Infrastructure Deterioration: Stochastic Duration Models and the Incorporation of Explanatory Variables

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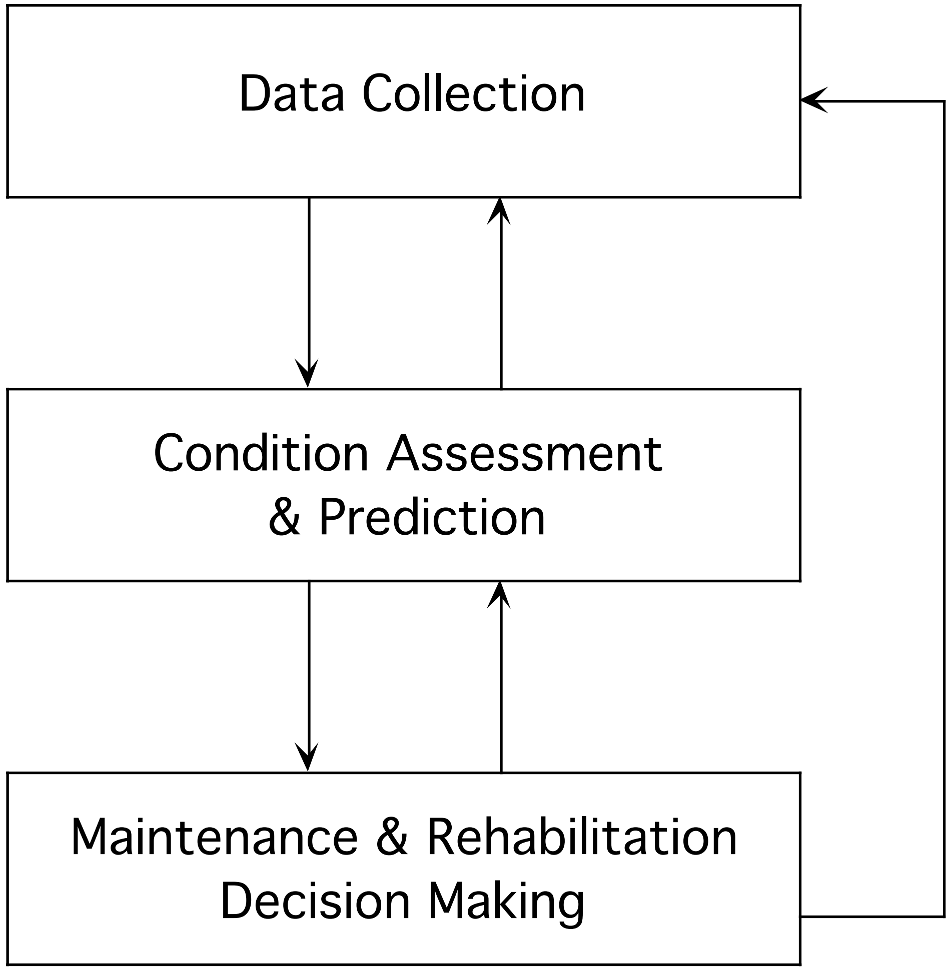
Infrastructure Management



Deterioration

- Types of failure
  - physical
  - functional
  - economic
- Physical condition is inherently continuous
- Deterioration is a stochastic process influenced by
  - design attributes
  - usage (traffic loading)
  - environment
  - age
  - maintenance history

Deterioration (cont'd)

- Measured or assessed condition can be
  - continuous
  - discrete
- Discrete condition ratings
  - reduce the condition state space
  - render decision-making more manageable

