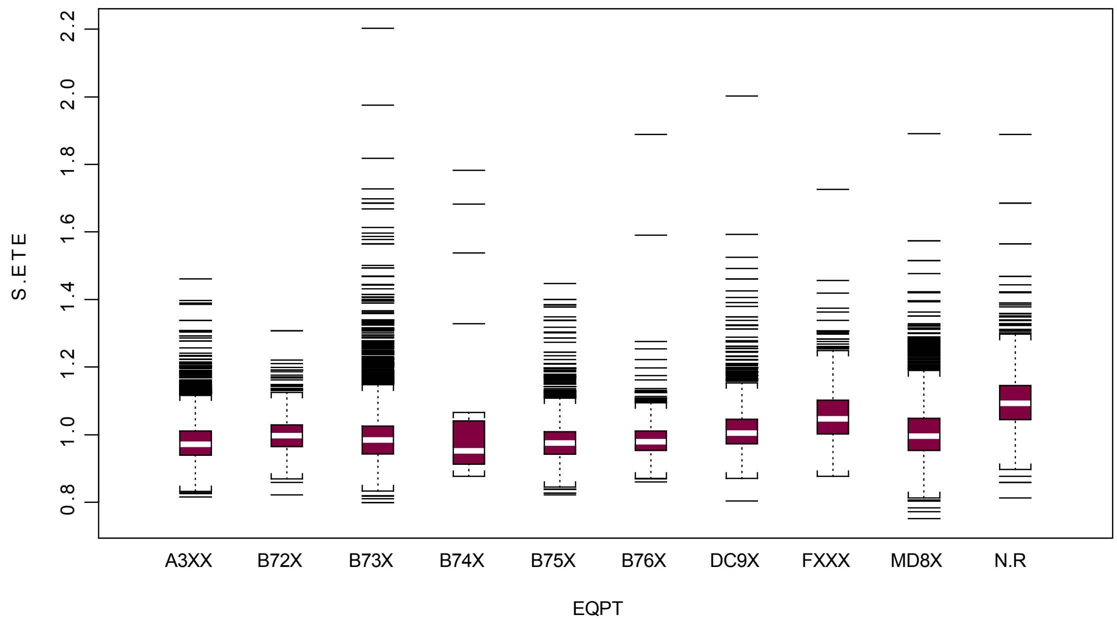
Month	**200201**	**200202**	**200203**	**200204**	**200205**
Mean	0.9981	0.9950	1.0126	0.9974	0.9966
SE of Mean	0.0007	0.0007	0.0007	0.0007	0.0007
75%Quantile	1.0395	1.0288	1.0470	1.0311	1.0278
50%Quantile	0.9870	0.9862	0.9981	0.9878	0.9859
25%Quantile	0.9496	0.9498	0.9629	0.9489	0.9472
IQR	0.0899	0.0790	0.0841	0.0822	0.0807
n	11600	10759	12025	12328	12754

Figure 4: Standardized ETE by hour of the day



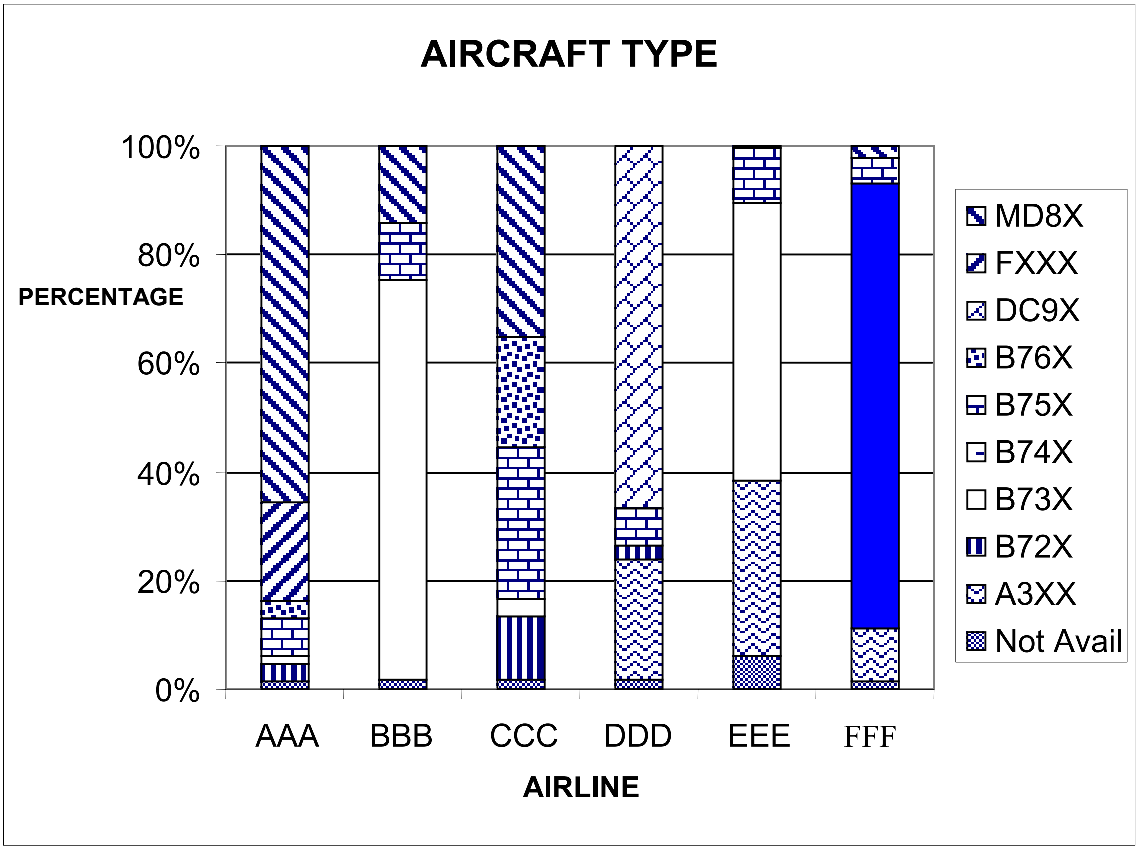
Hour	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
Mean	0.9976	0.9895	1.0030	0.9966	0.9938	0.9997	0.9942	1.0117	0.9932
SE of Mean	0.0013	0.0011	0.0010	0.0012	0.0012	0.0012	0.0009	0.0014	0.0013
75%Quantile	1.0334	1.0227	1.0384	1.0336	1.0304	1.0380	1.0311	1.0486	1.0218
50%Quantile	0.9875	0.9862	0.9919	0.9864	0.9868	0.9928	0.9875	0.9962	0.9864
25%Quantile	0.9498	0.9425	0.9537	0.9496	0.9489	0.9524	0.9498	0.9535	0.9501
IQR	0.0836	0.0802	0.0847	0.0840	0.0815	0.0856	0.0813	0.0951	0.0718
n	3342	3669	5291	3540	3351	3497	4441	4073	2834
Hour	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	
Mean	1.0028	1.0075	1.0060	1.0117	1.0052	0.9849	0.9913	0.9804	
SE of Mean	0.0011	0.0016	0.0011	0.0014	0.0014	0.0013	0.0020	0.0016	
75%Quantile	1.0348	1.0421	1.0395	1.0468	1.0446	1.0151	1.0229	1.0134	
50%Quantile	0.9898	0.9880	0.9913	0.9913	0.9888	0.9777	0.9859	0.9787	
25%Quantile	0.9532	0.9498	0.9536	0.9530	0.9496	0.9431	0.9496	0.9450	
IQR	0.0816	0.0923	0.0860	0.0938	0.0949	0.0720	0.0732	0.0684	
n	4738	3266	5191	3956	3392	2803	1146	936	

