# Optimization Models for Off-Loading Flow Constrained Areas

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# Introduction

## Flow Constrained Area (FCA)

- 3-dimensional volume of airspace
- specified along with flight filters and a time interval (start time $T_S$, end time $T_E$)
- The time interval is used to identify flights subject to capacity constraints.

## Offloading problem:

Partition the involved flights into two groups:

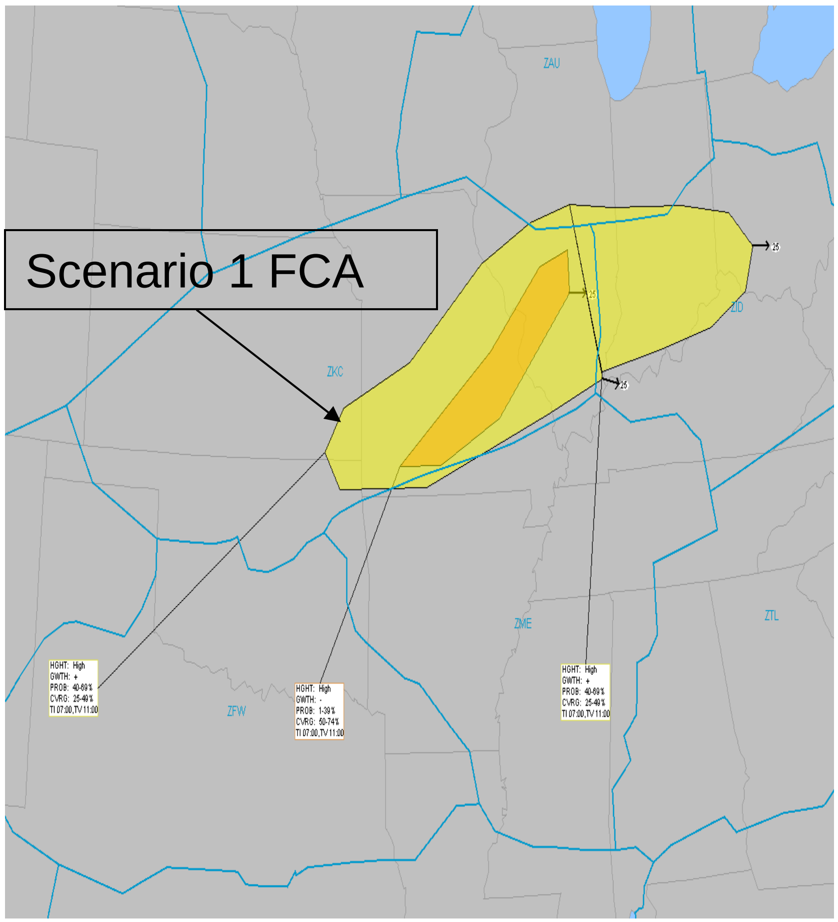
1) stick to original routes
2) reroute outside this area.