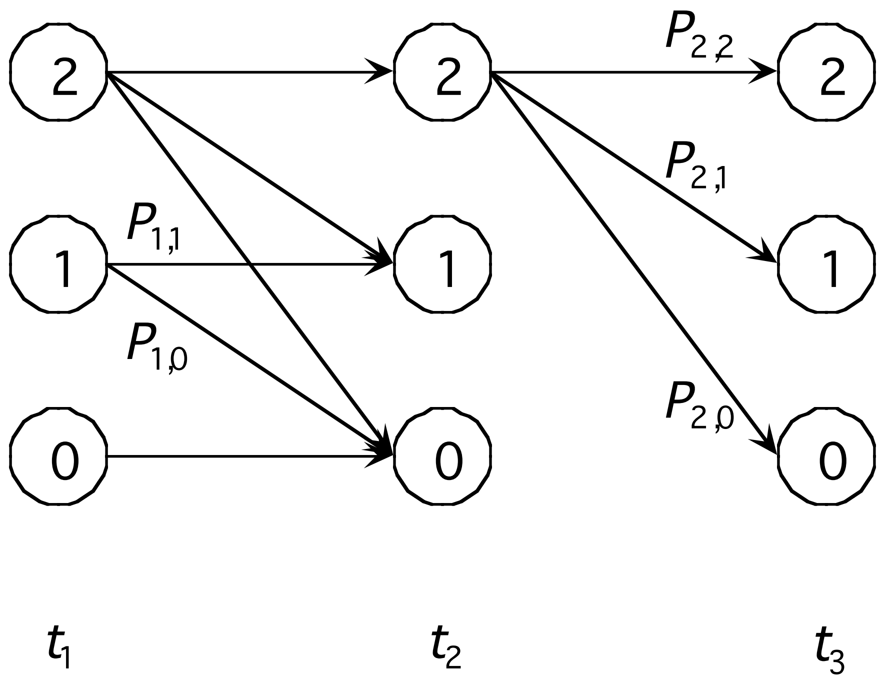
Types of Data

- Field data of in-service facilities
  - rich variety of conditions are captured
  - effect of maintenance actions which depend on condition have to be addressed
- Laboratory data (e.g., accelerated testing)
  - less varied set of conditions are possible
  - controlled experiments

Deterioration Modeling

- Probabilistic discrete state models
  - state-based (discrete-time)
  - time-based
- Discrete-time state-based models
  - Markov chain
  - transition probabilities from one condition state to other states

Discrete-Time State-Based Deterioration Model



Deterioration Modeling (cont'd)

- Time-based models
  - state duration
  - probability density function (pdf) of time spent in a condition state
- Can use one model to determine dependent variable of the other
- State-based models are predominantly used in decision-making

Transition Probability Estimation Literature

- Expected value method (Carnahan et al 1987, Jiang et al 1988)
  - minimizes distance between theoretical expected value of state and regression-based state prediction
  - **does not capture the effect of explanatory variables**
  - time segmentation to capture non-homogeneity is ad hoc
  - linear regression is inappropriate for discrete state data
- Ordered Probit method (Madanat, Mishalani, and Wan Ibrahim 1995)
  - estimate parameters of deterioration relationships
  - compute transition probabilities from such models
  - **independent and identically distributed observed condition states assumption does not hold**

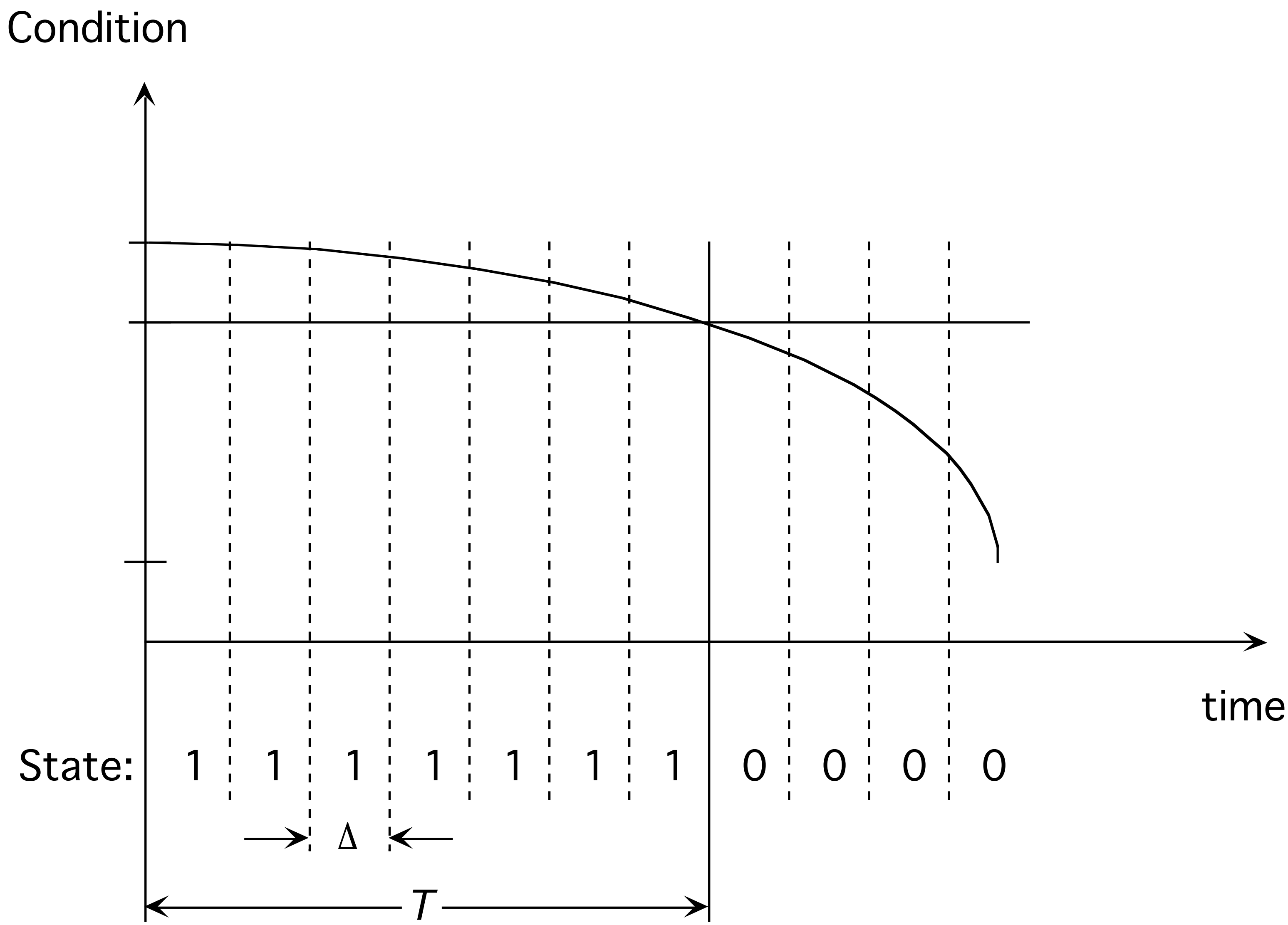
Transition Probability Estimation Literature (cont'd)