Figure 5: Standardized ETE by equipment



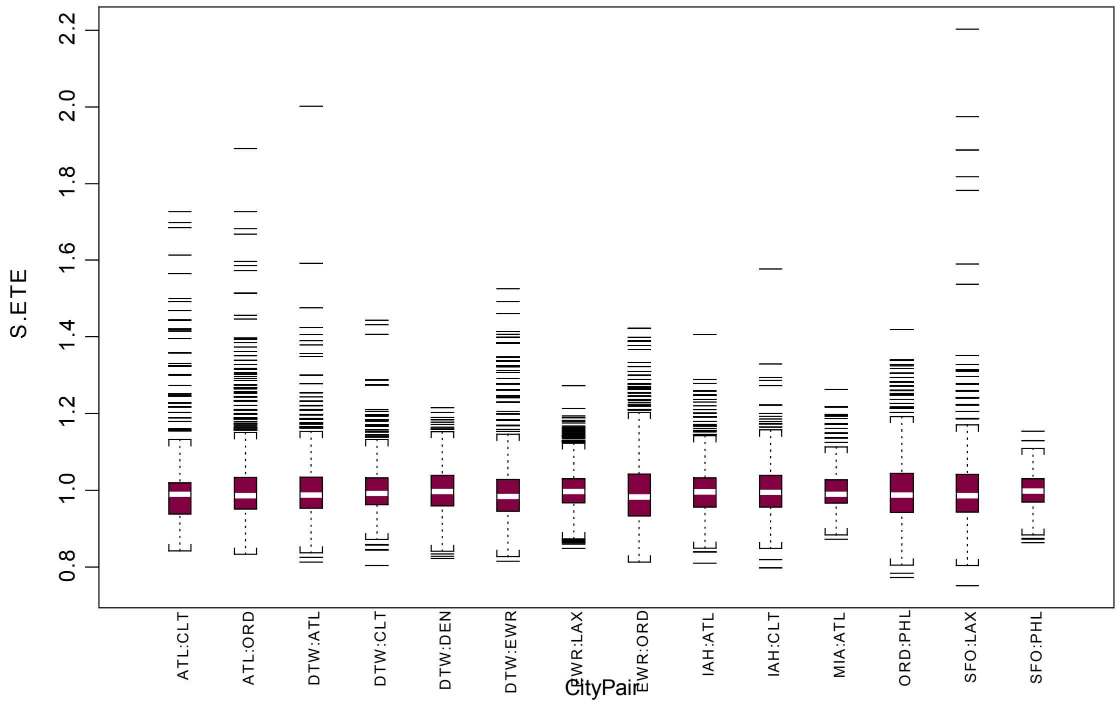
Aircraft Type	**A3XX**	**B72X**	**B73X**	**B74X**	**B75X**	**B76X**	**DC9X**	**FXXX**	**MD8X**	**Not Rec'd**
Mean	0.9791	0.9995	0.9943	1.0716	0.9795	0.9851	1.0161	1.0546	1.0069	1.1065
SE of Mean	0.0007	0.0012	0.0006	0.0620	0.0007	0.0009	0.0011	0.0016	0.0007	0.0021
75%Quantile	1.0110	1.0288	1.0251	1.0267	1.0082	1.0104	1.0458	1.1013	1.0486	1.1455
50%Quantile	0.9721	0.9966	0.9843	0.9525	0.9747	0.9784	1.0050	1.0470	0.9962	1.0927
25%Quantile	0.9394	0.9645	0.9437	0.9131	0.9415	0.9539	0.9725	1.0023	0.9537	1.0446
IQR	0.0716	0.0643	0.0814	0.1136	0.0667	0.0565	0.0733	0.0990	0.0949	0.1009
n	7417	1868	19449	20	7176	2794	4298	2056	12652	1736

Figure 6: Aircraft type by carrier



Equipment	AAA	BBB	CCC	DDD	EEE	FFF
A3XX				22%	32%	10%
B72X	3%		12%	2%		
B73X	2%	74%	3%		51%	82%
B74X				0%	0%	
B75X	7%	10%	28%	7%	10%	5%
B76X	3%	0%	20%		0%	
DC9X				67%		
FXXX	18%					0%
MD8X	65%	14%	35%			2%
Not Avail	1%	2%	2%	2%	6%	1%
n	11313	6693	11785	6432	16700	6543
ENTROPY	1.61	1.05	2.05	1.25	1.37	0.92

Figure 7: Standardized ETE by OD pair



Aircraft Type	ATL:CLT	ATL:ORD	DTW:ATL	DTW:CLT	DTW:DEN	DTW:EWR	EWR:LAX
Mean	1	1	1	1	1	1	1
SE of Mean	0.00158	0.00094	0.00110	0.00135	0.00154	0.00161	0.00091
75%Quantile	1.01889	1.03336	1.03448	1.03211	1.03926	1.02864	1.03055
50%Quantile	0.99059	0.98593	0.98799	0.99190	0.99702	0.98427	0.99707
25%Quantile	0.93887	0.95116	0.95312	0.96279	0.95963	0.94588	0.96731
IQR	0.08002	0.08220	0.08136	0.06932	0.07962	0.08276	0.06325
n	3765	6790	4301	2312	1542	3153	3360
	EWR:ORD	**IAH:ATL**	**IAH:CLT**	**MIA:ATL**	**ORD:PHL**	**SFO:LAX**	**SFO:PHL**
Mean	1	1	1	1	1	1	1
SE of Mean	0.00100	0.00101	0.00190	0.00082	0.00097	0.00085	0.00112
75%Quantile	1.04209	1.03213	1.03968	1.02731	1.04455	1.04095	1.03032
50%Quantile	0.98278	0.99585	0.99487	0.99062	0.98778	0.98617	0.99812
25%Quantile	0.93503	0.95680	0.95615	0.96616	0.94237	0.94373	0.96996
IQR	0.10706	0.07533	0.08353	0.06115	0.10218	0.09722	0.06036
n	7694	3731	1289	3955	7256	8770	1548

Figure 8: ETE for flights from ATL to CLT

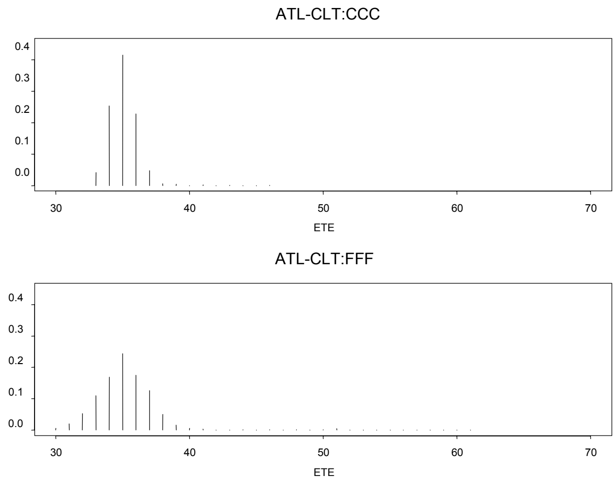
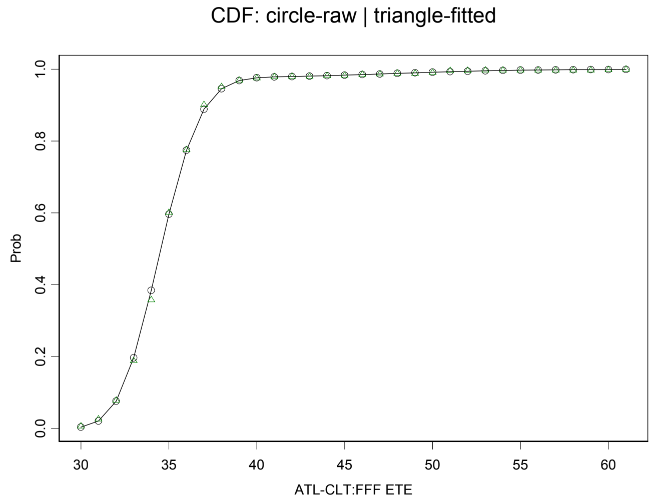
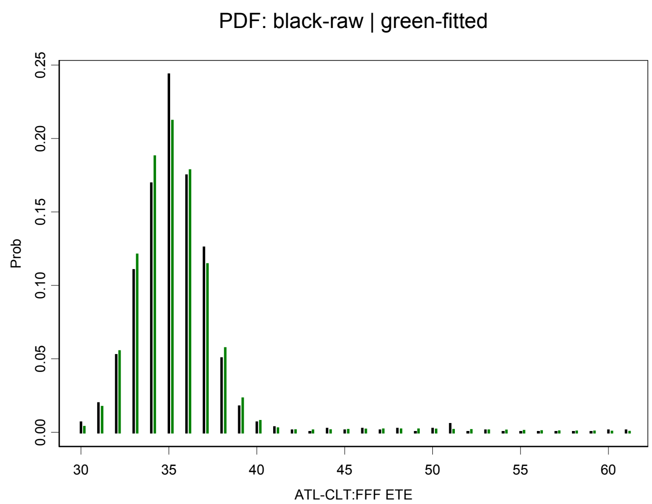


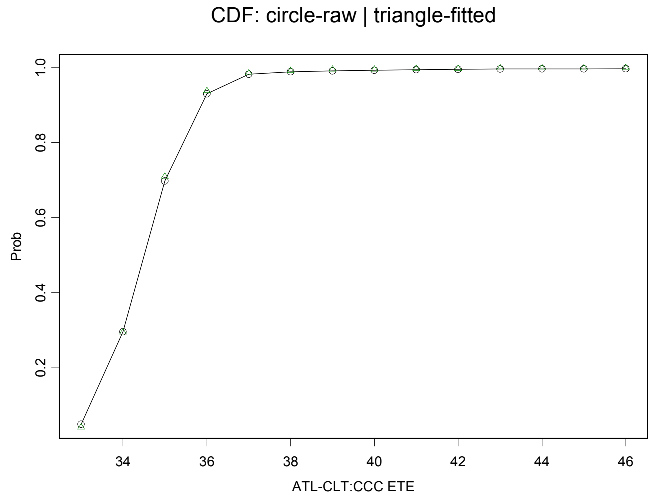
Figure 9: Mixture model fit to ETE for FFF flights from ATL to CLT