## Goals:

System-wise efficiency

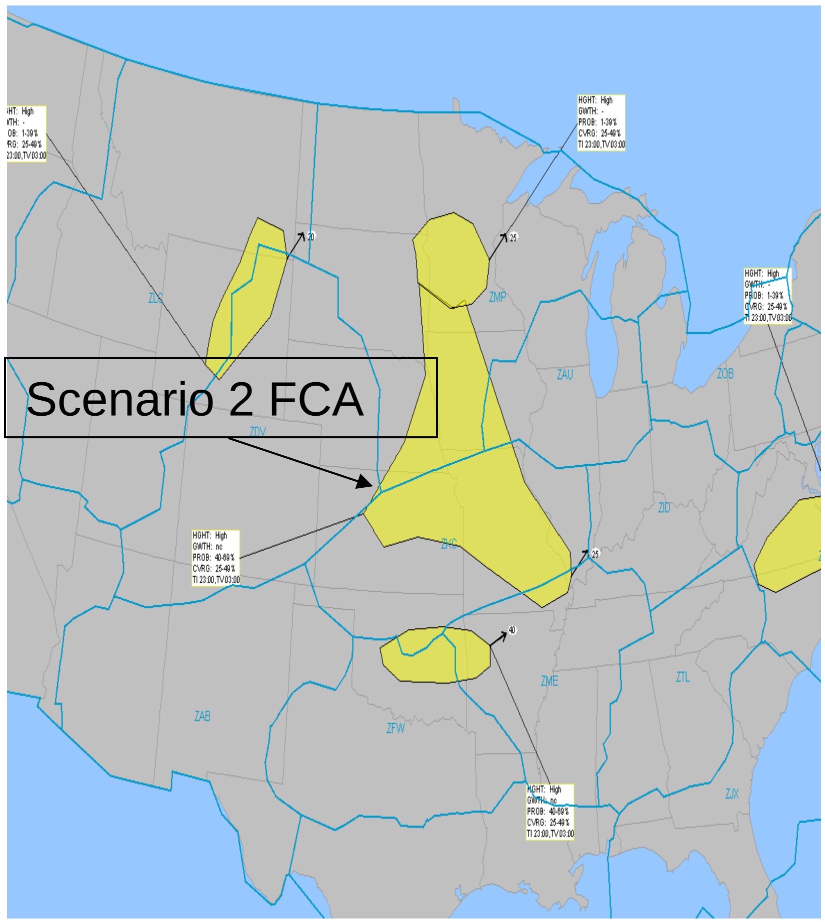
Equity among airline companies

# FCA scenarios



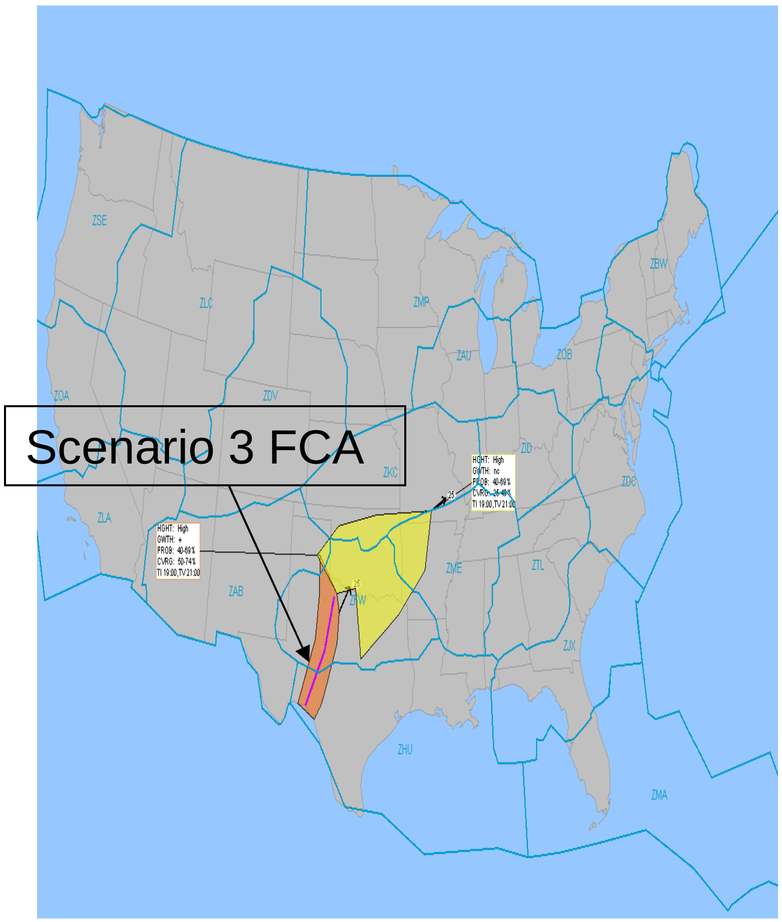

- FCA based on CCFP data
- Only the large yellow area is defined as the FCA
  - –Date: 05/07/2002, Tuesday
  - –Issue time: 0700, Valid time: 1100
  - –Direction: west to east @ 20-25 kph
  - –Occurrence: 40-69%
  - –Coverage: 25-49% (yellow)
  - –Height: High
  - –Growth: Positive