- Time–based model (DeStefano & Grivas 1998)
  - motivated by need for state transition probabilities
  - **does not capture the effect of explanatory variables**
- Consequences
  - biased state transition probabilities
  - poor condition predictions
  - suboptimal maintenance decision making

Methodology

- Objectives
  - develop time-based model that captures effect of explanatory variables
  - derive a method to compute discrete-time state-based transition probabilities
- Recognize deterioration is a continuous stochastic process
- Δ = inspection period
- Observed condition states: 1, 1, ..., 1, 0, 0, ..., 0 (bivariate case)
- Observations depend on threshold defining the states

Continuous Deterioration Process



Methodology (cont'd)

- t = time since most recent rehabilitation or construction
- T = duration of a given state (state 1)
- Approach
  - estimate pdf of T, $f(t)$, as a function of explanatory variables
  - compute transition probabilities based on $f(t)$ and Δ

Hazard Rate Function

- $R(t, \Delta)$ = transition probability out of state 1 at time t
- $R(t, \Delta) = \text{Prob}\left(t < T < t + \Delta \middle| T > t\right) = \dfrac{F(t + \Delta) - F(t)}{S(t)}$

  $F(t)$ = cumulative distribution function of T
  $S(t) = 1 - F(t)$, survival function

- Hazard rate function $\lambda(t) = \lim\limits_{\Delta \to 0} \dfrac{R(t, \Delta)}{\Delta} = \dfrac{f(t)}{S(t)}$

  (instantaneous rate of transition out of current state after time t)

- interpretation of $\lambda(t)$
  - constant $\Rightarrow$ duration independence
  - decreasing $\Rightarrow$ negative duration dependence
  - increasing $\Rightarrow$ positive duration dependence

Model Specification

- T follows the Weibull pdf

  $\lambda(t) = p\lambda^p t^{p-1}$ (λ, p = parameters)

- If $0 < p < 1$, $\lambda(t)$ is decreasing (negative duration dependence)
  If $p = 1$, $\lambda(t) = \lambda$ is constant (duration independence)
  If $p > 1$, $\lambda(t)$ is increasing (positive duration dependence)

- Capturing the effect of explanatory variables

  $\lambda = e^{-\beta X}$

  X = column vector of explanatory variables
  β = row vector of parameters

- Estimation of parameters β and p

  - Maximum Likelihood

  - effect of censoring in observing T is accounted for

Transition Probability Computation: Bivariate Case

- Probability of the transition from state 1 to state 0

$$P_{1,0} = R_1(t,\Delta) = 1 - \frac{\exp[-\lambda_1^{p_1}(t+\Delta)^{p_1}]}{\exp[-(\lambda_1 t)^{p_1}]}$$