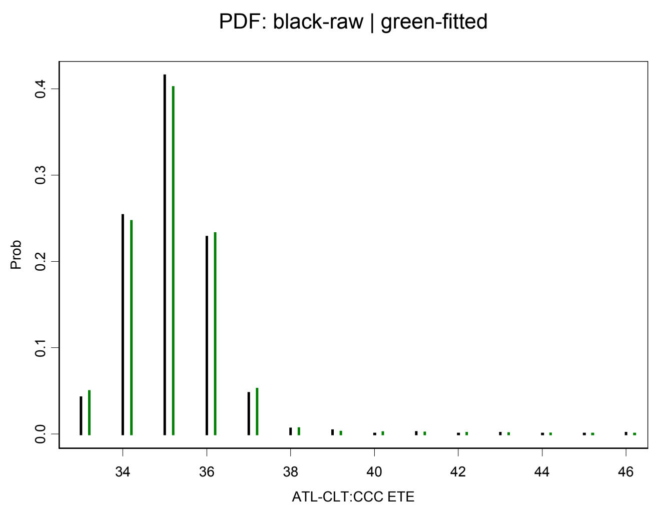




Estimated parameter values		
	p	0.977919
	sigma1	0.051967
	mu1	3.555017
	sd	1.82193
regular	avg	35.03569
	sigma2	0.106622
	mu2	3.881981
	sd	5.217626
irregular	avg	48.79682

Figure 10: Mixture model fit to ETE for CCC flights from ATL to CLT





Estimated parameter values		
	p	0.984316
	sigma1	0.026725
	mu1	3.554761
	sd	0.935329
regular	avg	34.99195
	sigma2	0.075
	mu2	3.65318
	sd	2.907031
irregular	avg	38.70592

Figure 11: ETE for flights from CLT to ATL

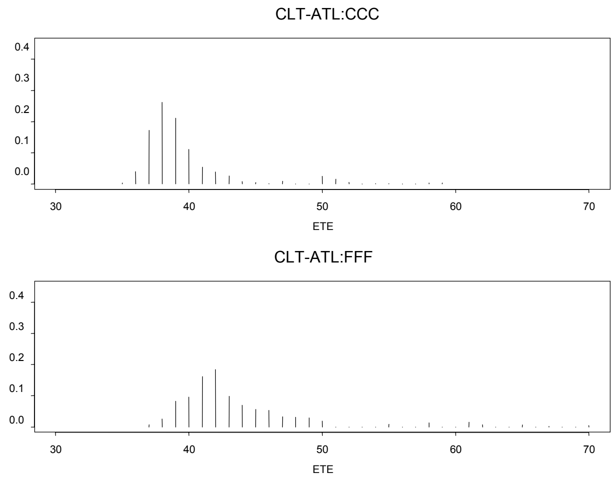
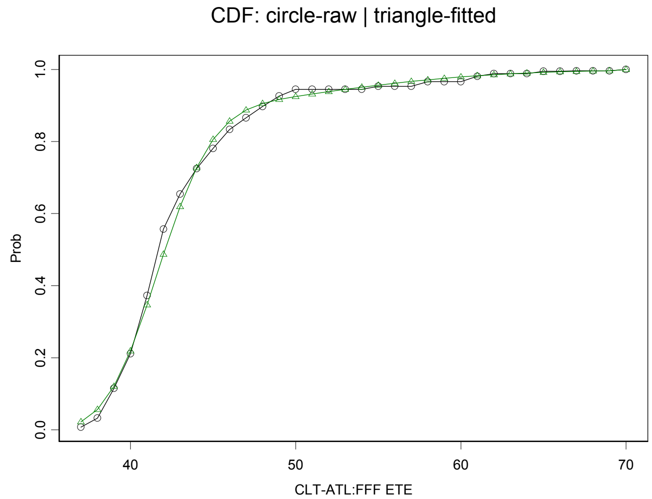
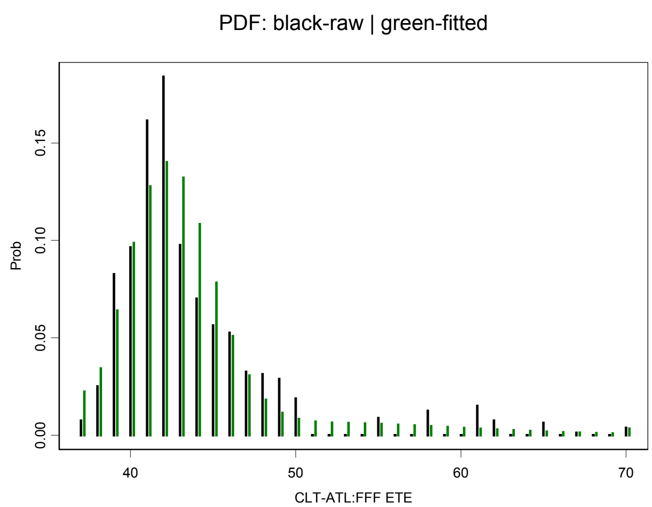


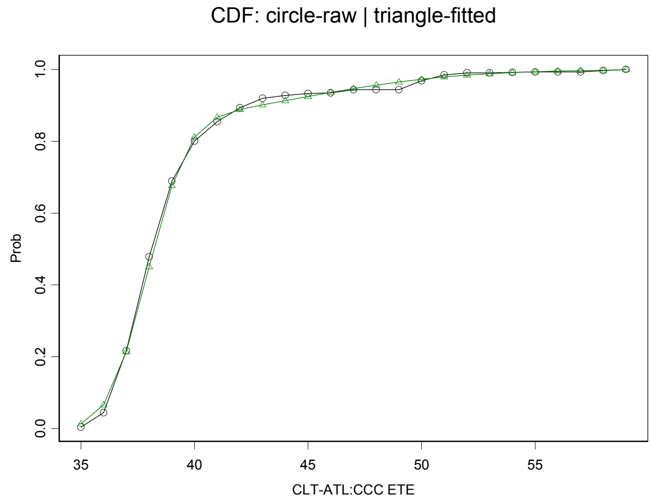
Figure 12: Mixture model fit to ETE for FFF flights from CLT to ATL

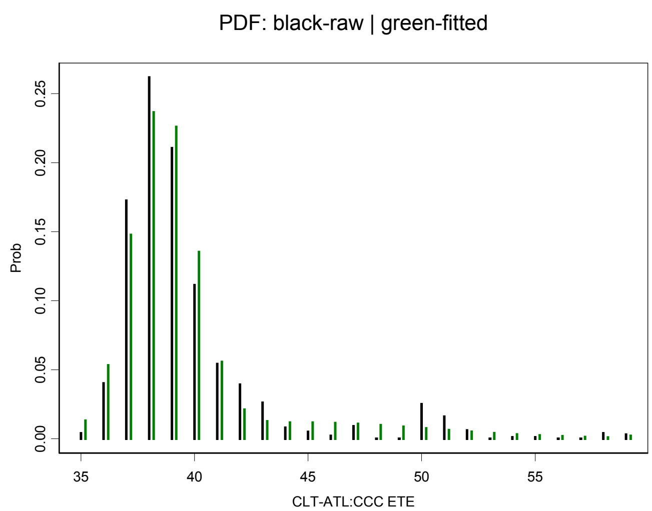




Estimated parameter values		
regular	p	0.884321
	sigma1	0.060155
	mu1	3.743005
	sd	2.546939
	avg	42.30115
irregular	sigma2	0.142474
	mu2	3.971521
	sd	7.676461
	avg	53.60651

Figure 13: Mixture model fit to ETEs for CCC flights from CLT to ATL





Estimated parameter values		
regular	p	0.849703
	sigma1	0.035661
	mu1	3.649076
	sd	1.372091
	avg	38.46358
irregular	sigma2	0.11366
	mu2	3.818118
	sd	5.224031
	avg	45.81343