# Scenario 2




- FCA based on CCFP data
- Only the large yellow area is defined as the FCA
  - Date: 05/07/2002, Tuesday
  - Issue time: 2300, Valid time: 0300
  - Direction: west to east @ 20-25 kph
  - Occurrence: 40-69%
  - Coverage: 25-49%
  - Height: High
  - Growth: No change

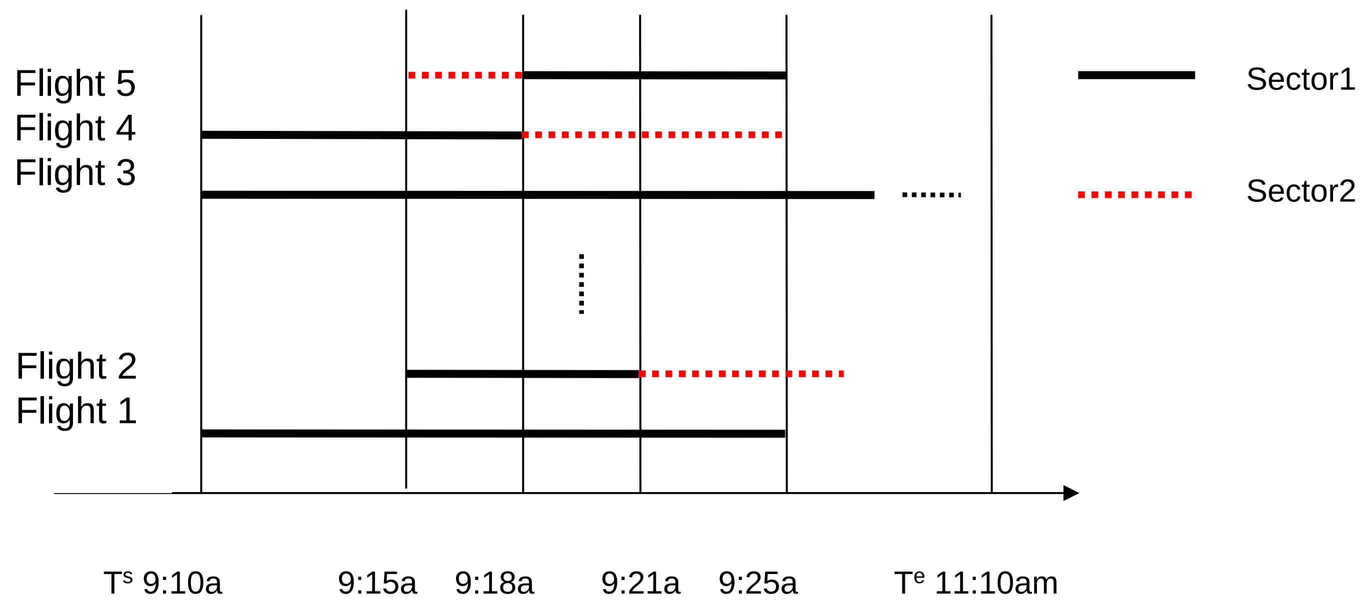
# Scenario 3




- FCA based on CCFP data
- Only the orange area is defined as the FCA
  - Date: 04/07/2002, Sunday
  - Issue time: 1900, Valid time: 2100
  - Direction: west to east @ 25 kph
  - Occurrence: 40-69%
  - Coverage: 50-74%
  - Height: High
  - Growth: Positive

# Example

- Partition the whole FCA time period into small time intervals according to the flight's entering/leaving time for each sector.



# Mathematical Formulation

**Decision variables:**

$x_f$ = 1, if this flight *f* stays in this area using original schedule,
0 otherwise.

$$r_i = \frac{\sum_{f \in C_i} x_f p_f}{\sum_{f \in C_i} p_f}$$

**Objectives:** **(1)** System-wide efficiency:

$$\text{Max} \sum_{f=1}^{N} x_f \qquad \text{Max} \sum_{f=1}^{N} p_f x_f \qquad \text{Min} \sum_{i \in I} \sum_{f \in C_i} (1 - X_f) D_f$$

**(2)** Equity issue:

$$\text{Max Min } r_i \qquad \text{Min} \sum_{i=1}^{M} \left| r_i - \overline{r_i} \right|$$