- Probability of remaining in the same state 1

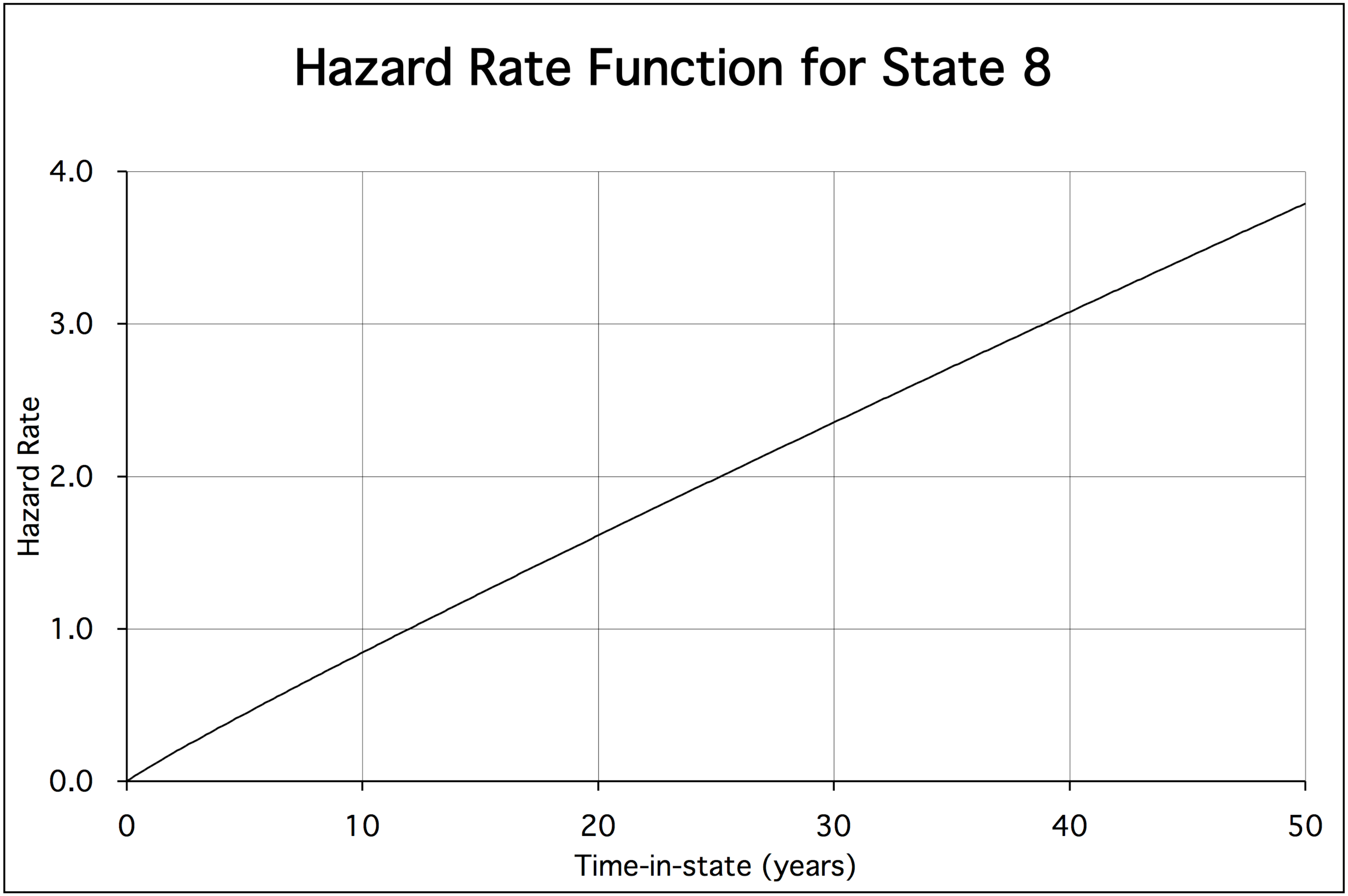
$$P_{1,1} = 1 - R_1(t,\Delta) = \frac{\exp[-\lambda_1^{p_1}(t+\Delta)^{p_1}]}{\exp[-(\lambda_1 t)^{p_1}]}$$