Figure 14: ETE for flights from SFO to LAX

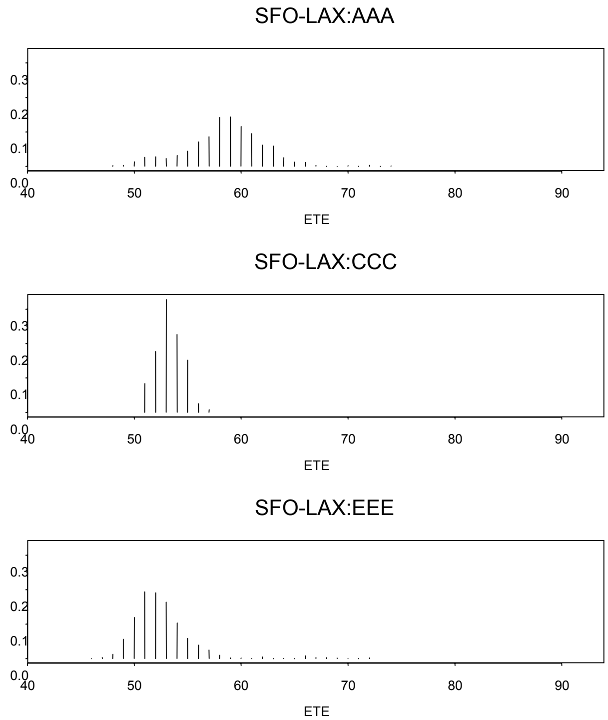
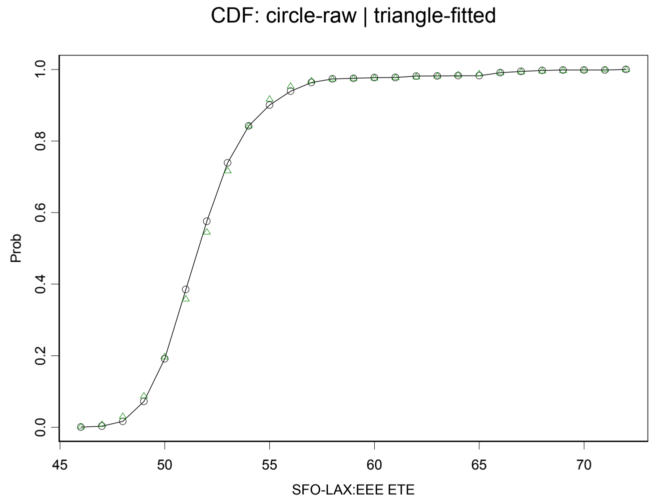
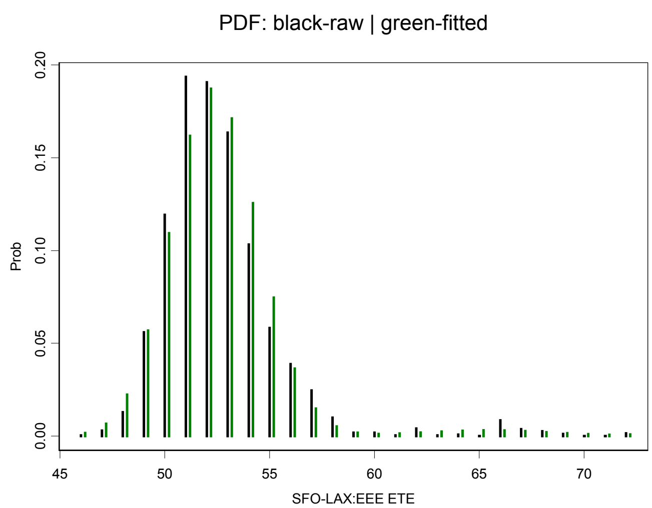


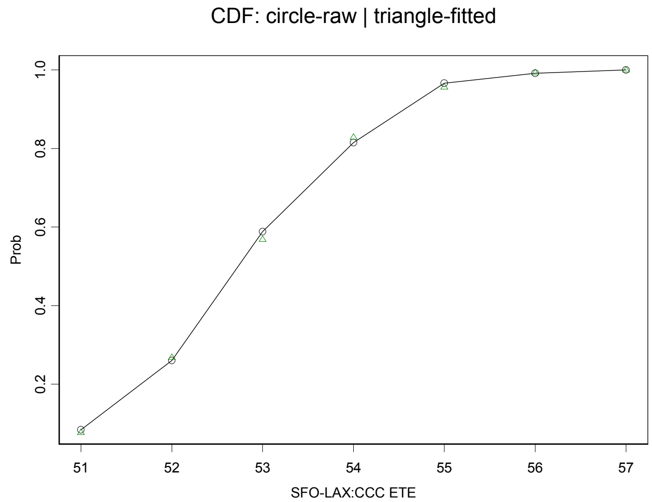
Figure 15: Mixture model fit to ETEs for EEE flights from SFO to LAX

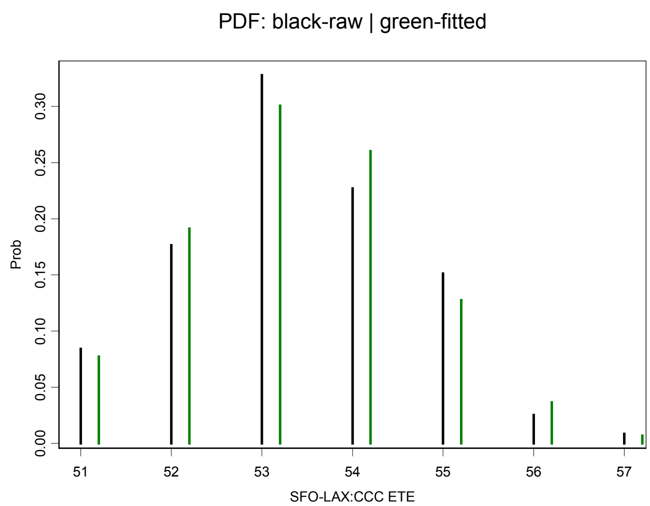




Estimated parameter values		
	p	0.975083
	sigma1	0.039406
	mu1	3.954885
	sd	2.058993
regular	avg	52.23023
	sigma2	0.049261
	mu2	4.179384
	sd	3.223841
irregular	avg	65.40491

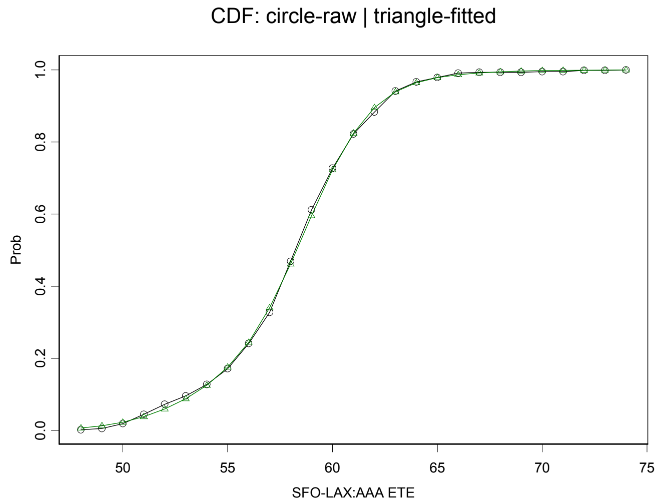
Figure 16: Mixture model fit to ETE for CCC flights from SFO to LAX

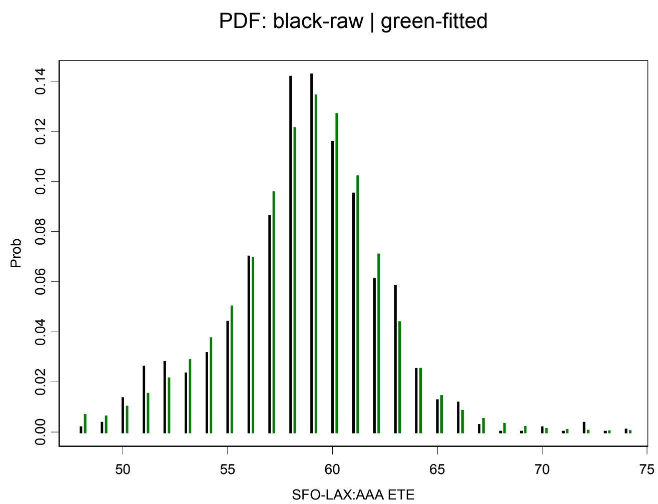




Estimated parameter values		
regular	p	1
	sigma1	0.023844
	mu1	3.97554
	sd	1.270924
	avg	53.29403
irregular	sigma2	-
	mu2	-
	sd	-
	avg	-

Figure 17: Mixture model fit to ETE for AAA flights from SFO to LAX





Estimated parameter values		
regular	p	0.519542
	sigma1	3.61E-02
	mu1	4.084797
	sd	2.145401
	avg	59.46853
irregular	sigma2	7.62E-02
	mu2	4.049128
	sd	4.391197
	avg	57.51433