# Mathematical Formulation (cont'd)

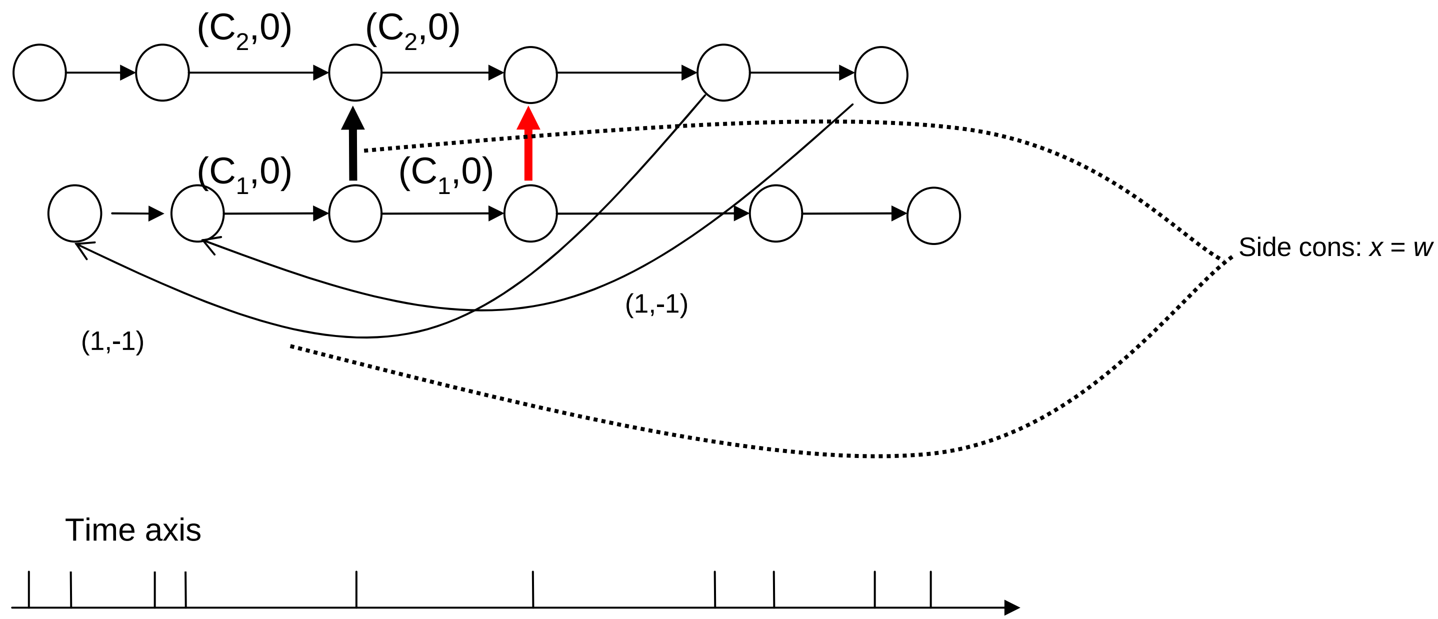
- **Constraints:**
  - Partition the total reduced capacity period into time intervals constraint:

$$\sum_{f \in U_{j,k}} x_f \leq c_k \qquad \forall j, \forall k$$

- **Model structure:**
  - Single sector: each column in the coefficient matrix have consecutive "1"s; the problem is totally unimodular (TU); it can be modeled as a min cost network flow model.
  - Multiple sector: "0"holes among consecutive "1"s; the problem is not TU; it can be modeled as a  min cost network flow model with side constraints.

# Network Flow Representation in Multi-Sector Case

- Network flow representation



# Conclusion and future work

- The optimization problem has a network flow model structure.
- Construct a test bed to investigate possible solutions.
- The optimization model can be extended to allow flexible time intervals.
- Incorporate route choice options for the involved flights.

# Decentralized model

- **Goal:**
  - Encourage airlines' interactions in decision making process
  - Give partial decision right to airline companies

- **Decentralization procedure:**
  - Stage 1: assign a set of capacity constraints to airline companies;
  - Stage 2: airline companies can solve their own optimization models;
  - Stage 3: collect the local solutions from airline companies and globally solve a smaller optimization model by using the residual capacity.

- Thank you!

# Questions

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