Empirical Analysis

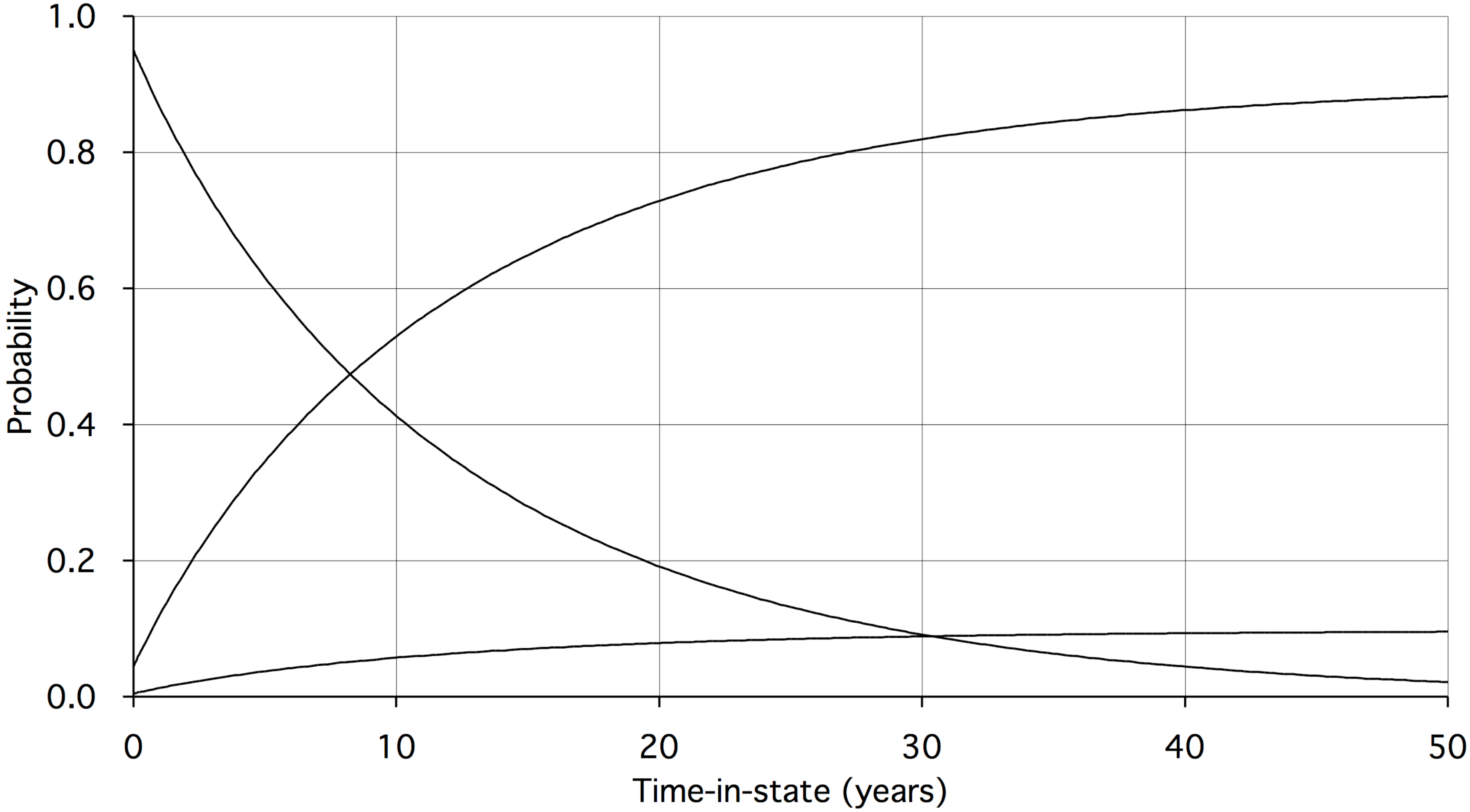
- Indiana Bridge Inventory (IBI)—part of National Bridge Inventory (NBI)
- Reinforced concrete bridge deck observations
- FHWA condition ratings: 9 (best) to 0 (worst)
- 1974–1984
- Number of observations for state 8: 368 (71% censored)
- Number of observations for state 7: 1,092 (80% censored)

Parameter Estimates for State 8

Variable	Estimated parameter	t-statistic
Constant	2.13	9.44
Age	0.15	6.31
Region	– 0.84	– 7.06
Type2	0.38	3.09
HWClass1	– 0.64	– 2.89
HWClass3	– 0.55	– 3.62
HWClass5	– 0.57	– 3.36
WearSurf1	– 0.80	– 4.73
$1/p$	0.52	12.01