Figure 18: ETE for flights from LAX to SFO

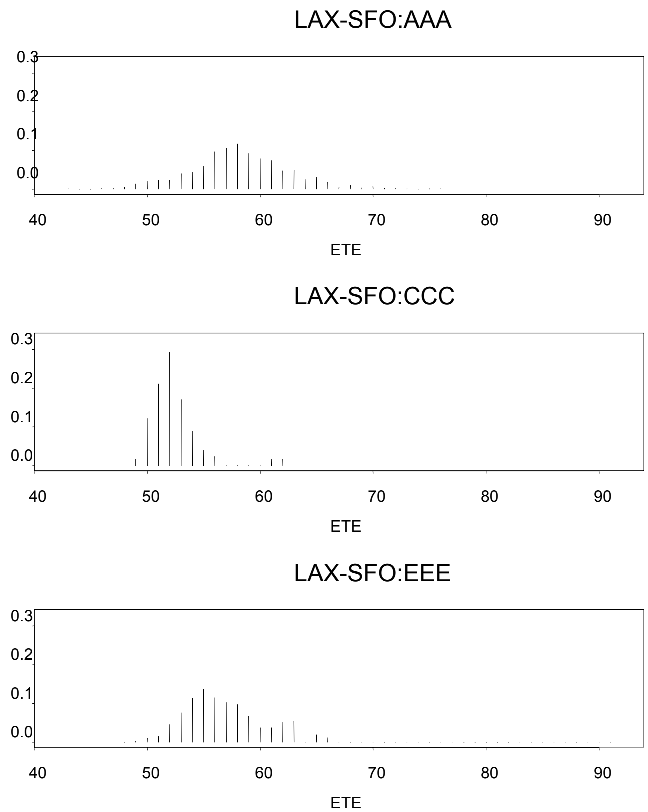
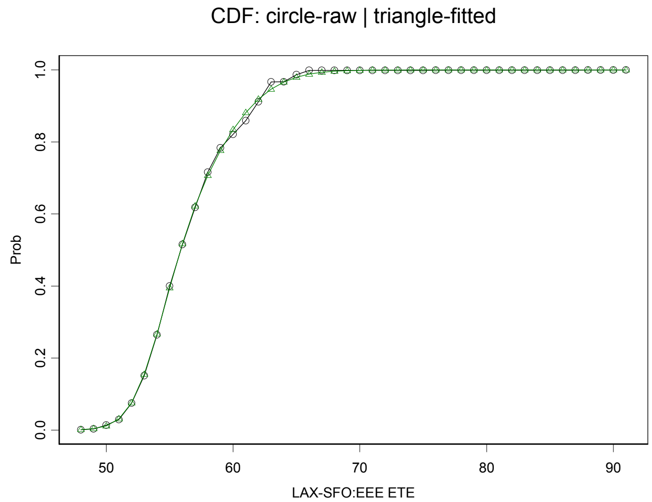
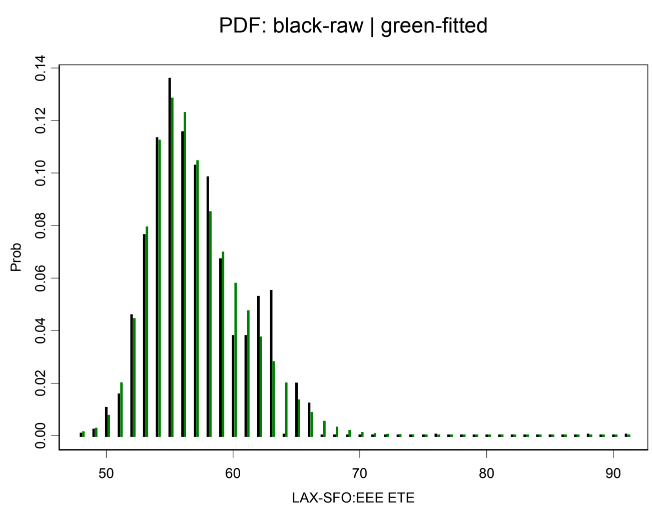


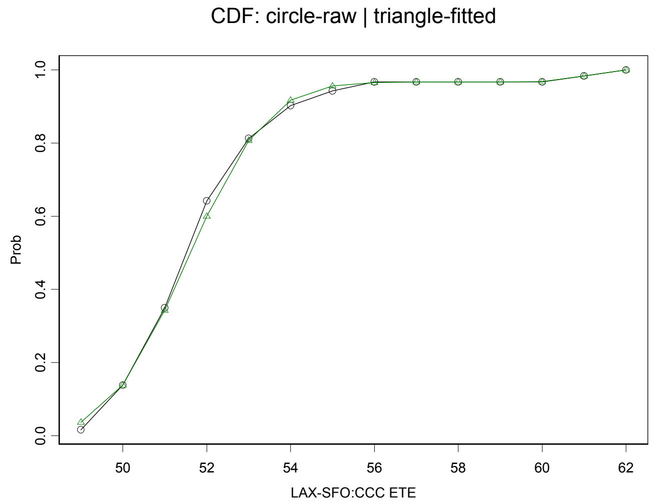
Figure 19: Mixture model fit to ETE for EEE flights from LAX to SFO

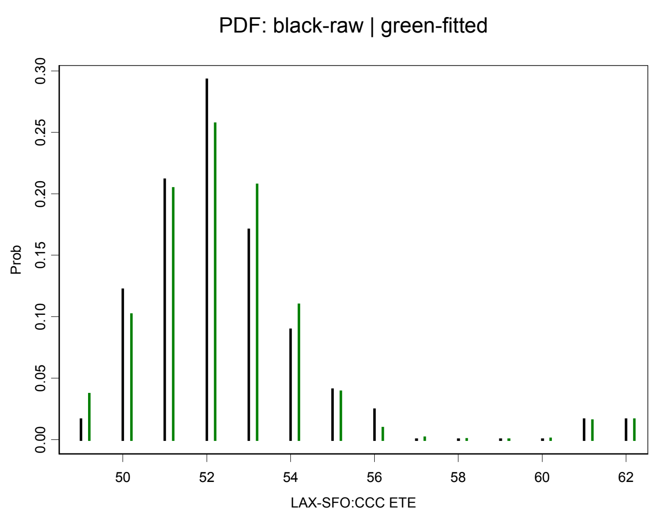




Estimated parameter values		
	p	0.42076
	sigma1	0.03522
	mu1	4.003817
	sd	1.932078
regular	avg	54.84095
	sigma2	0.063813
	mu2	4.066244
	sd	3.734061
irregular	avg	58.45633

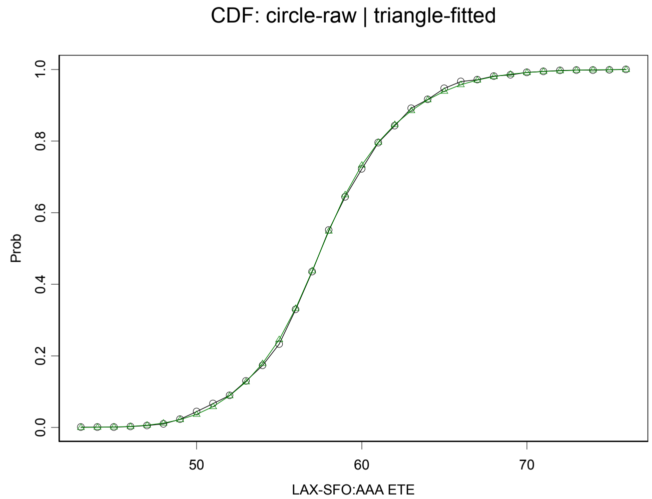
Figure 20: Mixture model fit to ETE for CCC flights from LAX to SFO

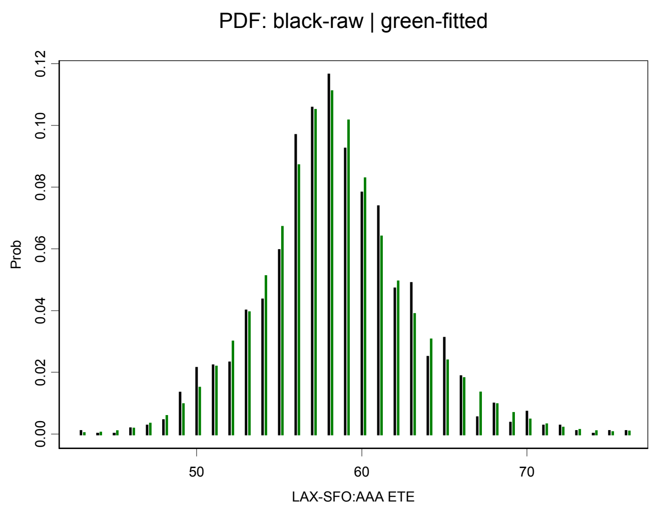




Estimated parameter values		
	p	0.96748
	sigma1	0.028305
	mu1	3.952133
	sd	1.474049
regular	avg	52.06712
	sigma2	0.00813
	mu2	4.119003
	sd	0.500017
irregular	avg	61.49993

Figure 21: Mixture model fit to ETE for AAA flights from LAX to SFO





Estimated parameter values		
	p	0.214213
	sigma1	0.031191
	mu1	4.059798
	sd	1.80923
regular	avg	57.9908
	sigma2	0.083817
	mu2	4.062672
	sd	4.897999
irregular	avg	58.33397