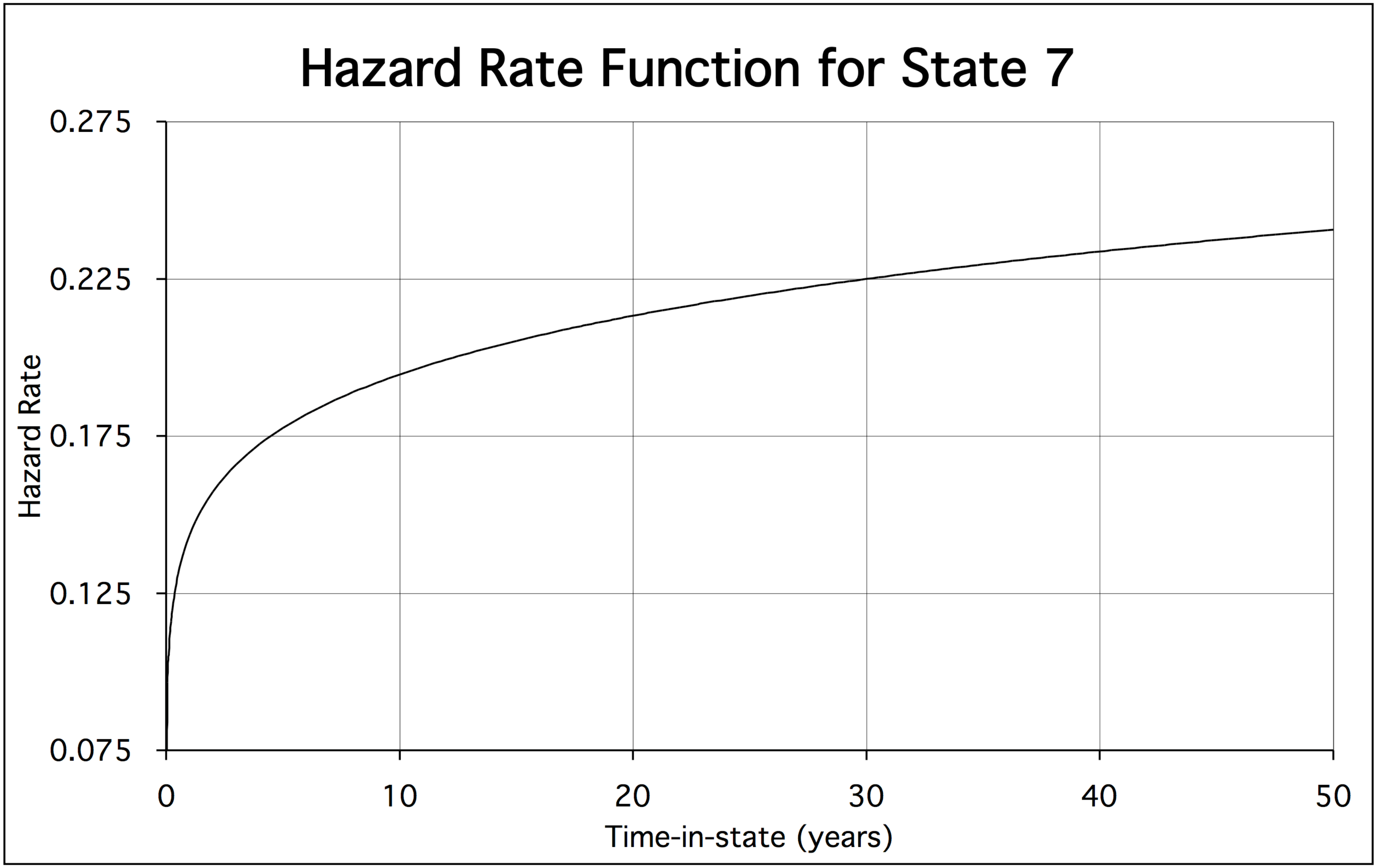

Hazard Rate Function for State 8
Hazard Rate
4.0
3.0
2.0
1.0
0.0
0
10
20
30
40
50
Time-in-state (years)

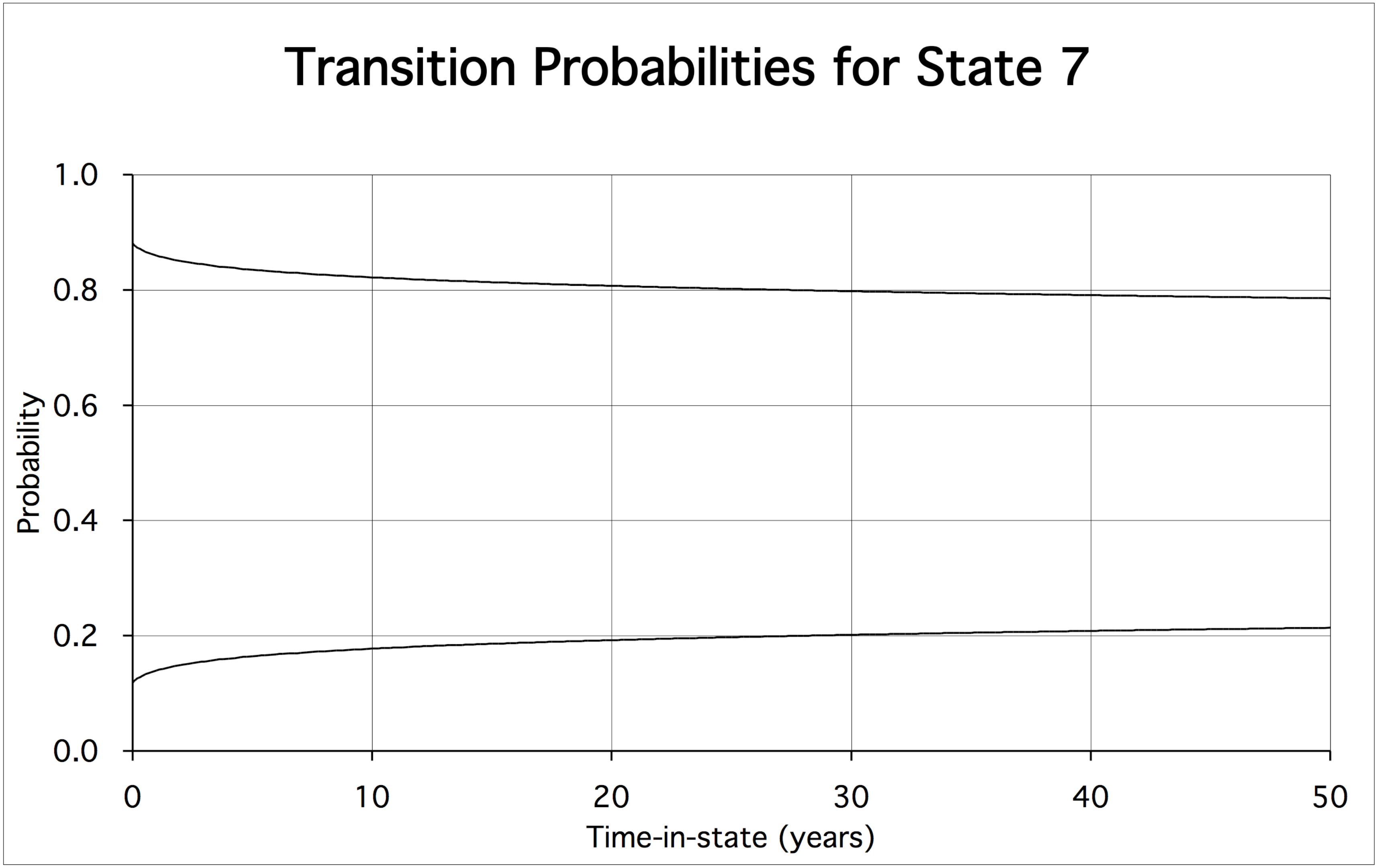
Transition Probabilities for State 8



Parameter Estimates for State 7

Variable	Estimated parameter	t-statistic
Constant	5.07	11.30
AvgADT	-2.64×10^{-5}	-2.70
Region	-0.83	-5.57
HWClass1	-1.02	-3.87
HWClass3	-1.30	-5.15
HWClass5	-1.14	-4.57
WearSurf1	-0.99	-3.74
WearSurf2	-1.15	-4.23
$1/p$	0.88	11.26


Hazard Rate Function for State 7
Hazard Rate
0.275
0.225
0.175
0.125
0.075
0
10
20
30
40
50
Time-in-state (years)


Transition Probabilities for State 7
1.0
0.8
0.6
0.4
0.2
0.0
Probability
0
10
20
30
40
50
Time-in-state (years)

Summary

- Critiqued state-based model transition probability estimation methods
- Developed time-based stochastic duration model that takes into account the effects of causal (explanatory) variables
- Derived a methodology for computing discrete-time state-based transition probabilities from time-based duration models
- Demonstrated the methodology using bridge deck condition data
- Statistically tested for state-dependence and age-heterogeneity using the time-based models

Issues for Future Research

- Definition of discrete condition states
- Network of interdependent components
- Functional form of hazard rate function (monotonic vs. non–monotonic)
- Identification of pertinent explanatory variables

Transition Probability Computation: Multivariate Case

- Probability of remaining in the same state 2

$$P_{2,2} = 1 - R_2(t,\Delta) = \frac{\exp[-\lambda_2^{p_2}(t+\Delta)^{p_2}]}{\exp[-(\lambda_2 t)^{p_2}]}$$