Figure 22: ETE for flights from EWR to LAX

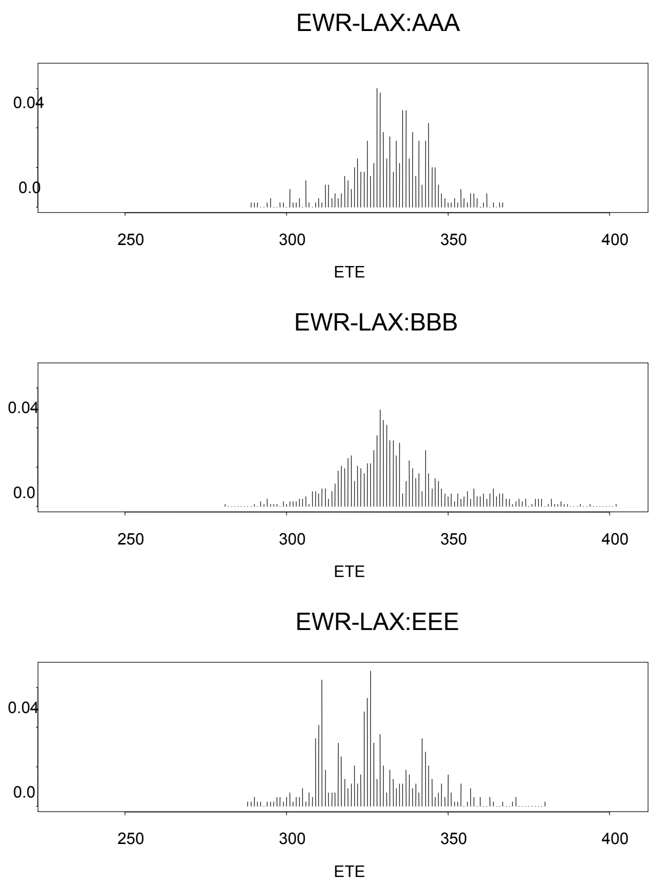
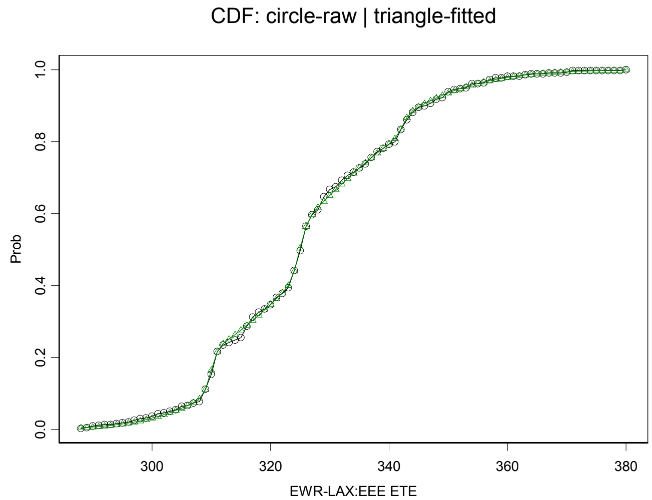
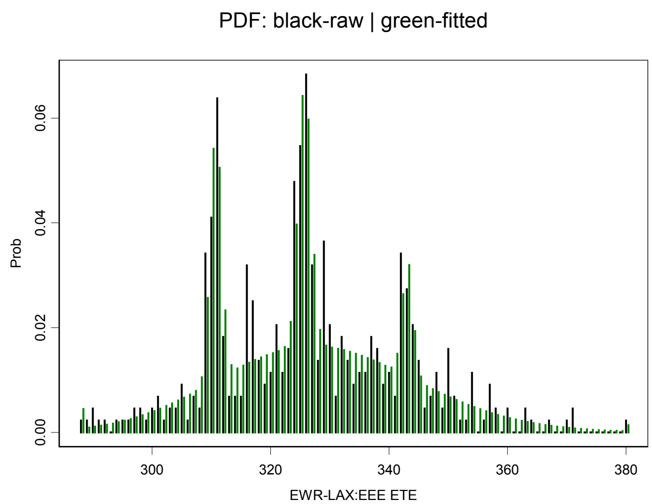


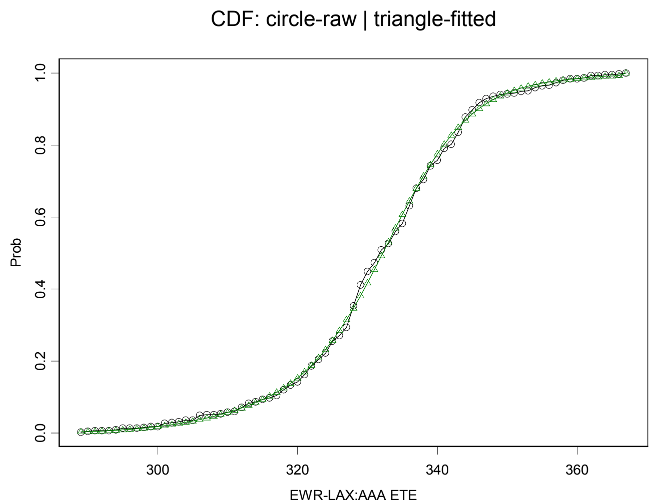
Figure 23: Mixture model fit to ETE for EEE flights from EWR to LAX

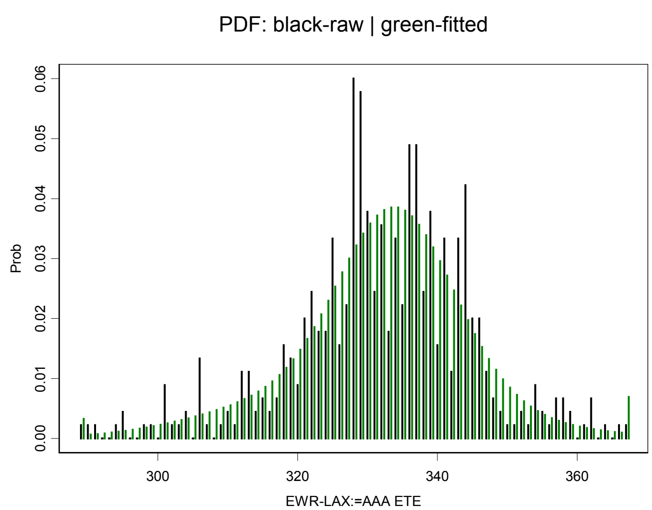




p1	0.140897		
mu1	5.784989	avg	325.3802
sigma1	0.003289	sd	1.070205
p2	0.05122		
mu2	5.837123	avg	342.7928
sigma2	0.002567	sd	0.880079
p3	0.117079		
mu3	5.737888	avg	310.4095
sigma3	0.00295	sd	0.915777
p4	0.690804		
mu4	5.791812	avg	328.0344
sigma4	0.051114	sd	16.77823

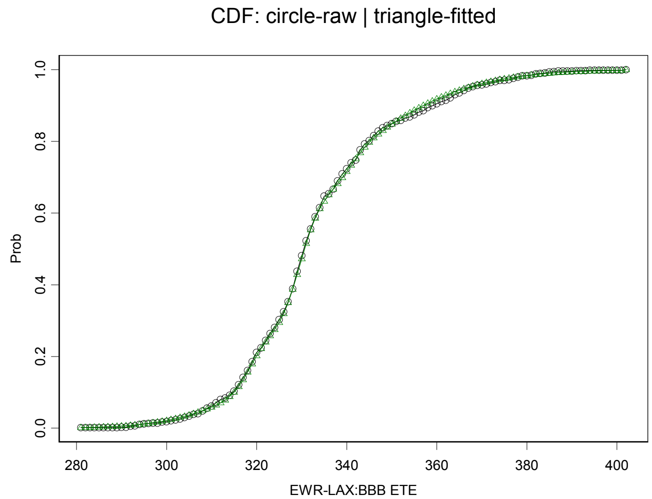
Figure 24: Mixture model fit to ETE for AAA flights from EWR to LAX

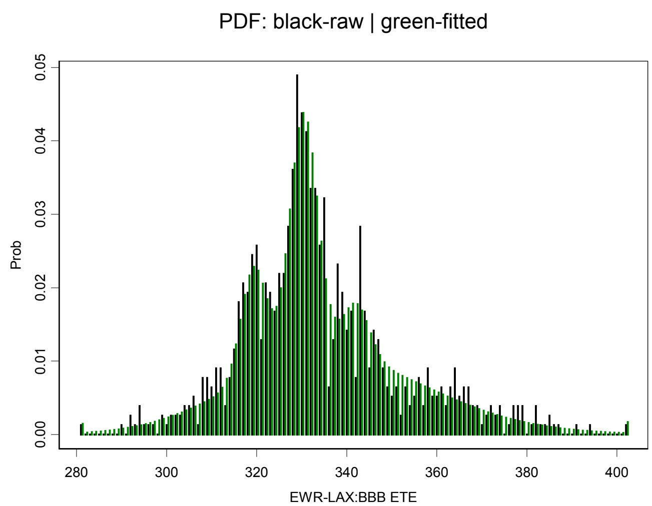




p1	0.612634		
mu1	5.811181	avg	334.1115
sigma1	0.02424	sd	8.099922
p2	0.387366		
mu2	5.793632	avg	328.6552
sigma2	0.052481	sd	17.26015

Figure 25: Mixture model fit to ETE for BBB flights from EWR to LAX





p1	0.108153		
mu1	5.7652	avg	319.0157
sigma1	0.008986	sd	2.866591
p2	0.058124		
mu2	5.833866	avg	341.6903
sigma2	0.008806	sd	3.008818
p3	0.262608		
mu3	5.79943	avg	330.126
sigma3	0.00943	sd	3.113105
p4	0.571115		
mu4	5.819215	avg	337.3975
sigma4	0.063981	sd	21.60927

Figure 26: ETE for flights from LAX to EWR

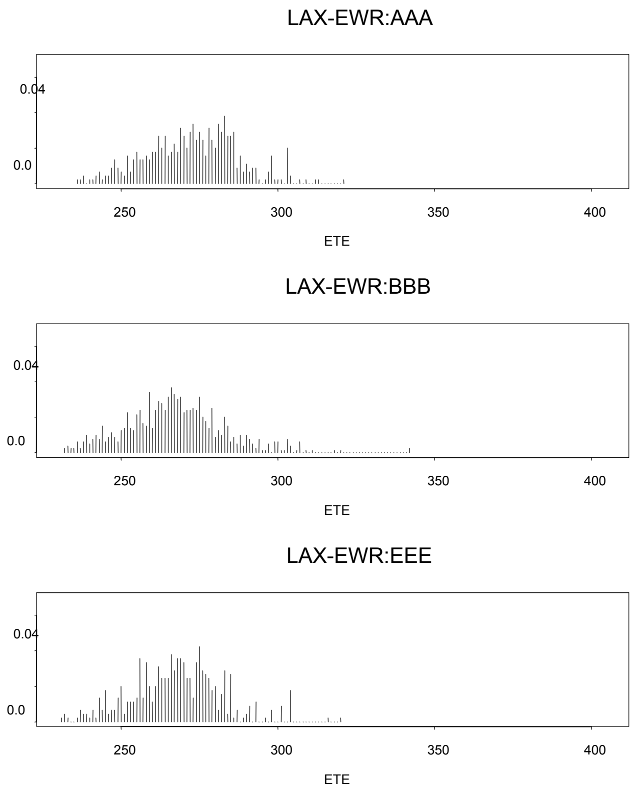
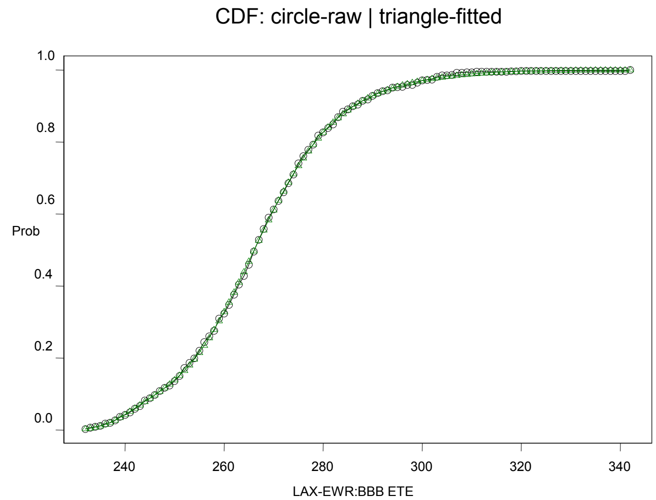
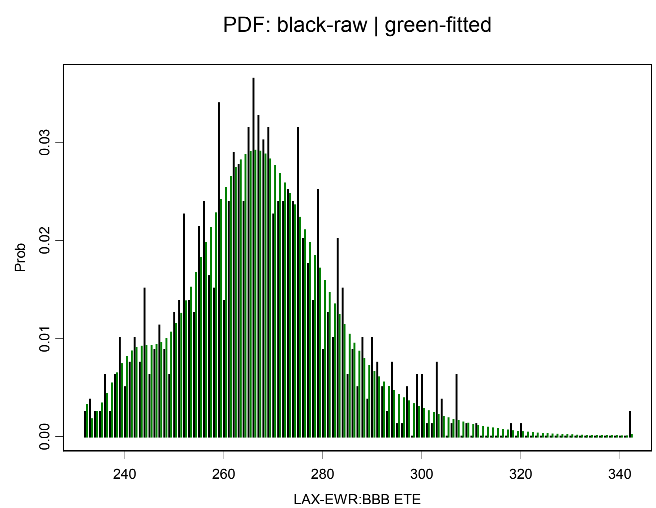


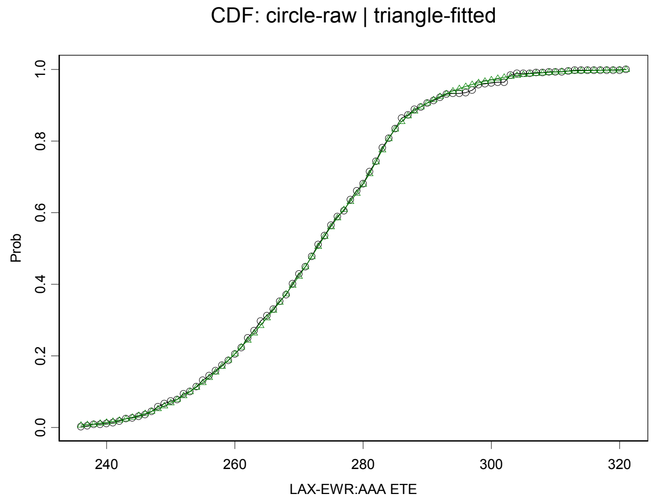
Figure 27: Mixture model fit to ETE for BBB flights from LAX to EWR

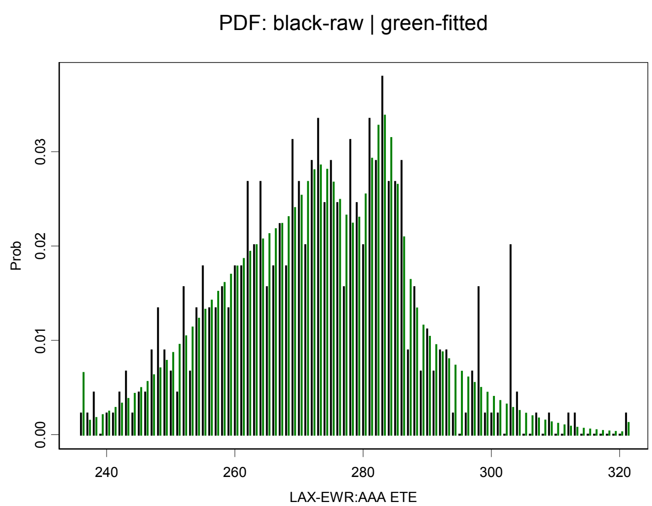




p1	0.076681		
mu1	5.484766	avg	241.0378
sigma1	0.019377	sd	4.671122
p2	0.205067		
mu2	5.636961	avg	281.1579
sigma2	0.062535	sd	17.59954
p3	0.718252		
mu3	5.582504	avg	265.9689
sigma3	0.041843	sd	11.13372

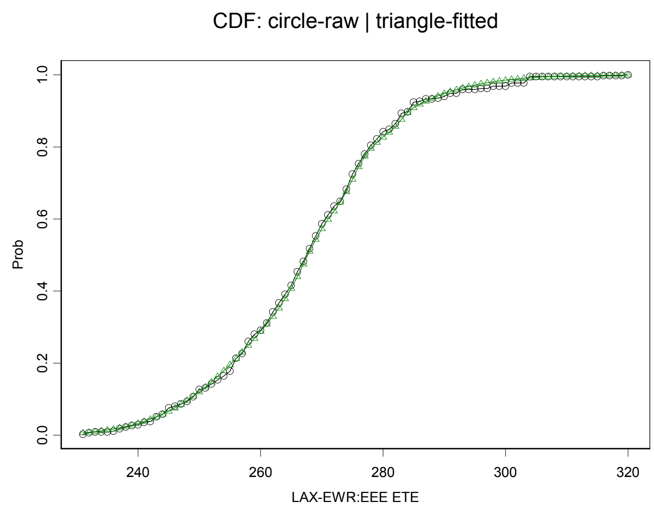
Figure 28: Mixture model fit to ETE for AAA flights from LAX to EWR

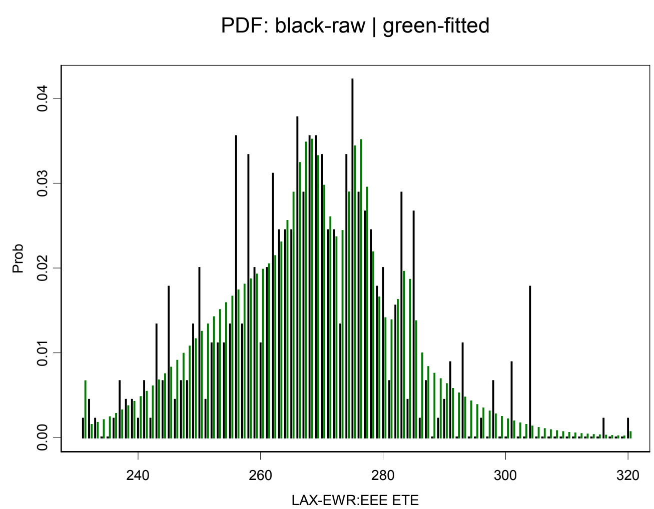




p1	0.041667		
mu1	5.610755	avg	273.3627
sigma1	0.009417	sd	2.574185
p2	0.098247		
mu2	5.645708	avg	283.0824
sigma2	0.007743	sd	2.192073
p3	0.860087		
mu3	5.602402	avg	271.5053
sigma3	0.05621	sd	15.27332