- Probability of the transition from state 2 to state 1

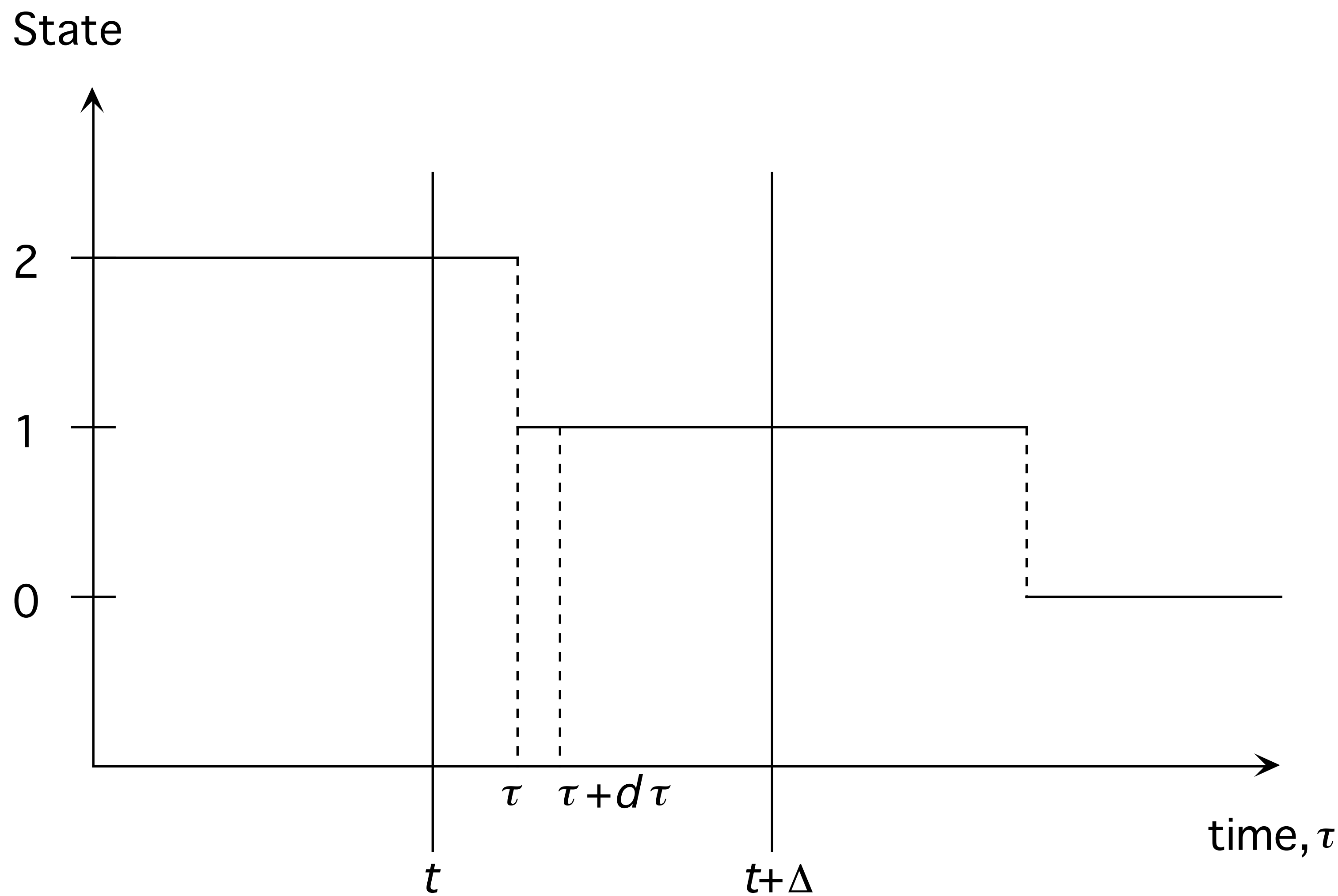
$$P_{2,1} = \mathrm{Prob}\left(\text{state} = 1 \text{ at time } \tau = t + \Delta \middle| T_2 > t\right)$$

$$= \int_t^{t+\Delta} \mathrm{Prob}\left(\tau < T_2 < \tau + d\tau \middle| T_2 > t\right) \cdot \mathrm{Prob}\left(T_1 > t + \Delta - \tau\right)$$

$$= \int_t^{t+\Delta} \frac{\lambda_2(\tau) \cdot S_2(\tau) \cdot S_1(t+\Delta-\tau)}{S_2(t)} d\tau$$

$$= p_2 \lambda_2^{p_2} \exp[(\lambda_2 t)^{p_2}] \int_t^{t+\Delta} \tau^{p_2 - 1} \exp[-(\lambda_2 t)^{p_2} - \lambda_1^{p_1}(t+\Delta-\tau)^{p_1}] d\tau$$

Transition from State 2 to State 1



Transition Probability Computation: Multivariate Case (cont'd)

- Probability of the transition from state 2 to state 0

$$P_{2,0} = 1 - P_{2,2} - P_{2,1} = R_2(t, \Delta) - P_{2,1}$$