Figure 29: Mixture model fit to ETE for EEE flights from LAX to EWR

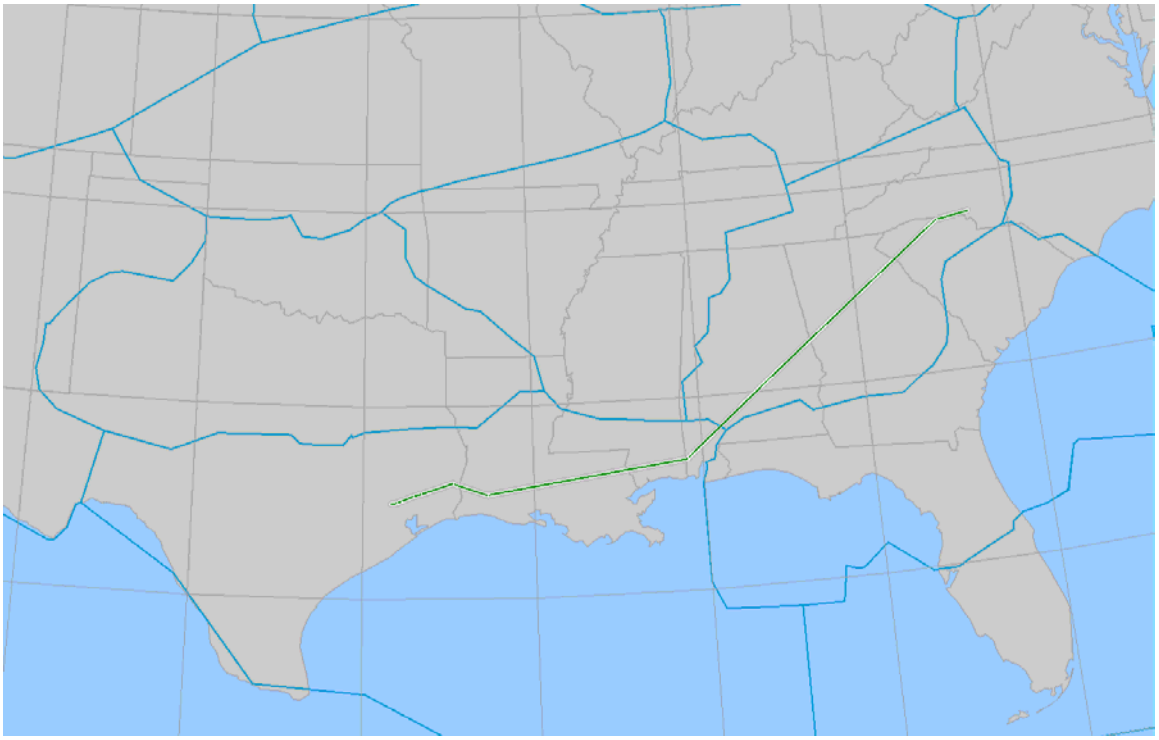




p1	0.075668		
mu1	5.619449	avg	275.7419
sigma1	0.005698	sd	1.571082
p2	0.027824		
mu2	5.647111	avg	283.4738
sigma2	0.004162	sd	1.179857
p3	0.092602		
mu3	5.590088	avg	267.7707
sigma3	0.009287	sd	2.486738
p4	0.803907		
mu4	5.583569	avg	266.4667
sigma4	0.057971	sd	15.46021

Figure 30: Filed flight paths from CLT to IAH by BBB and FFF

CLT-IAH BBB



CLT-IAH FFF

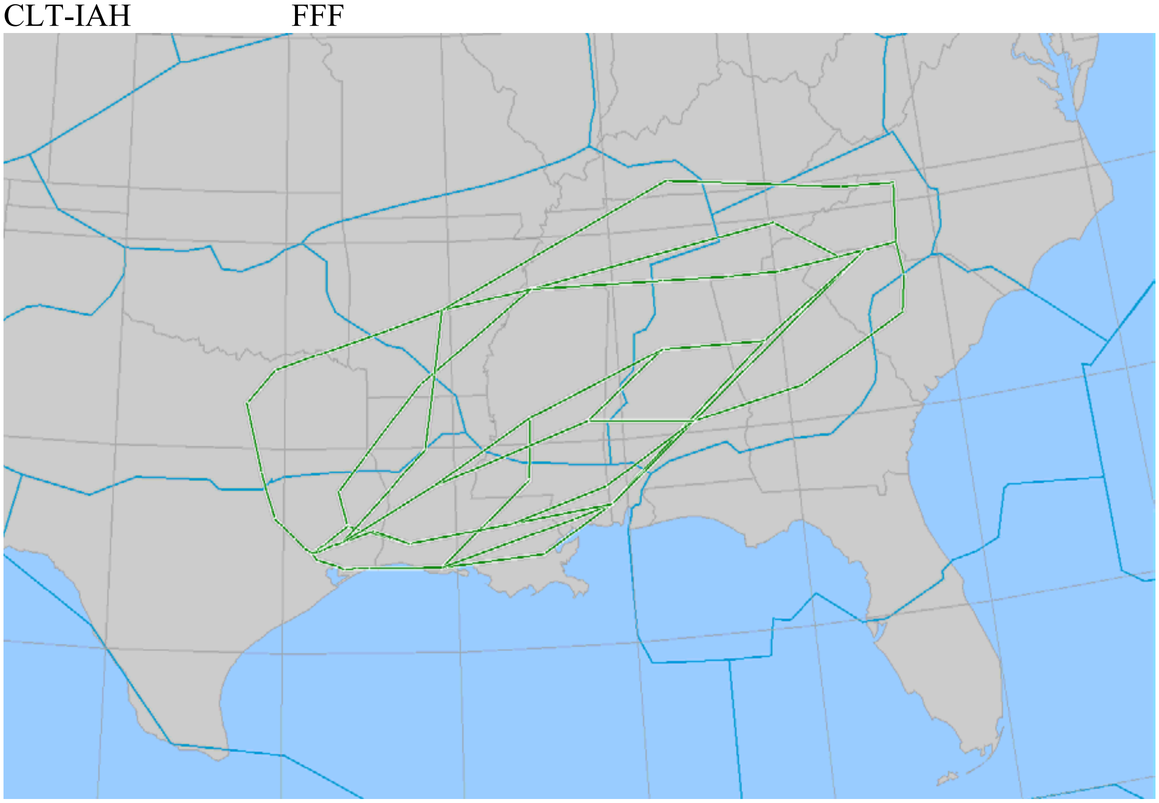
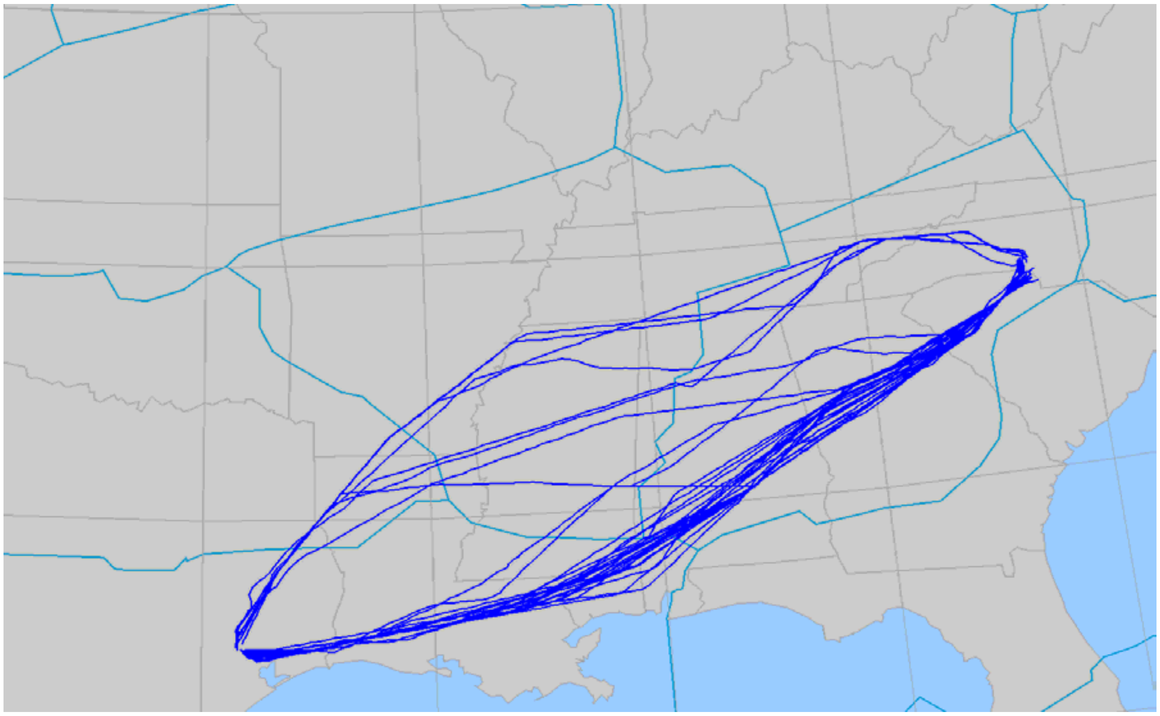


Figure 31: Flight paths actually flown from CLT to IAH by BBB and FFF

CLT-IAH BBB



CLT-IAH FFF